सत्यमेव जयते

DELHI COLLEGE OF ENGINEERING

LIBRARY

CLASS NO. 620.1

BOOK NO. H.E.

ACCESSION NO. [illegible]

DATE DUE

For each day's delay after the due date a fine of 3 P. per Vol. shall be changed for the first week, and 25 P. per Vol. per day for subsequent days.

Borrower's No.	Date Due	Borrower's No.	Date Due

PROCESS EQUIPMENT DESIGN

By

HERMAN C. HESSE

Dean of Engineering, Valparaiso University

AND

J. HENRY RUSHTON

Head, Department of Chemical Engineering,
Illinois Institute of Technology

(AN EAST-WEST EDITION)

D. VAN NOSTRAND COMPANY, INC.

TORONTO — PRINCETON, NEW JERSEY / NEW YORK — LONDON

AFFILIATED EAST-WEST PRESS PVT. LTD.
NEW DELHI

D. VAN NOSTRAND COMPANY, INC.
120 Alexander St., Princeton, New Jersey (*Principal office*)
24 West 40 Street, New York 18, New York

D. VAN NOSTRAND COMPANY, LTD.
358, Kensington High Street, London, W.14, England

D. VAN NOSTRAND COMPANY (Canada), LTD.
25 Hollinger Road, Toronto 16, Canada

COPYRIGHT © 1945
BY
D. VAN NOSTRAND COMPANY, INC.

No reproduction in any form of this book, in whole or in part (except for brief quotation in critical articles or reviews), may be made without written authorization from the publishers.

First Published January 1945
Reprinted June 1945; July 1946; February 1947; December 1949; August 1953; January 1957; September 1959; May 1962

East-West Student Edition - 1964
Affiliated East - West Press P. Ltd.
Second East - West Reprint-1967

Price in India Rs. 15.00
Rest of Asia $ 2.80
Sales territory: Asia

Reprinted in India with the special permission of the original Publishers, The D. Van Nostrand Company Inc., Princeton, New Jersey. U S.A. and the copyright holders

This book has been published with the assistance of the Joint Indian-American Standard Works Programme.

Published by W.D. Ten Broeck for AFFILIATED EAST-WEST PRESS PRIVATE LTD., C 57 Defence Colony New Delhi 3, India, and printed by S. M. Balsaver at USHA PRINTERS, National House, Tulloch Road, Bombay 1, India.

PREFACE

This book presents the fundamentals of mechanics, machine and structural elements, and economic and manufacturing considerations related to the design of process equipment, particularly for the chemical industries. Although few engineers are ever called upon for complete designs of pieces of equipment, they must be familiar with all of the foregoing factors.

The book is based upon five years of lecture and laboratory work with senior engineering students and is adaptable to advanced classes familiar with industrial process design, with physical metallurgy, and with the unit operations of chemical engineering. Graduate engineers, who may not have studied the elements of design as related especially to the process industries, will find the book to be an effective aid in their work. For college curricula, in which courses in mechanics and strength of materials are taught prior to the design course, Chapters 1, 2, and 5 may be omitted or used for review.

The treatment resolves equipment into structural elements in the order of their increasing analytical complexity. Rigorous methods of stress analysis, followed by empirical considerations and modifications, are logically applied to the design of process industry equipment and auxiliary structures and devices. Emphasis is placed upon formal methods of stress analysis to insure proper perspective in relation to governing codes. An abstract or a summation of a code follows the presentation of the theory underlying its application to develop a knowledge of the possibilities and limitations of the code and the manner in which a more appropriate or a less restricting code may be used.

Frequent reference is made to catalogs and trade literature, as well as to handbooks and other sources of design data. Ease and economy of fabrication and protection against chemical or corrosive action have been emphasized as of major importance in design and the selection of materials.

We are indebted to former students and to colleagues both in the educational and industrial fields for their suggestions and criticisms. We wish especially to acknowledge the help of F. W. Wheeler, of the University of Virginia, and W. E. Lobo, of the M. W. Kellogg Company, and to thank Mrs. Allan L. Campbell and Miss Virginia B. McLellon for their work with the manuscript.

Charlottesville, Va.
October, 1944

H.C.H.
J.H.R.

CONTENTS

CHAPTER I

MATERIALS OF CONSTRUCTION

CHOICE OF MATERIALS

1-1. In designing equipment for the handling and manufacturing of chemicals it is necessary to consider many factors whose roots lie in the theories of chemistry, physics, and economics. The choice of materials used to construct a piece of equipment are primarily dictated by considerations such as resistance to chemical reaction, strength to resist loads and stresses, and relative costs of two or more materials equally acceptable. The chemistry involved in a process leads to evaluation of the chemical resistance of the containing equipment, and the temperature and pressure required or resulting from the reaction define the limits wherein relative inertness of the material of construction must be achieved. It is necessary that equipment provide the proper environment—space, time, temperature, and pressure—to allow reactions to take place of their own accord. It must also provide the mechanism whereby energy may be supplied or removed as required to maintain equilibrium. With a knowledge of chemistry, physical chemistry, thermodynamics, and the unit operations of chemical engineering it is possible to evaluate quantitatively the best environment and the necessary energy requirements for a reaction. To establish limits of space for a reaction, the material used to confine the chemicals involved must have sufficient strength and must remain inert, or nearly so, at the extremes of pressures and temperatures that may be encountered. A temperature limitation, for example, would be dictated by the highest temperature used in the equipment, and could be either the maximum temperature of the reaction or the temperature required to produce the necessary temperature gradient to insure adequate heat flow. The best theoretical adaptation of a material of construction is appreciated only after consideration of the chemistry and energy relations involved.

1-2. Chemical Factors. Considering first the chemistry involved, account must be taken of the effect of the confined reactants on the confining metal and also the effect of the confining metal on the reactants. While it is essential that reactants do not corrode or otherwise injure the equipment, it is likewise often essential that no impurities be taken up by the reactants from the equipment. This latter is especially true when minute quantities of metals taken from the equipment might cause undesirable catalytic effects. Of the many materials of construction, metals are the most common. Wood, stoneware, plastics and other non-metallic substances are often found in chemical equipment. While the principal part of the equipment may be made of one type of material, it is

common to find some two or more metals present in a piece of metal equipment or to find a metal part in some non-metal equipment. Thus the action of several metals must usually be considered in connection with the resistance to corrosion and the development of impurities in the reactants.

In most cases, the action of reactants on a material of construction is thought of as corrosion. It is often extremely difficult to evaluate corrosion even when all the impurities in the reactants and the metals are known. Laboratory tests on the corrosion of metals in simple solution are usually not only inadequate but often directly misleading, mainly because it is exceedingly difficult to duplicate exact conditions which will be encountered in the equipment during operation. The direct solubility of metals in various fluids is not always a good index of corrosion or inertness, for there is always the question of the presence of dissimilar metals all of which cause corrosive action due to the formation of electrical couples. On the other hand it is possible to safeguard equipment by the judicious use of dissimilar metals which will develop a couple in such a way that the polarity of the principal metal will act to repress its solubility and thus eliminate corrosion. An instance of this phenomenon may be cited in the case of aluminum equipment, which may be protected anodically by the use of other metals in contact. Other methods to avoid galvanic action are the use of gaskets or other structural forms that will electrically insulate two dissimilar metals. Paint is often desirable, not only to form a protective coat, but also to reduce the effective cathodic areas. Nickel, Monel, copper, brass, bronze and similar alloys may often present serious corrosion problems due to galvanic action when used with iron, low-alloy steels, aluminum, and zinc. But nickel and its alloys usually may be used advantageously with copper and its alloys, or either with high-alloy steels, without causing galvanic corrosion. And iron and low-alloy steels may be used in combination with nickel and high nickel alloys when the area of the ferrous metal is large compared to that of the more noble metal. For a summary of fundamental causes of corrosion and their prevention see references 50 and 38.

Aside from the actual chemical reactions of metals there are a number of cases, especially in high temperature, high pressure gas reactions, where the metals used must withstand penetration by gas molecules. Even at low temperatures or pressures gas embrittlement may become an important factor. Thus as complete a knowledge as possible of all the reactants on all parts of the construction material should be obtained. All metals exert a solution pressure when in contact with liquids, so it is theoretically impossible to prevent some action between liquids and metals; but in practice it is possible to reduce the reaction to an economical minimum.

The design engineer is always faced with the problem of using the best material for the lowest net cost. If an almost inert metal is very costly, whereas a relatively inert metal is cheap, it may be possible to use the cheaper metal, provided the initial cost and replacement cost for the expected life of the equip-

ment add up to less or no more than the cost of the superior metal. There are many places, however, where very expensive or perhaps rare metals must be used in equipment. These expensive metals may be used alone (for example, nickel for an evaporator), or they may be used as lining material backed up by some less expensive metal that will provide the necessary strength. Tantalum linings illustrate this principle. Sometimes an inert metal will not stand the load to be applied. For example, lead is often too soft and of too low tensile strength for many uses, but it can be used as a lining. Lead and other metals that do not alloy well with iron, copper, aluminum, etc., are often used by themselves as linings rather than in alloys to give corrosive resistance.

Recently considerable attention has been given to the use of inhibitors to prevent certain types of corrosion. Data on inhibitors are widely scattered and a rational basis for correlation of results of these phenomena has not been found. Undoubtedly much more use will be made of specific inhibitors in the future.

The ferrous metals are in very common use for many classes of equipment. They are relatively inert to many substances, are inexpensive and easily fabricated. Proper alloying gives them resistance to many substances under various conditions of temperature and pressure. Non-ferrous metals are usually more expensive than ferrous alloys and thus are used only where ferrous metals would corrode too rapidly or where the products of corrosion would present undesirable impurities in the reactants.

A detailed description of the properties of the many materials of construction is beyond the scope of this text. The most comprehensive survey of the chemical and physical properties of both metallic and non-metallic materials of construction is given in the journal of Chemical and Metallurgical Engineering, where tabulations of data are published each year. The 11th Materials of Construction Issue is dated September 1944.[22] For other specific data and recommendations for use of materials consult the Chemical Engineers Handbook.[47]

1-3. Physical Factors. The physical properties of materials of construction that must be considered are the following:

Strength is often the most important physical property to be taken into account in the choice of a material. It must be known whether the material is to withstand tensile or other stresses, and whether these stresses are uniform and continuous or whether they are alternating. Due allowance must be made for the possibility of heavy peak loads. And the stress must be known for the material at the operating temperature involved. Many non-metallic substances, such as wood, leather, and plastic materials, have strength and other physical properties that depend upon their exposure to water, oil, air and other environments.

The *expansion* of materials due to applied stress or changes in temperature must be considered. This is especially true when several metals are in contact, each having different thermal expansion coefficients. Expansion, as temperatures

are changed, will produce forces over and above the normal force obtaining in the system.

Conductivity of heat, light, and electricity are physical properties that are often deciding factors in the choice of material. Substances with low conductivity, or insulators, may be thought of as conductors with very low conductivity coefficients.

Hardness and softness are properties that are difficult to define but are of importance in practically all equipment where resistance to erosion and abrasion are essential.

The *elasticity* of a material is an important property of both metallic and non-metallic substances. The elasticity of metals is relatively unaffected by environment; the elasticity of non-metals varies considerably, depending upon the effect of any contacting fluids.

Some materials are advantageous because of their *porosity or non-porosity* while others may be useful in view of *high or low density.*

In addition to the preceding properties any choice must be modified by the ease of fabrication of the material. Certain durirons are so hard that they cannot be machined successfully. Other materials may require careful and painstaking heat treatment before successful welding operations can be carried out. In others machining may cause a change in the mechanical structure so that subsequent heat treatment is essential.

Machinability is a measure of the ease with which material can be cut. It is a very complex property which is undoubtedly dependent upon physical structure but not easily characterized otherwise. For example, cast iron is harder and more brittle than copper but easier to machine. Duriron is harder and more brittle than cast iron but is more difficult to work. Sulfur, in general, imparts machinability to ferrous alloys. Some plastics, like hard rubber, are sometimes more difficult to machine than metals. Finally, the ease of machining is a function of both the tool and the material worked. Machinability ratings of a few steels are included in Table 2-2. Sometimes very large pieces of equipment must be made of materials that can be only partly fabricated in the factory and final fabrication done on the erection site. Ease of fabrication of a particular piece of equipment is dependent upon the materials used, the form of the piece desired, and many other individual characteristics. The problem must be considered separately for each piece of equipment.

1-4. Economic Considerations. The final decision for choice of a material is always an economic consideration.[19,27] The initial cost of a material compared with the expected life is of fundamental importance, and initial cost of a piece of equipment includes the cost not only of the materials of which it is made but also of machining and fabricating. The net cost of a piece of equipment is not necessarily dependent upon use of low cost material. Inexpensive materials may be costly to fabricate and may in addition have a comparatively short life. A minimum net cost can often be obtained by using an expensive

raw material where the fabrication cost is low and the life is long. In all cases the net cost is the important one. The chemical and physical properties can be interpreted in cash values and thus reduced to a basis whereby they can be balanced with the costs mentioned above. The decision in cases involving these variables would depend upon the total cost of maintaining the equipment at top performance over a given period of time. This period of time should be that of the useful life of the equipment. In some cases it is cheaper to use inexpensive materials, discard major portions of the equipment after a short period of use, and replace the discarded part by another of the same material, than to use longer lived and more expensive materials of construction. Such limited economic consideration must not, however, cover up such aspects as decreased safety to workmen and equipment, or the increase in impurity of the product, either of which may result when parts are not replaced at intervals upon which the design was based. In the sense that safety, workmen's health, and cost of accidents can be estimated on a dollar basis, these factors can be weighed and considered as an economic factor in the choice of the material of construction.

Economic conditions, as indexed by cost, are in a constant state of flux regardless of whether the economic system is stabilized during political periods of "conservative policies," "sound money" eras, "new deal" eras, or war periods. The engineer must design processes and equipment under the existing balances of governmental, labor, and management policies, no matter how complete the planned economy. Costs of at least some materials are invariably fixed by fiat rather than worth as regards their chemical and physical properties for particular equipment uses. It is due to such unnatural or subsidized costs that otherwise uneconomical materials become useful. Similarly, periods occur during which whole new industries become practical and can operate by direct or indirect subsidy to produce products that could not otherwise be sold in competition with materials produced under a freer economy. Under stress of changing world economy balances, materials change value not in proportion to dollar value but rather in relation to their procurability or their desirability for one or more political reasons. The engineer is only acutely conscious of these economic problems when adjustment is being made from one economic period to another within a short space of time. During an engineer's lifetime, wars, government, labor, and management policy changes often provide several periods where shifts occur to new economic equilibria.

1-5. Fabrication. Aside from the actual cost and ease of fabrication of parts there are other criteria for choosing materials on a fabrication basis. Account must often be taken of adaptability of parts, of future adjustments in design which may be necessary, or of replacement. Many chemical processes that have been worked out on a small scale in pilot-plant and semi-works require equipment to be designed on the basis of information obtained in these development stages. Due to the fact that large-sized equipment is designed from data obtained from small equipment, higher local stresses, higher local temperature

or heat fluxes, and less uniformity of chemical composition may be encountered than may have been anticipated. The astute design engineer will build into his equipment sufficient leeway so that relatively small unforeseen deviations in these factors can be taken care of without major changes in construction.

Other factors in fabrication, construction, and life of a material are also important. Equipment should be so "streamlined" that pockets, crevices, and ridges are absent, and thus corrosion is not encouraged by the presence of stagnant fluids. A design which results in too high or too low a rate of flow (especially through piping and fittings on vessels) may be the principal cause of failure of equipment. The actual locations of feed lines, reflux lines, and vapor connections are often causes for failure of materials which might be suitable if care were taken to prevent drip and channeling along metal walls and to insure direct mixing of such streams.

Provision should also be made for cleaning purposes. Proper washing and rinsing and even dilution of liquids will allow for greater life. Cleanliness (absence of foreign matter or stagnant accumulations of ingredients) is of utmost importance in the life of any piece of equipment whether it is made of ordinary steels, stainless steels, copper alloys, or nickel alloys.

Another factor of extreme importance in considering fabrication is the ease of replacement of parts or of substitution of different materials. This is also a function of the availability of a material which, in the case of a war economy, may become extremely important. Since the ability to make small parts will determine the ability to make complex equipment units, the availability or the ease of replacement of small parts may often be the deciding factor in the choice of the material of construction of major structural pieces.

Generalized Properties and Fields of Application of Metals

1-6. There are hundreds of possible metals, with their alloy combinations, which are used in chemical equipment for one reason or another. To summarize or outline all the theories or correlations between chemical resistance, strength, and other physical requirements with the chemical composition and molecular structure of the alloys is beyond the scope of this text; to do so would require a detailed survey of the chemical literature on corrosion and the literature on physical metallurgy. Only a few brief generalities are given here to call attention to certain of the more useful or unique correlations between chemical and metallurgical structure and adaptability. For detailed treatment of corrosion and corrosion resistance, see references 50, 38 and 6. For detailed recommendations regarding use of various materials for construction in particular chemical processes, see references 22 and 47. These sources list hundreds of materials showing composition, certain physical properties, and forms available. They do not, however, supply the underlying theory by which we attempt to explain why small amounts of one chemical element will modify the corrosion

resistance characteristics of another element when alloyed with it, or why variations in characteristics of such an alloy can be made by heat treatment or mechanical working. This type of theory is taken up in the study of physical metallurgy, for which consult metallurgical texts, such as references 29 and 57.

In a large majority of the many uses of metals, the temperatures met with are at or near those occurring under normal atmospheric conditions. There are an increasing number of processes, however, which require metals to withstand extremely high temperatures or extremely low temperatures. Data for the physical properties of metals at such extremes of temperature are not too widely available, nor are the techniques for evaluation of properties and performance at these temperatures too well correlated with experience. Both chemical and physical composition of alloys affect the properties at extreme temperatures and they have been related to creep data (especially in high temperature work) and to impact, resistance and tensile properties (at low temperatures). For compilations of such data reference is made to the publications of the American Society for Testing Materials.[10,11]

Aside from the actual chemical modifying action of alloying elements, the temperature-time history and the phase equilibria either obtaining or possible are factors which have tremendous influence on chemical inertness, tensile strength, shock resistance and, in fact, on all the physical properties of metals. As an illustration, it is generally true that solid solution alloys, being homogeneous throughout, are more resistant to corrosion than alloys of the same constituents made up of eutectic-compound or solid solution-compound. Lack of homogeneity may be caused in many ways, as in deoxidizing processes (in treatment of steels with ferromanganese in the Bessemer process), in heat treatment, in cold working, etc. A hypereutectoid steel can show wide variations in resistance to attack by acids and other corroding substances, depending upon the temperature-time history of the steel. Cold worked metals are more apt to corrode than unstrained metals of the same composition. Members subjected to alternating stresses may develop a change of structure in surface layers resulting in more rapid corrosion than normal; this phenomenon is known as "corrosion fatigue."

It must be borne in mind that each of the interrelated factors mentioned in the preceding paragraph—homogeneity, crystalline aggregation, cold working, and fatigue—not only affect the chemical activity of the alloy, but also have a profound effect upon the physical properties. To illustrate, a solid solution alloy may have a much greater tensile strength or hardness, and much lower electrical conductivity, than any of its constituents, and quantitatively the property is not in direct proportion to the composition. On the other hand, eutectic alloys show direct quantitative relations between tensile strength, hardness, and electrical conductivity and the composition of the alloy. Further, the presence of intermetallic compounds has marked but irregular effect on these same properties. Again referring to the structure of a hypereutectoid steel, martensitic structures

give the hardest steel, while troostitic and sorbitic structures result in softer metals, with sorbitic structure the softest. It is possible to quench a low carbon steel under controlled agitation conditions to produce armor plate equivalent in effectiveness to high alloy steels. Cold working generally strengthens a metal while at the same time making it harder and more brittle. Continuous vibrating produces fatigue and results in marked lowering (often as much as 25%) of the tensile strength.

1-7. Ferrous Metals. Iron and its alloys make up the bulk of the material used in equipment manufacture. There are such a wide variety of iron alloys that it is difficult to give general properties. The physical and chemical properties of the ferrous alloys differ greatly although most of them contain 95% or more iron.

The three general groups of ferrous metals are cast irons, wrought irons, and steels. Pig iron as produced in the reduction of iron ores usually contains 2.5 to 4.5% carbon. When pig iron is cast to form in sand molds, it is called cast iron. Malleable cast iron is made by annealing cast iron. These cast irons all contain carbon (above 1.7%) and may be modified by the addition of silicon, nickel, and several other elements. When pig iron is melted and treated to remove carbon, wrought irons and steels are produced. Wrought irons are malleable and have a grain, caused by slag enclosures and working, which give them characteristic properties; they usually contain less than 0.04% carbon. The steels are the iron alloys whose carbon content is low (usually less than 2.5%) and whose properties are governed principally by the amount of carbon present. When the distinctive properties are chiefly due to the carbon present, the steel is called a plain, simple, or ordinary carbon steel. When the properties of a steel are modified by small amounts of other metals or metalloids (such as phosphorus), they are spoken of as low-alloy steels. When relatively large quantities of other metals are added, so that they are chiefly responsible for the distinctive properties, the steels are called high-alloy steels.

1-8. Cast Irons. These high iron-carbon alloys usually contain small amounts of sulfur, silicon, manganese, and phosphorus. They are usually hard, brittle, and porous, but machinability and other physical properties can be modified appreciably by the use of metals and metalloids, and by controlled cooling techniques. Depending upon the time-temperature history during solidification, the iron carbide may or may not have become dissociated into iron and graphite and there will result "gray" or "white" cast iron. Most cast irons are "gray." Gray cast irons are cheap, easily machined, not very brittle, have high compressive strength and damping capacity. They do not have as high tensile strength nor are they as tough or ductile as steel, but can be produced with tensile strengths as high as 55,000 psi. White cast irons are formed when silicon content is low and the rate of chill has been high. They are very hard and brittle and almost unmachinable. They have a use in the production of malleable iron parts to replace steel forgings.

Alloy cast irons are made by the addition of chromium, nickel, molybdenum, copper, etc., alone or in various combinations, in order to improve the mechanical, corrosion, and heat resistant properties of gray cast iron. Each of the alloying metals mentioned has specific effects, but it is possible to employ combinations that will impart unusually good resistance to oxidizing action up to temperatures in the neighborhood of 1500° F.

High silicon cast irons (with 14 to 15% Si) are very hard, somewhat brittle, and very resistant to corrosion and mechanical abrasion. Tensile strengths are medium, but the castings cannot be machined. They can, however, be welded and ground. In general they are limited to use with pressures below 100 psi. New processes make possible the "siliconizing" of iron and steel, whereby low silicon alloys can be machined and then surface treated to the desired surface silicon content.

1-9. Wrought Iron. Wrought iron normally contains 0.02 to 0.04% carbon, and contains 1.0 to 2.0% ferrous silicate slag introduced because of the method of production. The slag accounts for much of the corrosion resistance of wrought iron. According to the type of working it receives, the slag is distributed as various lengths of slender fibers. The resultant structure is such as to give good welding characteristics and a considerable difference between longitudinal and transverse tensile strength. Except for special uses in chains, stay bolts, rivets, hooks, boiler tubes, and some pipe, wrought iron finds little use compared to low-carbon steels, largely due to its higher cost.

1-10. Plain Carbon Steels. While plain carbon steels may contain as much as 1.0% manganese and 0.2% silicon and various small amounts of sulfur, nitrogen, hydrogen, copper, nickel, chromium, aluminum, tin, lead, arsenic, and molybdenum, the important alloying element controlling its characteristics is carbon. The relation between the carbon content and the microstructure and properties of these steels is essentially dependent on the heat treatment. As carbon content increases to about 0.85% (other constituents and variables constant) there is a corresponding increase in tensile strength, yield strength, and hardness, and a decrease in ductility and impact resistance. An increase of carbon content above 0.85% has a much less marked effect on the strength and other properties. The fatigue strength increases with tensile strength but at a lower rate.

Small quantities of phosphorus, sulfur, and manganese may have beneficial or detrimental effects depending on their quantity and the amount of carbon present. Aluminum and titanium used along with phosphorus tend to give good resistance to atmospheric corrosion. Phosphorus, sulfur, manganese and silicon are used as deoxidizers also. Silicon increases tensile strength. Oxygen, hydrogen, and nitrogen exert considerable influence when present even in very small amounts. Great care is necessary to control the amount of these elements.

Many steels are formed to shape by cold working, giving physical properties that cannot be attained by alloying. Also these products of cold work may be annealed or heat treated to efface the result of the cold work, or to modify its extreme effects.

1-11. Classification of Steels. Steels are indexed by the SAE (Society of Automotive Engineers)[49] as in Table 1-1. Carbon steels are designated by a four-digit figure (1xxx), the first digit being 1. The last two digits show the carbon content in hundredths of one per cent. Thus a plain carbon steel containing 0.35 to 0.45% carbon (average 0.40%) would be an SAE 1040 steel, or a forty-point carbon steel. The corresponding permissible content of manganese, sulfur, and phosphorus is also limited by the specifications corresponding to the index number (see Table 1-2). Further, SAE 1112 is a free-cutting steel of 0.12% (0.08 to 0.16) carbon, and SAE X1325 is a free-cutting manganese steel containing 0.20 to 0.30% carbon with specified ranges of manganese and sulfur contents.

TABLE 1-1.—SAE NUMERAL INDEX SYSTEM FOR STEELS

Type of Steel	Numerals (and Digits)
Carbon Steels	1xxx
Plain carbon	10xx
Free cutting (screw stock)	11xx
Free cutting, manganese	X13xx
High-manganese Steels	T13xx
Nickel Steels	2xxx
0.50% nickel	20xx
1.50% nickel	21xx
3.50% nickel	23xx
5.00% nickel	25xx
Nickel-chromium Steels	3xxx
1.25% nickel, 0.60% chromium	31xx
1.75% nickel, 1.00% chromium	32xx
3.50% nickel, 1.50% chromium	33xx
3.00% nickel, 0.80% chromium	34xx
Corrosion and heat-resisting steels	30xxx
Molybdenum Steels	4xxx
Chromium	41xx
Chromium-nickel	43xx
Nickel	46xx and 48xx
Chromium Steels	5xxx
Low-chromium	51xx
Medium-chromium	52xxx
Corrosion- and heat-resisting	51xxx
Chromium-vanadium Steels	6xxx
Tungsten Steels	7xxx and 7xxxx
Silicon-manganese Steels	9xxx

The AISI (The American Iron and Steel Institute) also classifies and specifies steels by composition.[5] This classification differs somewhat from that of the SAE. Table 1-2 is a compilation of only a very few of the steels classified by the two systems, and is intended only as an illustration of the comparison and type of information obtainable from them.

The Iron and Steel Branch of the War Production Board has set up a standard classification referred to as NE (National Emergency Standard Steel). The letters NE are used as a prefix to the numeral index system as used by the AISI and SAE, for example, an NE 1006 steel. These specifications call for only slightly different chemical ranges from those shown in Table 1-2. Exact data can be obtained from manufacturers' literature or from the War Production Board.

In addition to the SAE and the AISI classifications and specifications given in Tables 1-1 and 1-2, the American Society of Mechanical Engineers (ASME) and the American Society for Testing Materials (ASTM) have set up material specifications. These specifications give detailed information on composition; directions for sampling, testing and analyzing; working stresses; working temperatures; and many other details about procedures of handling, use and surveillance of construction in connection with the codes. The specification numbers of the metals for various uses are given in Table 1-3. For the details of these specifications see references 14 and 8. There is no direct correlation between ASME numbers and SAE numbers, although the ASME specifications prescribe chemical and physical compositions and tolerances. An SAE or AISI steel can, however, be found to fit each ASME and ASTM number and steels can be designated by means of any of the four numbering systems.

It is unfortunate that there are at the present time at least five separate classifications for some of the commoner steels. This situation has arisen because various organizations have independently initiated their own standards. A similar situation also prevails in other codes for the use of these steels, as will be mentioned later.

1-12. Uses of Carbon Steels. A wide variety of properties can be obtained for both different compositions and heat treatments of carbon steels. A few of the more important steels and their applications are given here purely as an indication of the many possibilities for their use.

SAE 1010-1025. These are used in large quantity for sheet, rod, plate, pipe, wire, and structural shapes. These steels are not heat treated to increase hardness and strength but usually to modify the effect of cold working. Most tin plate, galvanized sheet, fence wire, and pipe are made of SAE 1010. SAE 1020 and 1025 are used in boiler plate, pipe, and low strength structural parts, and are readily welded, brazed, and drawn.

SAE 1030-1050 are the medium-carbon steels and are usually heat treated. SAE 1030 is used for forged, machined, or cold-worked parts of fairly high physical properties; it is suitable for case hardening. SAE 1035 is used for

TABLE 1-2.—COMPARISON OF SAE-AISI SPECIFICATIONS FOR ONLY A FEW OF EACH OF SEVERAL TYPES OF STEELS

	SAE	AISI	C	Mn	P Max.	S Max.	Si	Ni	Cr	Mo	V
			Chemical Composition Limits, Per Cent								
Carbon Steels	1010		0.05/0.15	0.30/0.60	0.045	0.045					
		1006	0.08max.	0.25/0.40	0.04	0.05					
		1008	0.10max.	0.30/0.50	0.04	0.05					
		1010	0.08/0.13	0.30/0.50	0.05	0.05					
		1012	0.10/0.15	0.30/0.50	0.04	0.05					
	1020		0.15/0.25	0.30/0.60	0.045	0.055					
		1014	0.13/0.18	0.30/0.50	0.04	0.05					
		1015	0.13/0.18	0.40/0.60	0.04	0.05					
		1020	0.18/0.23	0.30/0.50	0.04	0.05					
		1023	0.20/0.25	0.30/0.50	0.04	0.05					
	1040		0.35/0.45	0.60/0.90	0.045	0.055					
		1035	0.32/0.38	0.60/0.90	0.04	0.05					
		1038	0.35/0.42	0.60/0.90	0.04	0.05					
		1040	0.37/0.44	0.60/0.90	0.04	0.05					
		1042	0.40/0.45	0.60/0.90	0.04	0.05					
		1043	0.40/0.47	0.70/1.00	0.04	0.05					
	1070		0.65/0.80	0.60/0.90	0.040	0.055					
		1074	0.70/0.80	0.50/0.70	0.04	0.05					
		1080	0.70/0.85	0.60/0.90	0.04	0.05					
Free Cutting Steels	1112		0.08/0.16	0.60/0.90	0.09/0.13	0.10/0.20					
		1111	0.08/0.13	0.60/0.90	0.09/0.13	0.10/0.15					
		1112	0.08/0.13	0.60/0.90	0.09/0.13	0.16/0.23					
	X1314		0.10/0.20	1.00/1.30	0.045	0.10/0.20					
		1112	0.10/0.16	1.00/1.30	0.045	0.08/0.13					
		1117	0.14/0.20	1.00/1.30	0.045	0.08/0.13					
	X1335		0.30/0.40	1.35/1.65	0.045	0.10/0.20					
		1132	0.27/0.34	1.35/1.65	0.045	0.08/0.13					
		1137	0.32/0.39	1.35/1.65	0.045	0.08/0.13					
Manganese Steels	1330		0.25/0.35	1.60/1.90	0.040	0.050					
		1330	0.28/0.33	1.60/1.90	0.040	0.040	0.15/0.30				
Nickel Steels	2330		0.25/0.35	0.50/0.80	0.040	0.050		3.25/3.75			
		2330	0.28/0.33	0.60/0.80	0.040	0.040	0.15/0.30	3.25/3.75			
	2345		0.40/0.50	0.60/0.90	0.050	0.050		3.25/3.75			
		2340	0.38/0.45	0.70/0.90	0.040	0.040	0.15/0.30	3.25/3.75			

TABLE 1-2 —(*Continued*)

	SAE	AISI	Chemical Composition Limits, Per Cent								
			C	Mn	P Max	S Max.	Si	Ni	Cr	Mo	V
Nickel-Chromium Steels	3130		0.25/0.35	0.50/0.80	0.040	0.050		1.00/1.50	0.45/0.75		
		3130	0.28/0.33	0.60/0.80	0.040	0.040	0.15/0.30	1.10/1.40	0.55/0.75		
	3145		0.40/0.50	0.60/0.90	0.040	0.050		1.00/1.50	0.45/0.75		
		3140	0.38/0.43	0.70/0.90	0.040	0.040	0.15/0.30	1.10/1.40	0.55/0.75		
		3145	0.43/0.48	0.70/0.90	0.040	0.040	0.15/0.30	1.10/1.40	0.70/0.90		
	3245		0.40/0.50	0.30/0.60	0.040	0.050		1.50/2.00	0.90/1.25		
		3240	0.38/0.45	0.40/0.60	0.040	0.040	0.15/0.30	1.65/2.00	0.90/1.20		
Molybdenum Steels	4140		0.35/0.45	0.60/0.90	0.040	0.050			0.80/1.10	0.15/0.25	
		4135	0.33/0.38	0.70/0.90	0.025	0.025	0.15/0.30		0.80/1.10	0.18/0.25	
		4137	0.35/0.40	0.70/0.90	0.025	0.025	0.15/0.30		0.80/1.10	0.18/0.25	
	4320		0.15/0.25	0.40/0.70	0.040	0.050		1.65/2.00	0.30/0.60	0.20/0.30	
		4320	0.17/0.22	0.45/0.65	0.040	0.040	0.15/0.30	1.65/2.00	0.40/0.60	0.20/0.30	
	4615		0.10/0.20	0.40/0.70	0.050	0.050		1.65/2.00		0.20/0.30	
		4615	0.13/0.18	0.45/0.65	0.040	0.050	0.15/0.30	1.65/2.00		0.20/0.30	
		4620	0.17/0.22	0.45/0.65	0.040	0.040	0.15/0.30	1.65/2.00		0.20/0.30	
	4640		0.35/0.45	0.50/0.80	0.040	0.050		1.65/2.00		0.20/0.30	
		4640	0.38/0.43	0.60/0.80	0.025	0.025	0.15/0.30	1.65/2.00		0.20/0.27	
Chromium Steels	5120		0.15/0.25	0.60/0.90	0.040	0.050			0.60/0.90		
		5120	0.17/0.22	0.70/0.90	0.040	0.040	0.15/0.30		0.70/0.90		
	52100		0.95/1.10	0.20/0.50	0.030	0.035			1.20/1.50		
		52098	0.90/1.05	0.30/0.50	0.025	0.025	0.20/0.35		1.00/1.25		
		52107	1.00/1.15	0.30/0.50	0.025	0.025	0.20/0.35		1.35/1.65		
Chromium Vanadium Steels	6150		0.45/0.55	0.60/0.90	0.040	0.050			0.80/1.10		0.15 min. 0.18 des
		6150	0.45/0.53	0.70/0.90	0.025	0.025	0.15/0.30		0.80/1.10		0.15 min.
		6152	0.48/0.55	0.70/0.90	0.040	0.040	0.15/0.30		0.80/1.10		0.10 min
Silicon Manganese Steels	9260		0.55/0.65	0.60/0.90	0.040	0.050	1.80/2.20				
		9262	0.55/0.65	0.70/0.90	0.040	0.040	1.80/2.20		0.20/0.30		

TABLE 1-3.—SUMMARY OF THE ASME BOILER CONSTRUCTION CODE

SECTION II. Material Specification Showing Corresponding ASTM Specification Numbers

FERROUS METALS

Boiler Steel Plates and Rivets

Specification No.		
ASME	ASTM	
S-1	A70-39	Carbon-steel plates for boilers and other pressure vessels.
S-2	A89-39	Low-tensile steel plates of flange and firebox qualities.
S-4		Seamless steel drum forgings.
S-5	A31-40	Boiler rivet steel, staybolt steel, and rivets.
S-7	A7-33	Steel bars.
S-10	A18-30	Carbon-steel and alloy-steel forgings.
S-25	A129-39	Open-hearth iron plates of flange quality.
S-28	A202-39	Chrome-manganese-silicon (CMS) alloy-steel plates for boilers and other pressure vessels.
S-42	A201-39	Carbon-silicon-steel plates of ordinary tensile ranges for fusion-welded boilers and other pressure vessels.
S-43	A203-39	Low-carbon nickel-steel plates for boilers and other pressure vessels.
S-44	A204-39	Molybdenum-steel plates for boilers and other pressure vessels.
S-53	A30-39	Boiler and firebox steel for locomotives.
S-55	A212-39	High-tensile strength carbon-silicon-steel plates for boilers and other pressure vessels.
S-60	A225-39T	Manganese-vanadium steel plates for boilers and other pressure vessels.
S-61	A176-39	Corrosion-resisting chromium steel sheet, strip, and plate.
S-62	A240-40T	Corrosion-resisting chromium-nickel steel sheet, strip, and plate for fusion-welded unfired pressure vessels.

Steel Castings

S-11	A27-39	Carbon-steel castings.
S-33	A157-39	Alloy-steel castings for valves, flanges, and fittings for service at temperatures from 750 to 1100° F.
S-56	A216-39T	Carbon-steel castings suitable for fusion welding for service at temperatures up to 850° F.
S-57	A217-39T	Alloy-steel castings suitable for fusion welding for service at temperatures from 750 to 1100° F.

Steel Tubes and Pipe

S-8	A105-39	Forged or rolled steel pipe flanges for high temperature service.
S-17	A83-38T	Lap-welded and seamless steel and lap-welded iron boiler tubes.
S-18	A53-36	Welded and seamless steel pipe.
S-32	A178-37	Electric-resistance-welded steel and open-hearth iron boiler tubes.
S-34	A158-38T	Seamless alloy-steel pipe for service at temperatures from 750 to 1100° F.
S-35	A182-39	Forged or rolled alloy-steel pipe flanges, forged fittings, and valves and parts for service at temperatures from 750 to 1100° F.
S-40	A192-38T	Seamless steel boiler tubes for high pressure service.
S-45	A206-39T	Seamless carbon-molybdenum alloy-steel pipe for service at temperatures from 750 to 1100° F.

TABLE 1-3.—(*Continued*)

Steel Tubes and Pipe—(Continued)

Specification No.		
ASME	ASTM	
S-48	A209-38T	Seamless carbon-molybdenum alloy-steel boiler and superheater tubes.
S-49	A210-38T	Medium-carbon seamless steel boiler and superheater tubes.
S-50	A181-37	Forged or rolled steel pipe flanges for general service.
S-52	A213-40T	Seamless alloy-steel boiler and superheater tubes.
S-58	A135-34	Electric-resistance-welded steel pipe.
		Steel Bolting Materials
S-9	A96-39	Alloy-steel boling material for high temperature service.
S-51	A194-39	Carbon and alloy-steel nuts for bolts for high pressure and high temperature service to 1100° F.
		Wrought Iron
S-16	A84-33	Staybolt wrought iron, solid.
S-17	A83-38T	Lap-welded and seamless steel and lap-welded iron boiler tubes.
S-19	A72-39	Welded wrought-iron pipe.
		Iron Castings
S-13	A48-36	Gray-iron castings.
S-15	A47-33	Malleable-iron castings.

NON-FERROUS METALS

Aluminum and Aluminum Alloys

S-38	B25-38T	Aluminum sheet and plates.
S-39	B126-39T	Aluminum-manganese alloy sheets and plates for use in welded pressure vessels.
		Copper and Copper-Alloy Castings
S-41	B61-36	Steam or valve bronze castings.
S-46	B62-36	Composition brass or ounce metal castings.
S-59	B57-27	Muntz metal condenser tube plates.
		Copper and Copper-Alloy Forgings, Bars, Rods, and Shapes
S-21	B12-33	Copper bars for staybolts.
S-22	B13-33	Seamless copper boiler tubes.

TABLE 1-3.—(*Continued*)

Copper and Copper-Alloy Forgings, Bars, Rods, and Shapes—(Continued)

Specification No.		
ASME	ASTM	
S-23	B42-33	Copper pipe, standard sizes.
S-24	B43-39T	Brass pipe, standard sizes.
S-37	B98-39	Copper-silicon alloy rods, bars, and shapes.
S-47	B111-39T	Copper and copper-alloy seamless condenser tubes and ferrule stock
		Copper and Copper-Alloy Pipe and Tubes
S-23	B42-33	Copper pipe, standard sizes.
S-24	B43-39T	Brass pipe, standard sizes.
S-47	B111-39T	Copper and copper-alloy seamless condenser tubes and ferrule stock.
		Copper and Copper-Alloy Sheets, Strip, and Wire
S-20	B11-33	Copper plates.
S-36	B96-39T	Copper-silicon alloy plate and sheets.
S-54	B127-39T	Nickel-copper alloy plate, sheet, and strip.
		Nickel
S-54	B127-39T	Nickel-copper alloy plate, sheet and strip.

medium-sized forgings, shafting, axles, gears, etc. SAE 1050 is used for the same purposes where the cross sections are large.

SAE 1060-1095, and even higher, are classed as plain high-carbon tool and die steels. High hardness can be obtained readily.

1-13. Alloy Steels. When elements like Mn, P, Si, Ni, Cu, Cr, Mo, V, and W are added to steel, not primarily for deoxidation, the products are known as alloy steels. Low-alloy steels contain relatively small quantities of the alloying elements, while high-alloy steels contain large amounts of these elements. Low-alloy steels make up the major tonnage of the total steel production. Over 75% of all alloy steel is used in the automotive and machinery industries.

Table 1-1 gives the SAE index system for alloy steels. The first digit indicates the type of steel: 1 for carbon steel, 2 for nickel steel, 3 for nickel-chromium steel, etc. The last two digits give the approximate percentage of carbon as hundredths of one per cent. For simple alloys the second place usually indicates the approximate percentage of the predominant alloying element. Thus SAE 2350 means a nickel steel of 3.25 to 3.75% Ni and 0.45 to 0.55% C, and 71360 indicates a tungsten steel of approximately 12 to 15% W and 0.50 to

0.70% C. See Table 1-2 for further illustrations and for a comparison of SAE and AISI specifications.

The principal effects of alloying elements in low-carbon steels are to harden and strengthen the ferrite of a low-carbon steel and make possible specific desirable properties on heat treatment. Plain-carbon steels have inherent properties, such as decrease in ductility with increase in carbon content, rapid deterioration of physical properties with increased temperature, and optimum heat treatment effects in small sections only. These properties may be offset within reasonable limits by *low-alloying*. Low-alloy, low-carbon, high-strength steels usually exhibit very good resistance to atmospheric corrosion. Some steels are produced which have strengths of twice that of low-carbon steel. All the low-alloy steels are heat treated to develop optimum physical properties and can be made to combine high ductility with high strength. In general, when these steels are compared with plain-carbon steels of the same tensile strength and hardness, it is found that they have yield strengths 30 to 40% higher, and impact resistances about 80 to 100% higher. They can be quenched more easily with less chance of distortion and development of high internal stresses than plain-carbon steels.

The principal elements used in *high-alloy* steels are carbon (up to 1.0%), silicon (up to 0.75%), manganese, chromium, tungsten, and nickel. Various amounts of these latter elements produce characteristics of hardness, strength, fatigue and creep resistance, heat and corrosion resistance, and other unique and specific properties. Heat treatment is essential in controlling the properties and in providing good fabrication characteristics.

High-hardness steels are formed by various combinations, one example of which is a 1% C and 12% Mn steel. Such a steel can have both high hardness (Brinell 220) and high tensile strength (150,000 psi.) and is useful for withstanding abrasion for applications in crushers, grinders, scoops, etc. High-chromium and high-tungsten steels develop similar properties. High-nickel steels have useful magnetic, electric, conductance, and thermal expansion characteristics. Nickel imparts excellent low temperature characteristics to steel.

1-14. Corrosion-Resisting and Heat-Resisting Steels. Chromium is such an important alloying metal, because of the specific properties it develops by itself and in the presence of other alloying elements, that corrosion- and heat-resisting steels are often classified according to chromium content. Low-chromium steels contain 4 to 10% Cr and possibly small amounts of Mo, Ni, Si, and W in various combinations. These steels are more corrosion- and heat-resistant than similar low-alloy steels and will resist oxidizing and reducing atmospheres up to 1100° F. They find use in oil refinery equipment, catalytic chambers, valves, etc., where high temperatures are encountered. Chromium has the specific property of reducing attack by hydrogen, thus greatly extending the temperature range over which steel can be used in atmospheres containing hydrogen.

Steels with more than 10% Cr and not more than 2% of another element form an important group. Of these the "stainless irons," having 12 to 18% Cr, less than 0.14% C, and small amounts of Mo and other elements, are used for turbine blades, valves, pistons, etc. Steels containing 12 to 17% Cr with 0.2 to 0.6% C are stainless cutlery steels. Another group of tool, die, and valve steels has 12 to 18% Cr and 0.7 to 2.0% C. All these steels will withstand moderate corrosion up to 1300° F. and can be heat treated to give high strength and hardness. The high-carbon content steels are often too hard to machine in final heat treated form. For excellent resistance to corrosion at high temperatures, steels containing 20 to 30% Cr with carbon content around 0.5% are available. On the other hand, cold working will generally decrease the corrosion resistance of these steels.

Steels containing more than 10% Cr and more than 2% of other elements form a group possessing superior properties of corrosion and heat resistance over a wide temperature range (2000° F. to −300° F.). The well-known "stainless steels" (18% Cr, 8% Ni, 0.1% C), or KA2S (a commercial designation), and modifications have an excellent combination of some of the most desirable engineering properties.

Nickel imparts corrosion-resistant properties and at the same time allows the steel to be reasonably ductile and very strong. In general, these alloys can be greatly hardened and strengthened by cold working, but such working will decrease the corrosion resistance.

1-15. Non-Ferrous Metals and Alloys. Due to the corrosion-resistant properties of many of the non-ferrous metals, their use in chemical equipment is widespread. A few metals, such as aluminum and magnesium, also find use as structural materials due to their low weight or density. The more important non-ferrous metals are copper, aluminum, zinc, lead, nickel, tin, and magnesium. These are not only important when used alone but are also the bases of many of the important alloys. Metals of secondary importance are mercury, silver, gold, platinum, cadmium, antimony, and bismuth. Other metals important because of their use as alloying elements are chromium, cobalt, manganese, vanadium, tungsten, molybdenum, beryllium, and titanium; also aluminum, zinc, lead, tin, and nickel of the primary group. Metalloids like phosphorus and silicon are also used in these alloys. Some of the more important characteristics of a few of these useful metals and alloys will be commented on briefly.

Copper and Its Alloys. The properties of copper that make it so useful are its electrical conductivity, tensile strength, and resistance to corrosion. The tensile properties are a function of its cold working and heat treating, tensile strength being increased with cold working up to roughly 65,000 psi. The many alloys of copper have tensile strengths usually lower than copper, but a 65% Cu, 45% Zn brass can reach a tensile strength of 75,000 psi.; while manganese bronzes can go as high as 110,000 psi. tensile strength.

The corrosion resistance of copper makes its use in chemical equipment widespread. It cannot be used in the presence of oxidizing agents or ammonia or carbon dioxide. In alloy form with tin (bronze) it is modified with zinc, phosphorus, lead, manganese, silicon, and aluminum to give added corrosion resistance or other specific properties. Usually the alloys are more resistant than the base metal and, in fact, most alloys which are in the form of solid solutions exhibit excellent corrosion-resistant properties, whereas those containing eutectic or hard precipitated grains are less resistant to corrosion. Brasses (copper with zinc) are strong and have good corrosion resistance, especially when alloyed with lead, aluminum, manganese, and iron. The principal fault of brasses is due to non-uniformity which allows a type of corrosion called dezincification. Copper-aluminum alloys make very useful corrosion-resisting metals, especially for pipes and containers for acid solutions. Silicon alloys with copper to form excellent corrosion-resisting alloys. Beryllium-copper alloys have the especially desirable property of high strength and corrosion resistance up to about 380° F., together with good fatigue resistance.

Aluminum and Its Alloys. Aluminum has low density, high strength, and electrical conductivity and is not attacked by certain of the common active acids to which copper is not too resistant. The tensile strength of alloys with zinc and copper is higher than that of aluminum alone, reaching as high as 60,000 psi. Corrosion and heat resistance are good for particular uses. Aluminum-silicon alloys have relatively low strength but are better in corrosion resistance than pure aluminum and are resistant to a wider variety of chemicals.

Zinc is resistant to the action of many atmospheric corrosive conditions and as such is used as a pure coating on iron. By itself zinc has little strength, but when used as the base of alloys the strength improves. Alloys of zinc are useful in die-casting because of the conveniently low melting temperatures.

Lead is very plastic and resistant to corrosion. Because of its low strength it is used mostly as a coating or lining, even when lightly alloyed with hardening elements such as zinc. Lead has specific anti-corrosive uses (e.g., H_2SO_4) and, when alloyed with tin, antimony, zinc, silver, etc., makes the extremely useful alloys known as solder, type metal, and the like.

Nickel and Its Alloys. Nickel is an excellent corrosion resister for atmospheric conditions, fresh water, salt water, neutral and alkaline salt solutions, and alkalies. Its mechanical properties are excellent, and thus it is a preferred material for equipment construction when its high price can be justified. It is used alone or as a lining on an iron base. The most interesting nickel alloy (Monel metal—66.7% Ni, 31.3% Cu) is useful for its corrosion resistance, general high tensile strength, and other desirable mechanical properties. Nickel-chromium-iron alloys are useful for resistance to corrosion at elevated temperatures.

Tin is very malleable, and when pure it is extremely resistant to corrosion. In an unbroken thin coating it is used as a protection for iron. Tin forms im-

portant alloys, like the lead-tin solders, bearing metals and die casting materials.

Magnesium is less dense than aluminum and so finds use in light-weight alloys in die-cast, rolled and drawn form. It is too reactive chemically to be corrosion resistant. It is alloyed with aluminum and manganese, but these alloys likewise are easily attacked.

GENERALIZED PROPERTIES AND FIELDS OF APPLICATION OF NON-METALS

1-16. *Wood* for chemical equipment finds use especially in tanks for storage and processing. Supports and structures of wood are, of course, common. Physical properties of some of the more important woods used for engineering construction are given in Chapter 13. In contact with chemical solutions different woods behave in a variety of ways, depending upon density, curing, grain, and resistance to bacterial and fungus attack. The tables in Chemical and Metallurgical Engineering give data on the condition of woods after contact with various chemicals.

Plastic impregnated wood and plasticized wood products are coming into considerable use in engineering construction. This type of material is undergoing rapid development and no generalized properties can be mentioned except that the chemical resistant properties are controlled by the plastic used, and the physical properties of the wood and plastic are both enhanced.

Stone is primarily useful or of interest in chemical manufacture as a foundation, support, or flooring material. In general its durability depends upon its porosity and composition. Stone has the ability to absorb appreciable quantities of liquid and thus is not resistant to frost. Its fire resistance is poor, and it takes a permanent distortion when cooled or heated over appreciable ranges. Its mechanical properties as applied to building stones are given in Chapter 12.

Stoneware. Clays, alumina, silica, cements and similar materials are used as the bases of a wide variety of chemical resistant materials. Molded stoneware, porcelain, tile, and acid-proof and refractory brick are made by high temperature firing of mixtures, and a wide variety of physical and chemical properties can be achieved. Typical physical data of several classes of chemical stoneware and porcelain are given in Chapter 12. These products find wide use as complete pieces of equipment and as linings for ball mills, kilns, furnaces, pipes, valves and fittings, stills, tanks, towers, and pumps.

Glass and Fused Silica. Glass finds considerable application as a construction material. Glass piping, reaction vessels, and pumps are in general use. Methods for joining and bonding glass are reliable, and the modern industrial glasses have expansion coefficients reasonably close to some metallic alloys, so that glass-lined equipment is thoroughly practical. Glass spray technique has been developed to a point where cracked and chipped glass surfaces can be repaired conveniently. Glass is such an inactive substance in contact with almost all acid bodies, gases, and organic materials that it is a preferred material. It should

not be used with alkalies. Due to its low strength, glass equipment must be well supported, or used only as a lining, and considerable care must be given to the design of supporting or enclosing members to prevent strain and to protect the glass from shock. Fused silica has excellent chemical resistance even up to temperatures of 2000° F.

Cements. Portland cement and concrete are the common binders used to hold stone, brick, etc., in position and to form floors and vats. Cements and concretes can be molded and set up to give a wide variety of tensile and compressive strengths. They are fairly resistant to salt solutions when properly set up and when they are made of high density. They are usually somewhat porous and thus often fail when crystallization of salts takes place in porous sections. Chemicals are sometimes incorporated with concrete, and proper cement compositions are used to produce the so-called acid-proof cements. High-silicate cements are resistant to dilute alkalies as well as to acids.

Other cementing materials for binding chemical stoneware, plastic materials, and the like are made from base materials such as sodium silicate, phenolic and other resins, rubber, and asphalt. Similar materials are used as linings in combination with a matrix or container made of concrete and give good chemical resistance.

Carbon. Carbon is inert to many chemicals, and recent developments in forming graphite and carbon articles have resulted in the availability of carbon of low porosity and sufficiently high strength to be useful mechanically. Carbon tubes, valves, fittings, and pumps are available. They are useful for both high and low temperature operation. Carbon and graphite products are resistant to most alkalies and to all but the oxidizing acids.

Natural and Synthetic Rubber. These materials find wide applications, especially for linings of equipment where specific corrosion or flexing problems are encountered. The chemical properties and resistance of the many forms of elastomers make it difficult to give summarized data about them. Natural rubber products, in general, deteriorate in contact with oxygen, heat, sunlight, and hydrocarbons and other oils. They are useful for most applications where such elements are absent. Of the synthetic or rubber-like plastics, those in most common use in equipment, as pipes, tubing, or linings, are types of Butyl rubber, the Bunas, Perbunan, Vistanex, Tygons, Thiokol, and Neoprene. All have distinctive properties and many will withstand action of oxygen, hydrocarbons, etc., to a much greater extent than natural rubber products. It is now possible to modify most of these classes of elastomers to give chemical resistance to most chemicals at ordinary or not too high temperatures and at the same time to have mechanical properties similar to rubber.

Plastics and Synthetic Resins. Recent developments have resulted in many plastic products which can be used as basic engineering materials of construction, as well as for cements, linings, and coatings. Up-to-date data on chemical and physical properties can only be obtained from manufacturers and from the

current literature. So many new plastics are being developed that new applications for them are appearing almost daily. The properties of plastics can be modified over wide ranges by the proper choice of fillers or by a combination of plastics with complementing characteristics. Paper, wood, cloth, or woven material can be "filled" or impregnated with plastics to form substances with unique mechanical and corrosion-resistant properties. Since plastics tend to flow under stress, it is essential that they be well formed and that large pieces be well backed to withstand strain.

1-17. The foregoing discussion of materials of construction and the data referred to are intended as guides, or indications, of the limits of application of the more common materials. More detailed information may be obtained from the references given and from current scientific journals. Substitute or alternate materials of construction should always be visualized in evolving a design. When there is great demand, curtailment of supply, or other difficulty of delivery, the design of equipment from the materials standpoint is always in a state of flux, and the designer must keep in contact with the possibilities of all kinds of engineering materials.

CHAPTER 2

MECHANICAL PROPERTIES AND STRENGTH OF MATERIALS

2-1. Stress. Mechanical properties of the materials of construction are related to the resistance of the materials to external forces under given operating conditions. Stress is this resistance; it is the intensity of the internally distributed forces or components of force that resist a change in the form of a body due to a loading force. Its dimensions are pounds per square inch (psi.) and it is referred to as unit stress (S). Total stress (lbs.) is the summation of the unit stresses. Forces may be exerted under a variety of loading conditions, and the shape and behavior of a body will vary under each. Stresses may be the result of static or gradually applied loads (static stresses), of impact or suddenly applied loads (impact stresses), of fluctuating or alternating loads (fatigue stresses), or of loads at high temperatures (creep stresses). It is convenient to classify stresses resulting from loads under the headings of simple and combined stresses. A simple stress is one where the stress acts in one direction. A combined stress is one in which the stress acts in several directions or where several stresses act in one or more directions.

There are three types of simple stresses, tensile, compressive, and shear. A tensile stress is developed when the external forces are coaxal and directed away from each other, Fig. 2-1. A compressive stress is developed when the external forces are coaxal and directed toward each other. A shear stress develops when applied forces cause or tend to cause two adjacent parts of a body to slide relative to each other in a direction parallel to their plane of contact.

The strength of a material is its ability to resist applied forces. Thus:

Tensile strength is the maximum tensile stress a material can develop under load, Fig. 2-1.

Compressive strength is the maximum compressive stress a material can develop under load, Fig. 2-2.

Shear strength is the maximum stress a material can develop under shearing forces, Fig. 2-3.

Bending or flexural strength is the maximum stress developed when forces are applied in parallel planes relatively far apart and in opposite direction, Fig. 2-4. Bending stresses can never be completely divorced from shear stresses.

2-2. Simple Tension. The working strengths or stresses used in design are in large measure based upon values determined from a simple tension test. A standard type of testing machine is used. Load is applied under prescribed conditions of rate and size of specimen of material [see ref. 7, 8, 25, 40 and Fig.

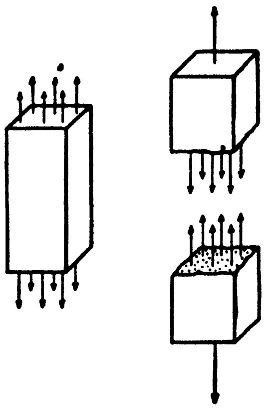


Fig. 2-1. Tension (tensile strength).

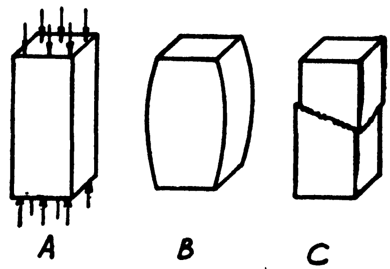


Fig. 2-2. Compression.

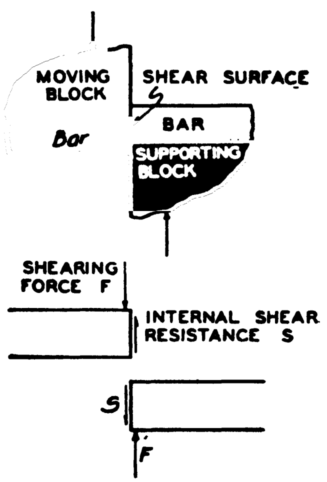


Fig. 2-3. Shear.

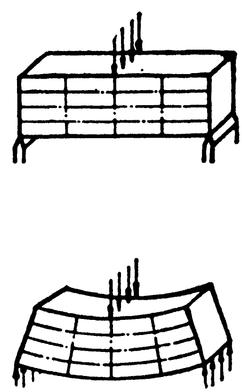


Fig. 2-4. Beam subjected to bending or flexual forces and stress.

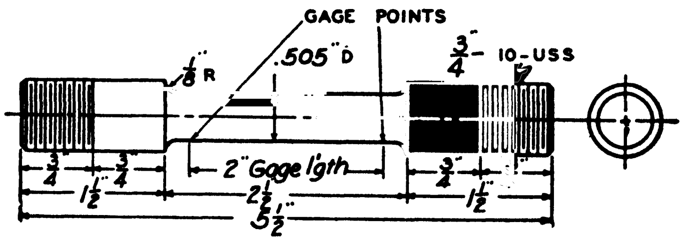


Fig. 2-5. Standard Tensile-strength Test Specimen.

2-5]. The specimen is placed in the testing machine with its ends secured in a vertical position. One end is attached to a weighing platform and the other to a mechanism for applying a load. For a tension test the load is applied and its amount measured by moving a counterweight along a graduated scale beam until it balances the pressure exerted through the specimen on the weighing platform. At various loadings, the force applied and the stress produced is computed as a unit stress by the relation

$$S = \frac{P}{A} \tag{2-1}$$

where P is the load in pounds, A the cross-sectional area in square inches of the test section before loading, and S the unit stress in psi.

Example 2-1. A ¾-in., 20-BWG standard condenser tube is subjected to an axial load of 5000 lbs. in tension, due to differences in coefficient of expansion between shell and tubes. Calculate the unit stress in the tube.

Solution. The wall thickness of a 20 BWG gage tube, from Table 9-2, is 0.035 in., and the inner diameter of the tube is 0.750 — (2) (0.035), or 0.680 in. The sectional area of the tube wall is $\pi(0.75^2 - 0.68^2)/4$, or 0.078 sq. in. Using Eq. 2-1,

$$S = \frac{5000}{0.078} = 64{,}000 \text{ psi.}$$

An elongation determination can be made at the same time a simple tension test is run. An extensometer is attached to the specimen to measure elongation between the two gage points. At increments of increasing load, readings are taken on the extensometer, and these data form the basis of estimating various important characteristics of the specimen. From the elongation, $L_2 - L_1$, for an original gage length L_1, the unit strain or deformation d is

$$d = \frac{L_2 - L_1}{L_1} \tag{2-2}$$

The units of d are usually inches per inch. The relation between S and d are most easily visualized by plotting the values to produce a stress-strain diagram. There are three general types of stress-strain curves for various types of materials, as illustrated in Fig. 2-6. Curves A and B are characteristic for ductile substances, and curve C is representative of brittle substances.

The behavior of a low carbon steel under increasing tensile loads will result in a stress-strain curve such as A in Fig. 2-6. When the load is applied to the carefully prepared specimen, which has dimensions as in Fig. 2-5, the elongation of the specimen is gradual, increasing uniformly with the applied load. Since the load is applied continuously the counterweight is moved continuously, and during the period between points O and E it will be necessary to move it uniformly. During this period, if stress is relieved by releasing the load, the metal will return to its original size and shape. Since elasticity is that property of a material which permits it to resume its original shape when deforming forces are removed, this region is called the elastic range, and the constant of propor-

tionality is called the modulus of elasticity. As load is applied above E, a point P is reached. P is called the proportional limit and may not necessarily correspond to the elastic limit E. At or beyond point P the elongation will be observed to increase more rapidly with increasing load until a point Y is reached, where the specimen will continue to stretch with little or no additional load. At this point it will be necessary to stop the motion of the counterweight to keep the weighing beam balanced, and even to move the counterweight back as required for balance. Under these conditions, point L is reached and the load is again applied regularly as the bar seems to recover strength. However, after this point, the elongation is much more rapid for the same increases in load until an ultimate strength U will be reached, after which the load must be decreased as the metal stretches until it breaks at R. During the application of load (between U and R) there is a change in transverse dimensions proportional to the change in elongation. At point U the stretching becomes localized and a "neck" is formed. Rupture will occur at the neck, and the transverse dimensions of the neck are usually appreciably less than the transverse dimensions of the original. This explains the apparent contradiction that the rupture stress is less than the ultimate.

The first part of the stress-strain curves A and B of Fig. 2-6 consists of straight lines from O to P showing that the unit stress is proportional to the unit strain. Expressed mathematically

$$S \propto d;\ \text{thus}\ S = Ed,\ \text{or}\ E = \frac{S}{d} \qquad (2\text{-}3)$$

where E is the modulus of elasticity in psi. This is known as Hooke's law and holds over the initial part of the curve where the slope is constant.

Example 2-2. If the tube of Example 1 is 8 ft. long and has a modulus of elasticity of 30,000,000 psi., what will be its length under the imposed load?

Solution. Using Eq. 2-3, $d = 64{,}000/(30 \times 10^6) = 0.0021$ in./in. The increase in length is 8(12)0.0021, or 0.201 in. The final length is 96.20 in.

The moduli of elasticity of various metals and their alloys are given in Table 2-1. It is interesting to note that, although other properties vary widely for any one metal and its alloys, the modulus of elasticity is practically constant for any one metal and its alloys. Therefore the generalities given in the table may be used with assurance.

When a structural part is made up of two materials as, for example, a clad steel used for a still, the unit stresses in each part are determined in the following way. Assuming that the two materials (subscripts 1 and 2) are bound or fastened so that they deform together, the unit deformations are equal, or $d_1 = d_2$. From Eq. 2-3, it follows that $S_1/E_1 = S_2/E_2$, and $S_2 = (E_2/E_1)S_1$. If n is the ratio of elastic moduli E_2/E_1, then $S_2 = nS_1$. The total imposed force

is $P = S_1A_1 + S_2A_2$, where A_1 and A_2 are the two cross-sectional areas. On substitution,

$$P = S_1A_1 + nS_1A_2, \text{ or } P = S_1(A_1 + nA_2) \qquad (2\text{-}4)$$

Here the quantity nA_2 is the equivalent area, or the area of material 1 which would have the same unit stress as that of material 2, and so could be substituted for it.

TABLE 2-1.—MODULI OF ELASTICITY

Material	Modulus of Tension Multiply by 10^6 psi.	Modulus in Shear Multiply by 10^6 psi.
Iron, alpha	30	12
Steel, all classes, even high-alloy	30	12
Cast irons, all classes	15	6
Malleable and wrought iron	25 to 28.5	10
Aluminum and its alloys	10.3	4
Magnesium and its alloys	6.3	2.5
Copper	16	6.4
Brass, all types	13	4.8
Bronze, all types	15.5	6.3
Nickel	30	12
Monel	25	10
Zinc die casting alloys	12 to 15	5
Lead	2.6	1
Silver	10.3	4

Example 2-3. A low-carbon steel sheet ⅜ in. thick, lined with aluminum ⅛ in. thick, is used to form a low pressure receiver 10 ft. in diameter. A tensile stress of 10,000 psi. is anticipated in the circumferential shell. Find the stress in both the steel and the aluminum.

Solution. Since the diameter is large it may be assumed that the outer surface of steel will stretch to the same extent as the inner surface of aluminum, and that the bond is complete. E_2 for the steel is 30×10^6 psi. and E_1 for aluminum is 10.3×10^6.

$$n = \frac{E_2}{E_1} = \frac{30}{10.3} = 2.92$$

Consider a 1-in. wide strip of the metal. The thickness of steel is ⅜ in. The cross-sectional area A_2 of the steel strip is (⅜) 1, or 0.375 sq. in. The cross-sectional area A_1 of the aluminum is 0.125 sq. in. Further, $nA_2 = 2.92(0.375) = 1.095$ sq. in., which is the area of aluminum required to withstand the same total load as 0.375 sq. in. of steel. The total equivalent and actual area for the aluminum is then $(0.125 + 1.095)$, or 1.22 sq. in. Substituting in Eq. 2-4, the stress in the aluminum is

$$\frac{10{,}000}{1.22} = 8200 \text{ psi.}$$

Also, the stress in the steel is $2.92(8{,}200) = 23{,}900$ psi. Further, the elongation d is $\frac{23{,}900}{30 \times 10^6} = 0.000797$ in./in., which is the same for either material.

2-3. Stress-Strain Relations. In considering strength see curves A, B, and C of Fig. 2-6. The proportional limit, point P, corresponds to the stress up to

which Hooke's law applies. The elastic limit, point E, corresponds to the stress up to which a material can be subjected without causing a permanent set. In other words, the elastic range reaches to this point, and the implication is that, whenever a stress within this range is removed, the material returns to its original size. Both the proportional and elastic limits are very difficult to determine accurately even for metals with characteristics similar to those of Fig. 2-6. The ASTM procedure should be followed rigorously when making tensile tests; but even then the interpretation of the test results must be viewed with caution. When the values of these limits are in question,

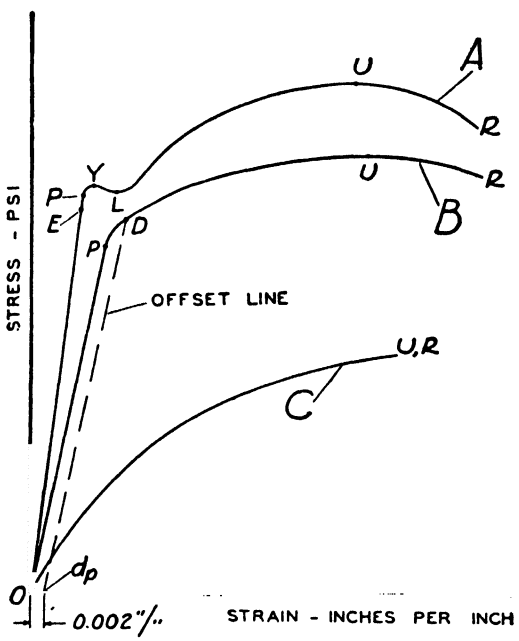


Fig. 2-6. Typical Stress-strain Curves.

the so-called yield points are useful. Point Y is known as the upper yield point, and L is the lower yield point. These points are at stresses above which the material stretches or yields appreciably with small changes in load. Small scratches on the test specimen, speed in loading, and other minor variations affect the determination of these points, but in general the lower yield point is more reliable. It is the value ordinarily used to define the elastic failure. Beyond the lower yield point the material is considered to be in the plastic range.

The portion of the stress-strain curve beyond the lower yield point represents a plastic range, where the deformation or elongation increases more rapidly with increased stresses, and where the metal will not return to its original size and shape when the load is released. *Plasticity* is the property of remaining in

a deformed shape without recovery of original shape when load is removed. Some substances are partly elastic and partly plastic; also under some conditions (e.g., elevated temperatures) it is difficult to detect the line of demarcation between elastic range and plastic range. For metals with these characteristics, strength is not easily defined by any one method. A limited amount of deformation is often required in design and thus the limiting stress corresponding to such deformation will be used instead of the ultimate strength.

When the stress-strain diagram is like curve *B* of Fig. 2-6, the proportional limit can be approximated at *P* as shown, but the elastic limit is not apparent and the yield points do not seem to exist. One way to approximate a yield point is to find the stress at which a permanent elongation d_p reaches a specified value, and the resulting point *D* on the curve (by line *AD* parallel to the straight part of the curve) is called the yield strength or proof stress. The value of d_p is usually taken as 0.002 in./in. (or 0.2% offset). This method is satisfactory, although it has no theoretical significance, especially as a basis for comparing the strengths of various materials under consideration for a particular application. For an approximation of the elastic limit for this type of material, Johnson's apparent elastic limit method can be used. *Johnson's apparent elastic limit* is a stress at which the rate of change of strain with respect to stress is 50% greater than at zero stress. To find this, a line is drawn whose slope is 0.67 times the slope of the stress-strain curve at zero stress, and its point of tangency to the curve is the value of the apparent elastic limit.

The ultimate strength is the one most commonly used to define the strength of a material, but sometimes for ductile materials the yield point strength is used. For brittle materials giving stress-strain diagrams, such as *C* in Fig. 2-6, the ultimate strength (point *U*) is the only measure of strength.

2-4. Factor of Safety. For design purposes the allowable stress, working stress, and design stress are synonymous and are expressions signifying that the stress is safe and that failure should not occur. The design stress is a fraction of either the ultimate stress or the stress at the elastic limit. Usually the ultimate stress is the more accurately known and is used unless otherwise noted. These design stresses are computed by dividing the ultimate stress by a so-called "factor of safety." The factor of safety is defined as the ratio of the ultimate to the working stress. The term "factor of safety" is an unfortunate one in that it implies safe working conditions, which is not necessarily the case and the true definition of the term should always be borne in mind. The reciprocal of the factor of safety (sometimes called the factor of utilization) is a more significant term since it indicates the fraction of the ultimate strength that is *utilized* as the working stress. Allowable stresses are established for various materials and under various conditions and services (and factors of safety subsequently calculated) in the codes set up by the ASME, AISI, and other bodies. The working stress must always be below the elastic limit, otherwise a permanent deformation would occur resulting in a stress redistribution

not considered in the original design. The factor of safety, therefore, should always be sufficient to produce a working stress below the elastic limit.

Other contingencies covered by the factor of safety are the non-uniformity of materials, overloads, uncertainties and non-uniformity of loads, wear and corrosion, shock and fatigue, or uncertainties in the true strength of the material. Each of these may be rationalized in terms of its effect on the allowable stress and thus the factor of safety. For example: if the ultimate strength of a steel were known to be 60,000 psi. and its elastic limit 36,000 lbs., a factor of safety greater than 60,000/36,000, or 1.67, would be required. If it were found that the non-uniformity were such that the elastic limit might vary by ±10%, a factor of safety greater than 60,000/0.9 × 36,000, or 1.85, is required. If, in addition, overloads or non-uniformity of loads might cause a 50% increase in unit stress, the factor should be greater than 1.85 × 1.50, or 2.78, or could be computed by 60,000/0.9 × 36,000 × 2/3. Further, if wear and corrosion could be allowed to a point where the cross-sectional area of the material would reach 75% of the initial area, the upper limit of the allowable unit stress would be 0.9 × 36,000 × 3/4 ÷ 1.50, or 16,200 psi., resulting in a factor of safety greater than 60,000/16,200, or 3.7. If the allowable working stress is equal to one half of the elastic limit, the factor of safety is 3.7 × 2, or 7.4. As it is customary to use rounded values, the 7.4 value just computed would be rounded to 8.

2-5. Various Physical Properties. Many mechanical properties can be correlated with the stress-strain relations as determined by the tensile test; they will be defined here in view of the preceding discussion.

Stiffness is the index of the rigidity of a material in the elastic range. It is measured by the modulus of elasticity E (Eq. 2-3) and is the rate of increase of stress to strain, or the slope of the straight initial part of the stress-strain curve. A high value of E indicates a material having a small deformation for a given stress or, in other words, a high degree of stiffness. The stiffness of a material that does not follow Hooke's law may be estimated as the slope of the tangent of the stress-strain curve at zero stress. Another estimation is the slope of a line between zero stress and a predetermined working stress value.

Ductility is a measure of deformation in the plastic range. Although metals used for constructing equipment are designed for stresses within the elastic range, a high degree of ductility is desired in engineering materials for several reasons: to safeguard against collapse due to excessive loads; to relieve localized loads produced by secondary stresses, and those produced in fabrication or erection; to protect against settling of foundations.

The *percentage elongation* and *percentage reduction in area* of a specimen at fracture are used for quantitative comparisons of ductility. The tensile test is used to find these percentages which are based upon the final values at the breaking point and the measurements of the original specimen. To measure the elongation of a test bar for per cent elongation calculation, the broken parts of the test bar are fitted together in the same position as before the break and

machined. An SAE 1020 cold drawn steel has a machinability rating of 60%, meaning that it can be worked with cutting tools with approximately 60% of the ease with which SAE 1112 steel can be worked. A machinability rating greater than 100% indicates a material more easily cut than SAE 1112.

The more important of the preceding properties are all dependent upon the composition, chemical and physical, of the metal. The physical

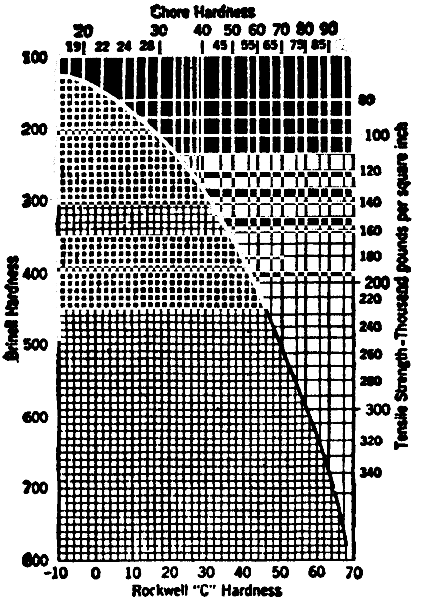


FIG. 2-7. Approximate Relation between Tensile Strength and Hardness.

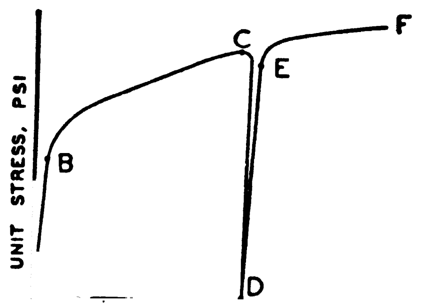


FIG. 2-8. Typical Stress-strain Diagram for Strain Hardening.

composition of alloys can be affected so markedly by heat treatment (including quenching) that a single alloy can have a wide range of mechanical properties. These are most often summarized in the form of a chart, called a Physical Properties Chart. Fig. 2-9 is such a chart for an SAE 3145 steel. For further data of this type consult manufacturers' literature.

2-6. Poisson's Ratio. When a tensile load is applied to a material and its axial dimension is lengthened, the transverse dimensions are shortened accordingly. Below the elastic limit the transverse deformation is proportional to the stress, just as the strain, measured axially, is proportional to the stress. So it may be considered that Hooke's law applies to both longitudinal and transverse stresses. The ratio of the unit transverse deformation to the unit axial deformation is constant and is known as Poisson's ratio. Thus

$$E_T = KE \tag{2-8}$$

where K is Poisson's constant, E_T the modulus of elasticity in a transverse direction, and E the modulus of elasticity in an axial direction. Values of K are

approximately 0.25 for cast iron, 0.30 for steels, and 0.33 for copper and its alloys.

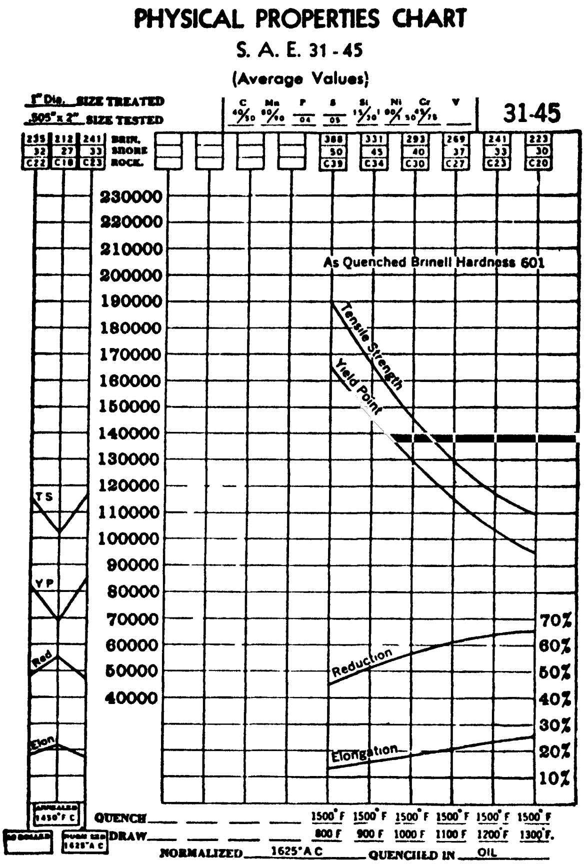


FIG. 2-9. Physical Properties Chart—Nickel-alloy Steel. (Courtesy Bethlehem Steel Co.)

Example 2-4. A steel bar 5 in. wide, 1 in. thick, and 12 in. long is stressed by a tensile load of 80,000 lbs. along the 12-in. axis. The modulus of elasticity for the steel is 30×10^6 psi. Calculate the dimensions of the stressed bar and the equivalent lateral stress.

Solution. See Fig. 2-10. The axial unit stress is $80{,}000/5 \times 1$, or 16,000 psi. Using Eq. 2-3, the axial elongation d is $16{,}000/3 \times 10^7$, or 0.00053 in./in. The total elongation is 12×0.00053, or 0.00636 in., and the new length is 12.00636 in. The value of K, Eq. 2-8, is 0.3, and thus E_T and the corresponding d_T will be 3/10 of the values of E and d. Therefore d_T is equal to 0.3×0.00053, or 0.000159 in./in. Since both transverse dimensions are reduced the new thickness becomes 0.999841 in. and the width becomes $5 - 5 \times 0.000159$, or 4.999205 in. The lateral stresses in each direction x and y are equal and are

$$S = (1.59 \times 10^{-4})(30 \times 10^6) = 4770 \text{ psi. in compression}$$

TABLE 2-2.—AVERAGE PHYSICAL PROPERTIES TABLE FOR STEELS

These are average results compiled from many tests. They are offered as a general guide to the probable physical expectancy of the steels listed.

SAE	AISI	Condition of Steel	Tensile Strength psi.	Yield Point psi.	% Elong. in 2 in.	% Red. of Area	Hardness Brinell	Hardness Rock-well "C"	% Machin-ability Rating
Mild Steel		Natural hot rolled	57,000	32,000	37	66	113	..	..
		Cold drawn	67,000	42,000	30	60	149	..	50
1015	1017	Natural hot rolled	65,000	40,000	32	65	137	..	50
		Cold drawn	67,000	43,000	30	62	143	..	50
1020	1017	Natural hot rolled	67,000	45,000	32	65	137	..	52
		Cold drawn	69,000	48,000	30	63	143	..	60
1025	1023	Natural hot rolled	70,000	41,000	31	58	130	..	58
		Cold drawn	80,000	67,000	18	48	162	..	65
1040	1042	Natural hot rolled	93,000	58,000	27	52	190	9	60
		Cold drawn	100,000	64,000	22	46	221	19	62
		1" Rd. water quenched 1525° F. Drawn 1000° F.	110,000	84,000	21	58	235	21	..
1095	1095	Hot rolled, annl'd	106,000	60,000	23	47	201	12	45
		1" Rd. oil quenched 1450° F. Drawn 1000° F.	178,000	122,000	12	37	363	39	..
B's'mr. 1112	1112	Natural hot rolled	67,000	40,000	27	47	140	..	..
		Cold drawn	80,000	62,500	16	43	170	6	100
B's'mr. X112	1113	Cold drawn	83,000	73,000	15	45	180	8	120 140
1115	1120	Natural hot rolled	69,000	36,000	32	55	117	..	..
		Cold drawn	80,000	62,500	20	50	170	4	80

TABLE 2-2.—(*Continued*)

These are average results compiled from many tests. They are offered as a general guide to the probable physical expectancy of the steels listed.

SAE	AISI	Condition of Steel	Tensile Strength psi.	Yield Point psi.	% Elong. in 2 in.	% Red. of Area	Hardness		% Machinability Rating
							Brinell	Rockwell "C"	
2315	2317	Natural hot rolled	85,000	56,000	29	60	163	..	50
		Cold drawn	95,000	75,000	25	58	197	12	..
		1" Rd. carburized 8 hrs. at 1700° F., cooled in box, reheated to 1500° F., oil quenched—core properties	168,000	122,000	14	40	363	38	..
2340	2340	Natural hot rolled	110,000	80,000	22	47	225	19	40
		An'ld and cold dr.	115,000	90,000	21	48	235	21	..
		1" Rd. oil quenched	1475° F.						
		Drawn 1000° F.	137,000	119,000	21	59	302	31	35
3140	3140	Hot rolled, an'ld	96,000	64,000	26	56	195	12	57
		An'ld and cold dr.	115,000	98,000	17	45	248	24	..
		1" Rd. oil quenched	1525° F.						
		Drawn 1000° F.	147,000	123,000	18	57	302	31	38
3250	.....	Hot rolled, an'ld	107,000	75,000	24	55	217	18	44
		An'ld and cold dr.	117,000	98,000	17	43	255	25	..
		1" Rd. oil quenched	1450° F.						
		Drawn 1000° F.	173,000	154,000	17	53	363	38	..
4150	.....	Hot rolled, an'ld	105,000	71,000	21	54	220	20	54
		An'ld and cold dr.	124,000	100,000	16	48	269	27	..
		1" Rd. oil quenched	1550° F.						
		Drawn 1000° F.	175,000	153,000	15	50	375	40	..

TABLE 2-2.—(*Continued*)

These are average results compiled from many tests. They are offered as a general guide to the probable physical expectancy of the steels listed.

SAE	AISI	Condition of Steel	Tensile Strength psi.	Yield Point psi.	% Elong. in 2 in.	% Red. of Area	Hardness		% Machinability Rating
							Brinell	Rock-well "C"	
4615	4615	Natural hot rolled	82,000	55,000	30	61	167	4	58
		Cold drawn	98,000	70,000	18	55	203	14	..
		1" Rd. carburized 8 hrs. at 1700° F., cooled in box, reheated to 1475° F., oil quenched—core properties	151,000	125,000	15	44	321	34	..
4640	4640	Hot rolled, an'ld	100,000	87,000	21	50	201	12	60
		An'ld and cold dr.	126,000	97,000	14	39	269	27	..
		1" Rd. oil quenched Drawn 1000° F.	1500° F. 161,000	145,000	17	54	341	36	..
52100	52100	Hot rolled, an'ld	109,000	80,000	25	57	235	22	45
		1" Rd. oil quenched Drawn 1000° F.	1550° F. 185,000	170,000	9	34	415	43	..
6150	6152	Hot rolled, an'ld	103,000	70,000	27	51	217	18	50
		An'ld and cold dr.	118,000	94,000	20	43	255	25	..
		1" Rd. oil quenched Drawn 1000° F.	1575° F. 179,000	160,000	15	49	388	41	..
9255	9255	Hot rolled	135,000	90,000	19	40	269	27	..
		1" Rd. oil quenched Drawn 1000° F.	1650° F. 182,000	160,000	15	32	363	39	..

2-7. Compression. Fig. 2-2A illustrates the forces applied in compression: Fig. 2-2B shows the bulging effect and type of failure on the ductile material; Fig. 2-2C illustrates the type of fracture resulting from compression of brittle material. The compressive strength of metals is closely allied to their tensile strengths. The results of

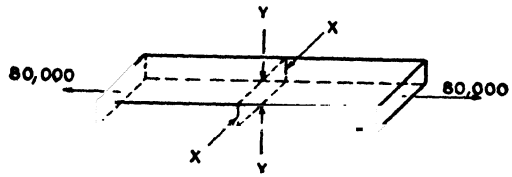


FIG. 2-10. Compressions Resulting from Tensile Forces.



FIG. 2-11. Column

compression tests can be plotted in the same way as the stress-strain diagrams for tension tests. Most of the properties revealed under tension are the same as for compression. For most structural materials the values of yield point, elasticity, and ultimate strength are identical for tension and compression. For materials, such as cast iron and concrete, however, this is not the case; for example, concrete has an ultimate compressive strength of about 10 times its ultimate tensile strength. The relations summarized in Eq. 2-1, 2-2, and 2-3 apply equally for tension and compression (see Example 2-4). The behavior of ductile materials under compression is often due not only to true compressive stresses but also to bending or buckling stresses, as illustrated in Fig. 2-11. Buckling tendency increases as the square of the length-area ratio increases. At high ratios the bending or flexural stress predominates and is the controlling stress. In compression loads where the sides of the material are confined, the stresses differ from those resulting from simple compression. Thus for ductile metals in compression a yield point equal to that in tension may be used, but the increase in lateral dimensions makes it impossible to predict the ultimate compressive strength, since the material may not break but flow to a flat disk. For brittle materials in compression it is usually impossible to determine the yield point, but a fracture type of failure will result, and then it is possible to determine the ultimate strength with fair accuracy.

Tests for compressive strength are made with testing machines and equipment such as those used for tensile tests. In the tests for compressive strength the loading is in the reverse direction and the test specimen should be cylindrical with a height equal to about two diameters. The ends of the cylinder should be carefully prepared to insure parallel and smooth surfaces. For concrete and

stone the length is usually three times the minimum lateral dimension. If the length of the specimen is too short, frictional forces interfere and modify test results. If too long, bending occurs and buckling failure results. For detailed directions for a compression test see ref. 8.

2-8. Flexure. Properties obtaining during the bending of a material are determined by a bending or flexure test, illustrated in Fig. 2-4. Here the deflections at mid-span are measured on a specimen supported only at the ends and having a concentrated load applied at the center. Load-deflection data are visualized by plotting as shown in Fig. 2-12 (compare with curve *B* in Fig. 2-6). From such data the modulus of elasticity, the point of rupture, and the yield

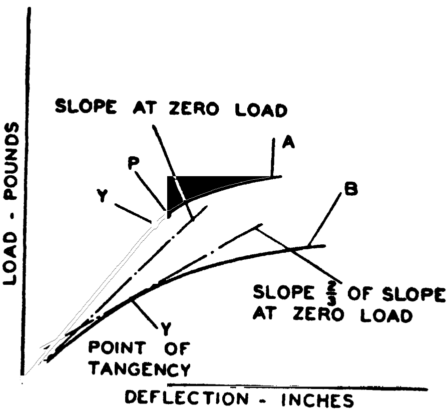


FIG. 2-12. Load-deflection Diagram.

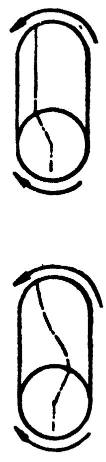


FIG. 2-13. Torsional Shear and Deflection.

point are obtained. The modulus of elasticity may be calculated on the assumption of Hooke's law. Such values are not exact and are usually lower than those obtained from tensile tests, because of the influence of shear at the loading points. However, the calculations are useful for comparing stiffness in bending of some materials.

With ductile materials having a linear load-deflection relation, the yield point can be found, as for example point *Y*, Fig. 2-12 curve *A*. When the load-deflection diagrams have curves deviating from a straight line (curve *B*), the method of Johnson (see Johnson's apparent elastic limit, section 2-3) may be used for finding the yield point. For bending, the load used is that for which the rate of change of deflection to load is 50% greater than at zero load. This results in point *Y*, Curve *B*, Fig. 2-12. Marin[37] has developed a relation between yield point in simple tension and that in bending, making use of the

offset-stress method. For details of bending test procedures, see ASTM Stds., 1939, Part II.

2-9. Shear and Torsion. Shearing forces, illustrated by Fig. 2-3, are such as to cause one section of material to slip over an adjacent section. Pure shear is encountered only in the case of twisting a bar of circular cross section about its axis (torsional shear), Fig. 2-13. Transverse shear unaccompanied by bending is almost never attained, but may be approximated in cases such as that of a rivet holding several plates from being pulled apart, as indicated in Fig. 3-8B.

Torsion tests are usually made on circular cross-section specimens to determine shear characteristics. They are more reliable than transverse shear tests, and are also more directly useful in many applications involving twist and torsion. The tests are made on torsion test machines under carefully prescribed conditions. The strength values obtained from torsion tests vary

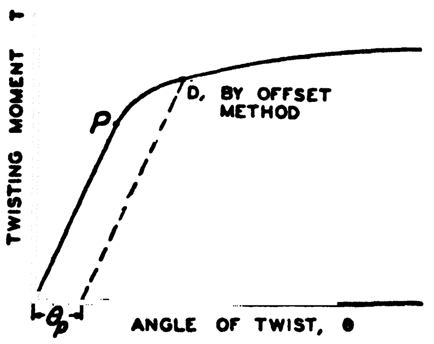


Fig. 2-14. Torque-twist Diagram.

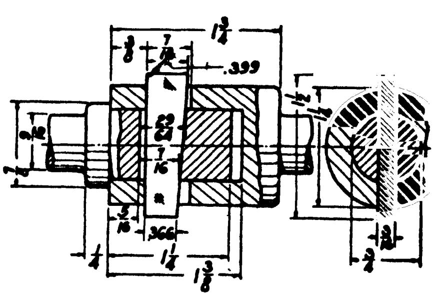


Fig. 2-15. Cottered Joint.

with the size and shape of the test piece. Since the maximum stresses occur on the outer parts of the metal during torsion, it is best to correlate torsion and tensile results for thin-walled cylinders, although hollow cylinders give lower values than solid cylinders. Solid cylinders can be used for comparative test purposes, but the values calculated will be somewhat lower than the true moduli because of the strengthening effect of inner sections of metal, since these sections are under less stress than the outer layers for which E more nearly applies.

Properties may be evaluated from torsion tests in a manner similar to that for tension tests. In a torsion test increments of the twisting effort or torque T are measured, and corresponding values of the angle of twist are observed for a given gage length L. These data are plotted to make a torque-twist diagram, Fig. 2-14. For materials following Hooke's law, as indicated by the uniform slope of the first part of the curve of Fig. 2-14, Eq. 16-2 (Chap. 16) may be used to calculate the yield-point stress in torsion by substituting the

yield-point torque for R. If the maximum torque is known, it may be substituted in the equation for T, and the value of S thus calculated is called the modulus of rupture, or ultimate strength in torsion. This value is not the actual stress, however, since Eq. 16-2 is based upon Hooke's law, which does not apply beyond the proportional limit. In general, for ductile materials the ultimate strength in shear is approximately ¾ of the strength in tension. Thus the shear stress is often the more important one to take into consideration in design. With a brittle material like cast iron, the shearing strength may be equal to the tensile strength or even up to 15% higher.

Stiffness in torsion is measured by the modulus of elasticity in shear E_s. The value of E_s can be calculated from the diagram or from its relation to E (the modulus under tension) and Poisson's constant K, which is

$$E_s = \frac{E}{2(1+K)} \tag{2-9}$$

For metals the modulus of elasticity in shear is about 40% of the value of the modulus of elasticity in tension or compression. For wood and concrete the modulus varies with the direction of the cross section. Values of moduli in shear are given in Table 2-1.

2-10. Stress Analysis. When a piece of equipment is designed, it should be studied to find the magnitude of various stresses at all points of contact of parts and for all connecting material. Such a study involves stress calculations and is known as a stress analysis. By means of it the weaker parts of the equipment may be located, and if stresses are found to be higher than working stresses, redesign can be made accordingly.

Fig. 2-15 shows a cottered joint used as a connection in a slow-speed, reciprocating steam engine. The end of the rod at the left has a slotted hole and a collar and is inserted into the socket attached to the right-hand rod. A tapered key or cotter is driven through a slot in the socket and rod as shown. Clearance is allowed in the socket and end slots so that the cotter may be driven in further if the joint becomes loose. The following example illustrates the various types of failure by simple stresses by making an analysis of such a joint.

Example 2-5. Assume that the cottered joint of Fig. 2-15 is subjected to a reversing load of 2000 lbs., that the cotter remains tight, and that no bending or torsion is present. Make a stress analysis.

Solution. Fig. 2-16 shows the various types of failures possible. Parts of the solution are lettered in accordance with the figure letters.

A. Failure by tension in the rod,

$$\text{Rod area} = \frac{\pi \times 0.563^2}{4} = 0.249 \text{ sq. in.}$$

$$S = \frac{2000}{0.249} = 8030 \text{ psi.}$$

B. Failure by compression in the rod. Same as *A*.

C. Failure by tension in the slotted portion of the rod,

$$\text{Area in tension} = \frac{\pi \times 0.75^2}{4} - 0.75 \times 0.188 = 0.301 \text{ sq. in.}$$

$$S = \frac{2000}{0.301} = 6660 \text{ psi.}$$

D. Failure by tension in the slotted portion of the socket,

$$\text{Area in tension} = \frac{\pi(1.25^2 - 0.75^2)}{4} - 0.188(1.25 - 0.75) = 0.691 \text{ sq. in}$$

$$S = \frac{2000}{0.691} = 2900 \text{ psi.}$$

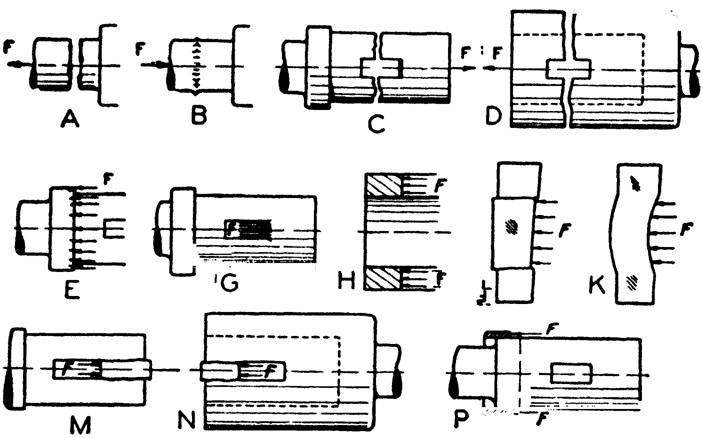


FIG. 2-16. Analysis of Stresses in Cottered Joint.

E. Failure by compression of the bearing surface of the rod collar,

$$\text{Area in compression} = \frac{\pi(0.88^2 - 0.75^2)}{4} = 0.159 \text{ sq. in.}$$

$$S = \frac{2000}{0.159} = 12{,}600 \text{ psi.}$$

G. Failure by compression of the bearing surface of the slot in the rod,

$$\text{Area in compression} = 0.75 \times 0.188 = 0.141 \text{ sq. in.}$$

$$S = \frac{2000}{0.141} = 14{,}200 \text{ psi.}$$

H. Failure by compression of the bearing surface of the slot in the socket,

$$\text{Area in compression} = (1.25 - 0.75)0.188 = 0.094 \text{ sq. in.}$$

$$S = \frac{2000}{0.094} = 21{,}300 \text{ psi.}$$

J. Failure by shear in the cotter,

$$\text{Area in shear} = \text{upper plus lower area}$$

$$\text{Upper area} = 0.188\left(0.399 - \frac{0.399 - 0.366}{4}\right) = 0.0733$$

$$\text{Lower area} = 0.188\left(0.399 - \frac{0.399 - 0.366}{4}\right) = 0.0703$$

$$\text{Total area} = 0.0733 + 0.0703 = 0.1436 \text{ sq. in.}$$

$$S = \frac{2000}{0.1436} = 13{,}900 \text{ psi.}$$

M. Failure by shearing out the end of the rod,

$$\text{Area in shear} = \frac{0.75(0.5 + 0.484)2}{2} = 0.762 \text{ sq. in.}$$

$$S = \frac{2000}{0.762} = 2620 \text{ psi.}$$

N. Failure by shearing out the end of the socket,

$$\text{Area in shear} = 2 \times 0.375(1.25 - 0.75) = 0.375 \text{ sq. in.}$$

$$S = \frac{2000}{0.188} = 10{,}600 \text{ psi.}$$

P. Failure by shear of collar on rod,

$$\text{Area in shear} = \pi \times 0.25 \times 0.75 = 0.589 \text{ sq. in.}$$

$$S = \frac{2000}{0.589} = 3400 \text{ psi.}$$

K. Failure by flexure, or beam action of the cotter, and failure by buckling of the rod are not simple stresses and will be considered in later chapters.

2-11. Impact. The ability of a material to resist suddenly applied, or impact, loads is of considerable importance both in machine and structural parts, and as a test for the physical metallurgical properties. Impact loads produce greater stresses than steady loads and result in different magnitudes of deformations for the force applied. Less deformation at rupture results, and the total energy for rupture may be considerably less, when impact is concentrated over a small area. The reduction in deformation accompanying impact will result in a reduction of ductility and in increased brittleness.

Impact tests are made solely for purposes of utility and comparison. They are used to detect the degree of brittleness or toughness of materials for use as rails, pipes, tubes, gears, etc. Such tests are made on wood, concrete, plastics, metals, and other materials on machines especially developed for the particular utility involved. Impact testing is also a simple and useful method of checking heat treatment of alloys, and as an overall check on whether or not the alloy is metallurgically satisfactory.

The so-called Standard Impact Tests [8,25] are made with well recognized machines, and the tests are performed under exact and prescribed conditions. The Charpy and Izod tests measure the foot-pounds of work necessary to fracture

a small specimen by impact. A weighted pendulum is used, raised to position, and released to strike the sample. The instrument is calibrated to the angle from which the pendulum arm is released. Specimens are usually notched to produce localization of fracture and uniformity of results.

2-12. Fatigue. A material subjected to repeated loads (in the order of millions), even though stressed below the elastic limit, may fail by rupturing. Failure from the repeated application of a load, whether the load acts in the same direction each time or is reversed, is called fatigue failure. Fatigue failure starts by a very gradual spreading of minute cracks and, when the effective area resisting the load is sufficiently reduced, rupture will occur suddenly without warning. Fatigue strength, or endurance limit, is in the neighborhood of one half or less of the ultimate strength, and theoretically is the stress at which fracture cannot be produced except at an infinite number of cycles. A general summary of the relation between tensile strength and fatigue strength is shown in Table 2-3.

TABLE 2-3.—PERCENTAGE REDUCTION OF TENSILE STRENGTH IN FATIGUE

Material	Nature of Stress % Reduction of Tensile Strength	
	Released	Reversed
Ductile	30 to 50	67
Brittle	60	75

Two types of repeated loadings are common, one giving a "released stress" and one producing a "reversed stress." A released stress is produced when the stress passes from zero to full load and returns to zero, completing one cycle without stress in the opposite direction. A reversed stress occurs when the stress varies between equal values of tension and compression, or has complete reversal of load in one cycle. This latter is the type of cycle most often used in fatigue testing.

One of the commonest methods used to show the endurance limit from test data is to plot the stress imposed against the number of cycles of stress required to cause rupture at that stress. Most plots of this nature on a logarithmic scale, Fig. 2-17, show a definite horizontal part. The point of transition of the curve to horizontal is the endurance limit. The figure shows that for untempered steel a stress of 74,000 psi. can be endured for an indefinite number of cycles (at least 10^7 as plotted), whereas if this steel were tempered at 800° F., a stress of 99,000 psi. could be endured for an indefinite number of cycles. Most metals have a fairly definite endurance limit, but some metals, notably duralumin, have curves that do not become horizontal, and thus the endurance limit is variable and must be carefully considered.[1]

Non-uniformity of material, method and rate and range of stress application, size of part, character of surface (polished, scratched, sharp projection), heat treatment, etc., will greatly influence the fatigue strength. There are many types of fatigue testing machines operating on the basis of alternating loads in tension and compression, and others of a rotating nature where bending loads are applied.

2-13. Stress Concentration. In all of the foregoing discussion it has been assumed that the stressed members have been of constant or gradually changing cross section. When there are notches, holes, or sharp angles, local stresses develop which are frequently much in excess of the average stresses. The stresses obtaining at sudden changes in cross section are not subject to analysis by ordinary methods. Often, the actual stresses caused by concentration are

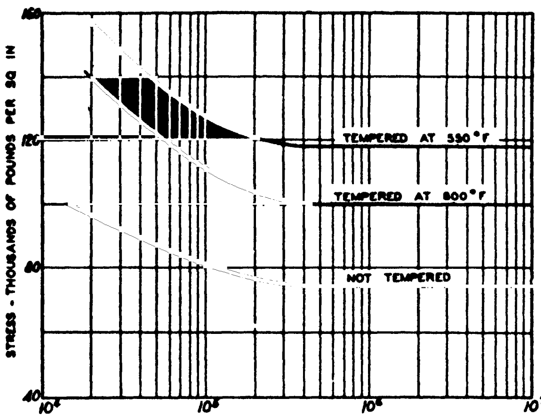


Fig. 2-17. Fatigue Resistance Data for an SAE 9387 Steel, Quenched at 1400° F. in Water, Tempered as Indicated

much higher than values computed by the areas involved. Occasionally, however, the reduction in strength due to stress concentration is not as serious as might be supposed. Ductile materials under steady load can adjust themselves in overstressed areas so that the stress is transferred to adjacent areas. This results in a minimizing of the deleterious effect of stress concentration. With brittle materials, on the other hand, stress concentration is always serious. Stress concentrations are especially serious in members subjected to fluctuating loads. and ductile materials under varying stresses often are subject to rapid deterioration when stress concentrations are present.

Many factors influence the magnitude of the stress concentration effect in the case of fatigue. The material, grain size, shape, indentation and projections, sharp corners or edges, internal flows, ultimate strength, size of specimen, and magnitude of load fluctuations all have been found to be of considerable importance.

2-14. Designing for Stress Concentration. In the case of static loads, the values of maximum stress at changes of cross-sectional areas can be determined experimentally or theoretically and may be symbolized by S_t. If then the average stress S is computed by Eq. 2-1 for the cross-sectional area in question, the ratio S_t/S is used to define the theoretical stress concentration factor k_t,

or
$$k_t = \frac{S_t}{S} \tag{2-10}$$

Most design cases of importance involving stress concentration have to do with alternating loads, and here the theoretical stress concentration factor is of little use, since no method is known for predicting S_t. It becomes necessary to make fatigue tests on specimens with and without discontinuities to determine actual stresses. With these data, S, the stress at the endurance limit for the specimen without stress concentration, and S_s, the stress at the endurance limit for the specimen having stress concentration, the term fatigue stress concentration factor k_f can be determined by

$$k_f = \frac{S}{S_s} \tag{2-11}$$

This factor is always 1 or greater. Many shapes have been investigated in this fashion and a series of k_f values are available in the literature for shafts with keyways, plates with holes, screw threads, and other common forms.

Sometimes equipment parts may absorb alternating loads and yet not be subject to varying stress. Bolts and tie rods are usually pulled up so tightly that they are under a uniform stress much greater than that which is caused by the alternating load. When this is the case the steady static load controls and the alternating load does not develop the fatigue stress concentration effect. When functionally practical, the best way to offset the harmful effects of stress concentrations is to provide interior fillets with generous radii and to avoid sharp corners and abrupt changes of section.

2-15. Temperature Stresses. When two materials having different coefficients of expansion are bound together, stresses will develop in the two materials when the temperature is changed. This is due to the restraining action of the low coefficient material tending to withhold expansion of the other. The stress induced is directly proportional to the modulus of elasticity E, the coefficient of expansion a, and the temperature difference $t_2 - t_1$. For a material firmly held to an original length (corresponding to t_1), the stress induced by a temperature change will be

$$S = Ea(t_2 - t_1) \tag{2-12}$$

These stresses may be relieved in the case of pipe lines by providing expansion joints, bends, sleeves, or similar devices, all of which provide the system with a yielding or flexible member.

2-16. Creep. Creep is the slow flow that takes place in solid bodies under sustained loads (in the elastic range) at elevated temperatures. Creep is not

to be confused with the normal decrease in yield point with increasing temperature, which phenomenon is characteristic of practically all substances. Some metals like lead and zinc will creep at normal temperatures at low stress. Lead sheets, for example, will flow gradually from the effect of its own weight at normal temperature. Continuing creep will result in distortion, and failure will occur. The whole body is affected, and failure will occur over a wide area, as distinguished from the localized type of rupture in fatigue. Materials possessing superior creep resistance are essential in modern petroleum, synthetic ammonia, and other catalytic synthetic organic processes, as well as high pressure power mechanisms (boilers, turbines, etc.). The working stress must be of such value that the creep deformation will be less than that estimated for the useful life span of the equipment.

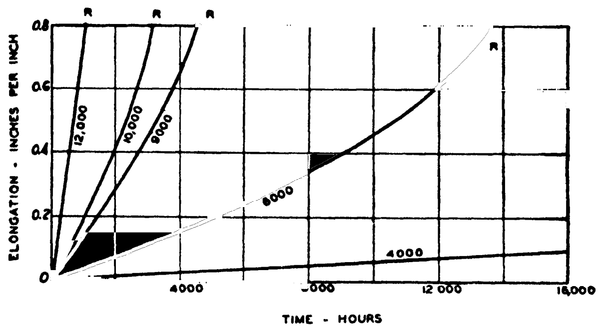


FIG. 2-18. Typical Time-elongation Curve for a Carbon Steel. Temperature Constant at 1000° F., Stressed as Indicated.

Four variables are involved in creep: stress, deformation, temperature, and time. Creep test data are often correlated by time-elongation curves as shown in Fig. 2-18. *R* indicates the rupture point. The slope of the curve is a measure of the rate of flow, or creep. There are three rather well defined stages in the rate of creep. In the first stage, a very short relative time, the creep rate decreases; in the second stage the rate is practically constant (as shown in the straight line portion of the curve); while in the third stage the rate increases rapidly. Increase of stress increases the elongation and the rate of creep. The time periods of the three stages decrease with increasing stress, but the fracture will not necessarily occur soon after the third stage of creep is entered. For the steel shown in Fig. 2-18, the loads of 6000 psi. and greater all show rapid creep for various intervals before rupture, but the 4000 psi. stress was still in the second stage of creep after 16,000 hours. To visualize the resistance of this metal to creep at 6000 psi. load, consider that failure did not occur at 1000° F. until almost 14,000 hours, or 19 months, and after 11% elongation; whereas failure occurred with 9000 psi. at 6½ months after 13% elongation.

The creep limit is the allowable stress in a metal when subjected to a given temperature, which will keep the deformation within a prescribed limit for a

certain number of months or years. Since it is not practical to conduct creep tests over such long periods for various temperatures and pressures, creep limits are found by extrapolation of data of the second stage of creep. Accurate creep data must be obtained from recent literature [10] and manufacturers' publications. Creep limits are, of course, covered by various structural codes, but in the past few years great advances have been made in the metallurgy of creep resistant steels even in corrosive and embrittling atmospheres. Thus, for design of equipment for the newer high-pressure, high-temperature processes, stresses are possible and practical by the use of newer materials that would be impossible under the restrictions of the well established design codes.

2-17. Warning. The strength and mechanical properties of materials depend on the mechanics of the atoms and crystals and on their interactions, from which we deduce cause and effect. Modern physics and thermodynamics indicate that the interrelation of forces and action can only be predicted on the basis of probability. Probability must be considered in terms of large numbers of atoms which then produce overall and average effects—the extensive properties of thermodynamics, for example. It must be borne in mind, therefore, that the application of quantitative properties, which are based upon probabilities, to an individual case may differ greatly from actual values. If too small a portion of a material is considered, for example the material at or very near a sharp edge or notch, the probable stress calculated from strength formulae may deviate appreciably from the actual stress. When a piece of material is taken which is large enough so that it contains an infinitely large number of atoms, then the most probable average or statistical value as calculated should approach the actual value.

It is for such considerations, as well as the probability that variations in homogeneity, elasticity, unpredictable effects of stresses set up in fabrication, and uncertainties in test values, that code values of allowable working stresses play so important a role in the design of a particular piece of equipment. The codes make use of factors of safety and, in effect, the factor of safety (or its reciprocal, the factor of utilization) is a moduli of statistical approximation.

PROBLEMS—CHAPTER 2

1. A bar 0.658 in. in diameter was tested in a tension test machine, giving the following results. The deformation was measured along a 4-in. length.

Load, Pounds	Elongation, Inches	Load, Pounds	Elongation, Inches
2,040	0.0008	11,080	0.0058
4,080	0.0016	12,200	0.0068
9,180	0.0036	13,540	0.0088
9,550	0.0039	14,120	0.0112
10,150	0.0044	11,960	0.0126
10,170	0.0050		

Plot the stress-strain diagram, and determine the following, marking them on the curve where possible: (a) yield point, (b) proportional limit, (c) ultimate strength, (d) breaking strength, (e) modulus of elasticity, (f) modulus of resilience.

2. A cylindrical bar ½ in. in diameter is held at its upper end. The bar is made of steel weighing 490 lbs. per cu. ft., and has a breaking strength of 76,000 psi. How long must the bar be for it to fail because of its own weight?

3. Two plates 3½ in. wide and ½ in. thick are lapped over 1 in., and are connected by two ⅝-in. diameter rivets 1¾ in. apart, and in alignment. If the plates are subjected to a pull of 5000 lbs., describe the possible modes of failure, and find the resultant stresses.

4. Find the necessary rivet diameter for the plates and joint of problem 3 if the strength of the plate at the net section is to be equal to the shearing strength of the rivets. Assume the tensile strength of the plate equal to 125% of the shearing strength of the rivets.

5. A 1½-in. diameter, SAE 1025, steel bar 8 in. long is subjected to a tensile load of 20,000 lbs. What is the apparent factor of safety, based upon: (a) the ultimate strength? (b) the yield point?

6. A 2-in. diameter steel bolt made of SAE 4615 steel is used to reinforce a frame for a high temperature reactor. The bolt is inserted, heated to a temperature of 500° F., and the nut set up snugly. If the length between the nut and head is 15 in., what unit stress is induced in the bolt after it cools to a room temperature of 70° F.?

7. A 1¼-in. diameter steel rod broke under a tensile load of 77,500 lbs. Estimate the approximate carbon content of the steel.

8. A short, hollow cylinder made of SAE 1040 steel, with an inner diameter equal to 75% of the outer diameter, is to support a load of 5 tons. What should be the outer diameter if the factor of safety is 2, based upon the yield point?

9. A bar of rectangular section is made of AISI 9255 steel, and is subjected to a tensile load of 8 tons. If the width of the bar is 1⅓ times the thickness, find the bar dimensions if the factor of utilization is 0.25, based upon the ultimate strength.

CHAPTER 3

RIVETED PRESSURE VESSELS

3-1. Pressure Vessel Types. Pressure vessels are containers for fluids subjected to pressure. Vessels, such as stills, autoclaves, and the like, may be fabricated in one piece or made of separate plates formed to shape and riveted or welded at the seams. Pressure vessels of spherical form permit the greatest volume for a given enveloping surface and are uniformly stressed in all directions, thereby affording the most economical utilization of material. Although they may be employed as containers for gases and volatile liquids, they are not in general use. Vessels of cylindrical form, with spherical, semi-spherical, or flat heads at each end, are preferred for liquids and gases because the plates may be more easily preformed, and because such vessels are easier to support than spherical vessels.

Two types of fastenings are employed in engineering construction—removable and permanent. Removable fastenings are those in which repeated assembly and disassembly are possible without injury either to the parts held together or to the fastening. Screws, bolts, keys, and dowels are examples of removable fastenings. Permanent fastenings are those in which either the fastening or the parts themselves must be destroyed in taking the device apart. Riveted, welded, soldered, and brazed joints are examples of permanent fastenings.

3-2. Riveted Fastenings. Riveting is a widely used method of constructing tanks, boilers, stills, and drums. A rivet is a cylindrical member, with one preformed head, inserted in coincident holes in two or more plates, holding them together by the pressure exerted between the preformed head and a fabricated head at the other end. The fabricated head may be formed, either hot or cold, by ordinary hammering, pneumatic hammering, or by a hydraulic pressure die. Rivets are made of soft steel or wrought iron and are hot-formed for all pressure vessel and structural applications. (Cold-formed rivets of soft iron, copper or brass are usually used for light machine construction.) Representative rivet head shapes are shown in Fig. 3-1. Rivet holes for pressure vessels are usually drilled to a diameter $\frac{1}{16}$ in. greater than the size of the rivet. Punched holes are forbidden by the standard pressure vessel construction codes unless the brittle portion of the plate surrounding the punched hole is removed by reaming after the punching operation.

Typical joints for the seams of cylindrical pressure vessels are shown in Figs. 3-2 to 3-6, which illustrate a portion of the longitudinal or axial joint at its juncture with the single-riveted lap joint G employed for the girth joint. S is the shell or vessel plate, and O the outer and N the inner butt straps.

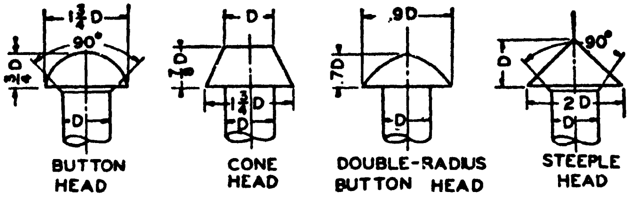


FIG. 3-1. Proportions of Rivet Heads for Pressure Vessels.

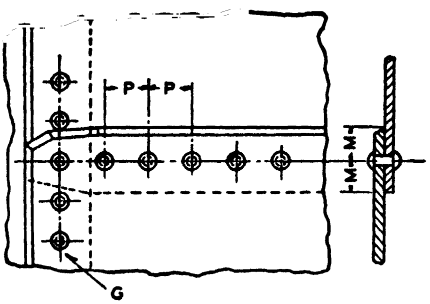


FIG. 3-2. Single-riveted Lap Joint—Longitudinal.

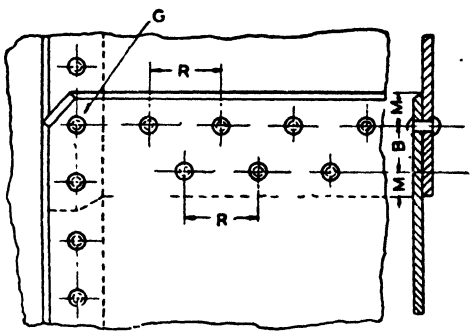


FIG. 3-3. Double-riveted Lap Joint—Longitudinal.

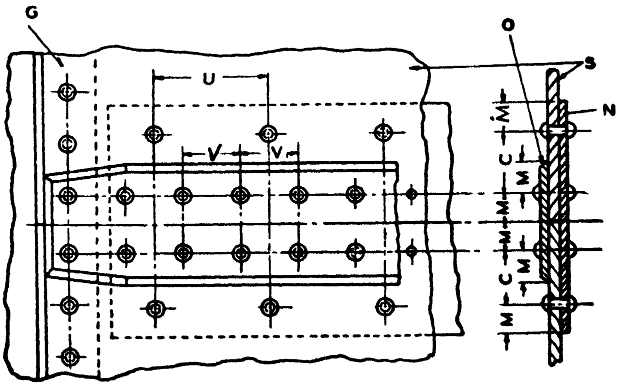


FIG. 3-4. Double-riveted Butt Joint—Longitudinal.

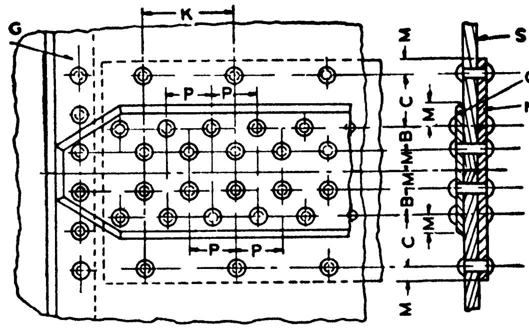


FIG. 3-5. Triple-riveted Butt Joint—Longitudinal.

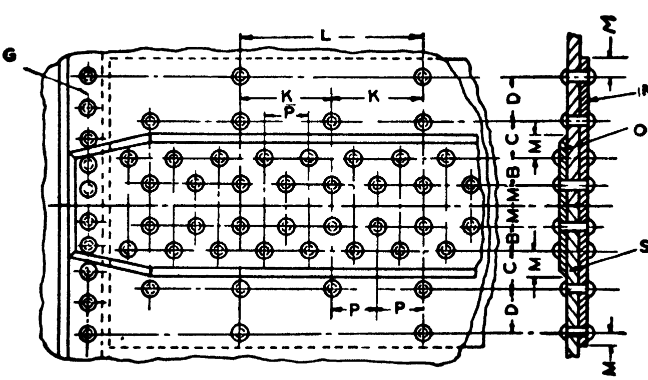


FIG. 3-6 Quadruple-riveted Butt Joint—Longitudinal.

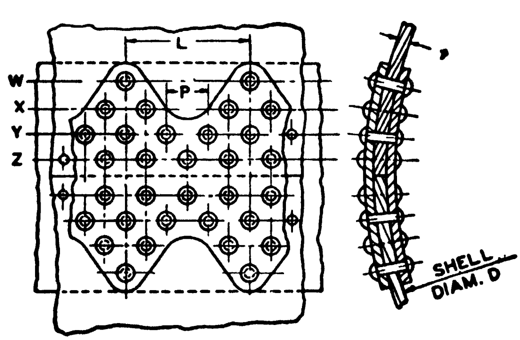


FIG. 3-7. Quadruple-riveted, Two Strap Butt Joint with Scalloped Outer Strap.

The pitch of a riveted joint is the distance between centerlines of adjacent rivets in rows parallel to the seam, and is represented by P and R for the longitudinal joints of Figs. 3-2 and 3-3. In Fig. 3-4 a pitch V and a long pitch U are used, while in Fig. 3-6 a pitch P (sometimes referred to as a short pitch), an intermediate pitch K, and a long pitch L are involved. (In this joint, $L = 2K = 4P$.) The distance between adjacent rows of rivets, B or C, is termed the back pitch, and the distance from a row of rivets to the edge of the plate or strap, M, is termed the marginal pitch.

3-3. Calking and Sealing. Riveted joints are ordinarily sealed against leakage by calking, where the beveled edge of one plate is driven into close contact with the plate beneath by hammering with a blunt or round tool. To make calking effective the rivets at the calking edge must be spaced sufficiently close to prevent the plate from springing out of contact. (One practical rule for a satisfactory pitch at the calking edge is that it shall be not more than eight times the thickness of the calked plate.) In Figs. 3-4, 3-5, and 3-6 the outer butt strap is of smaller width than the inner to permit utilization of the shorter pitch of the second row of rivets for holding the strap in calking. The joint illustrated in Fig. 3-7 is employed for vessels subjected to high pressures; the scalloped outer butt strap, although more expensive than the plain strap shown in Fig. 3-6, permits the utilization of all the rivets and can be effectively calked because of the comparatively short pitch of the rivets adjacent to the scalloped calking edge. In some vessels calking is accomplished by depositing a bead of welding material along the edge to be sealed. This process is termed seal welding; no credit for weld strength is permitted.

3-4. Modes of Failure of Riveted Joints. Riveted joints may fail in several ways, as shown in Fig. 3-8. A rivet may have a single area subjected to shear, as in Fig. 3-8A, termed *single shear,* or two (or more) areas subjected to shear, as at B, termed *double shear*. In practice, a properly headed rivet should hold the plates in such contact that the friction between them will prevent them from slipping, since the joint may leak if slipping occurs. Serious slippage does not occur if the joint is designed for adequate shearing strength, although the frictional resistance of the plates is disregarded in the computation. Lap joints, Figs. 3-8A and 3-8D, are subjected to non-collinear forces which form a couple and tend to bend the plate and the rivets, as shown in Fig. 3-8C, resulting in a tendency to split and tear out the plate, as shown at J. For this reason lap joints are rarely employed for longitudinal seams if the plate thickness exceeds ½ in. Lap joints are extensively employed, however, for girth seams, where the cylindrical form of the telescoping sections provides sufficient rigidity to eliminate this bending action.

Example 3-1. Analyze the stresses involved in a lap joint riveted as in Fig. 3-8A. The rivet diameter d is ½ in.; the plate width w is 2 in.; the plate thickness t is ¼ in.; and the marginal distance m is ¾ in. Assume the following ultimate stresses for the plate and rivet materials: S_t (tensile stress), 60,000 psi.; S_b or S_c (bearing or compressive stress), 90,000 psi.; S_s (shearing stress), 45,000 psi.

Solution. The joint may fail by one of the six ways illustrated in Figs. 3-8A and E to J. The force F required to cause failure in each case is found as follows:

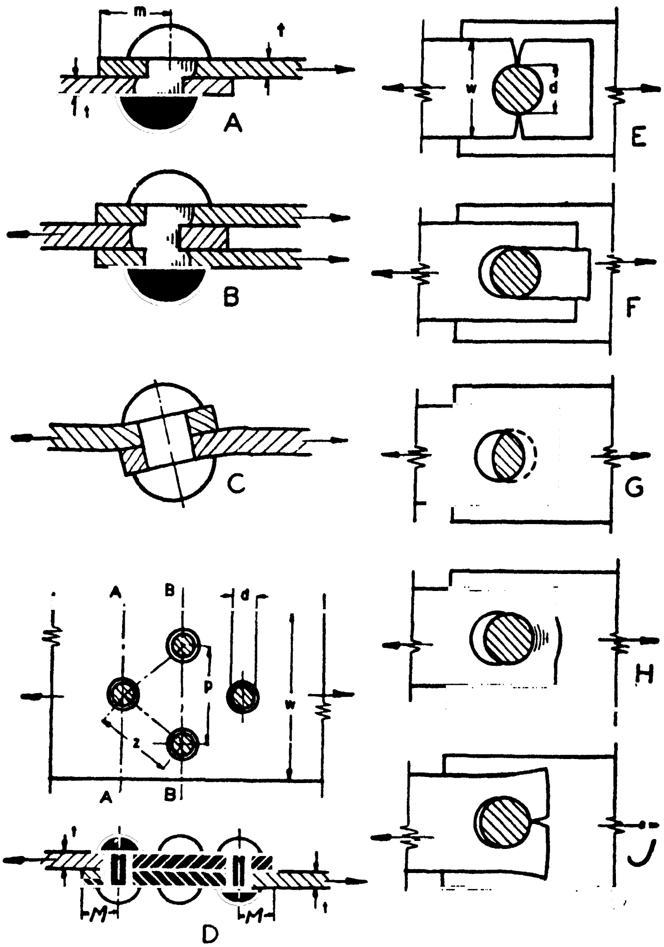


FIG. 3-8. Methods of failure of riveted joints.

Fig. 3-8A. Failure of rivet by single shear.

$$F = S_s \frac{\pi d^2}{4} = 45{,}000 \frac{\pi (0.50)^2}{4} = 8840 \text{ lbs.}$$

(In a butt joint, one half of which is shown in Fig. 3-8B, there would be two rivet areas in shear, with a consequent strength of 17,680 lbs.)

Fig. 3-8E. Failure by tearing of plate.

$$F = S_t t(w - d) = 60{,}000 \times 0.25 \times (2 - 0.50) = 22{,}500 \text{ lbs.}$$

Fig. 3-8F. Failure by shearing out end of plate.

$$F = S_s tm(2) = 45{,}000 \times 0.25 \times 0.75 \times 2 = 16{,}800 \text{ lbs.}$$

Figs. 3-8G and 3-8H. Failure by crushing rivet or plate.

$$F = S_c td = 90{,}000 \times 0.25 \times 0.50 = 11{,}200 \text{ lbs.}$$

(In this mode of failure, the "projected" area of the rivet in the plate, or the product td, is used.)

Fig. 3-8J. Failure by splitting, due to bending of plate.

The strength of the joint in this respect cannot be determined readily, and is usually disregarded; if failure by shearing out the end of the plate does not take place, the joint may be considered safe as far as splitting out the end is concerned.

3-5. Riveted Joint Efficiency. The efficiency of a riveted joint is a strength ratio of the weakest part of the joint and is obtained by dividing the load or force at which failure will occur by the force which would cause the solid plate to fail.

Example 3-2. What is the efficiency of the riveted joint of Example 3-1?

Solution. The strength of the solid plate is

$$F = S_t tw = 60{,}000 \times 0.25 \times 2.00 = 30{,}000 \text{ lbs.}$$

The individual efficiencies for each method of failure are:

$$\text{Shear of rivet} = \frac{8840}{30{,}000} = 0.295$$

$$\text{Tearing of plate} = \frac{22{,}500}{30{,}000} = 0.750$$

$$\text{Shearing out end} = \frac{16{,}800}{30{,}000} = 0.560$$

$$\text{Crushing of plate or rivet} = \frac{11{,}200}{30{,}000} = 0.373$$

The efficiency of the joint is the least of these, or 29.5%, and the joint may be expected to fail by shearing the rivet.

3-6. Vessel Joint Analysis. In pressure vessel joints of usual proportions, if the marginal distance m is not less than one and one half times the rivet diameter, the plate will be safe against both shearing and tearing by the rivet pressure, and the mode of failure illustrated in Fig 3-8F is usually disregarded. In addition to the modes of failure shown, however, failure of more complex joints may occur by a combination of stresses.

Consider a joint as illustrated by Fig. 3-8D. The upper plate may tear at either section AA or BB. A superficial consideration would give the impression that failure would occur at BB rather than AA, because the net area remaining in the plate after the rivet holes have been drilled is less at BB than at AA. A more careful examination will show that tearing at BB is impossible until the

rivet at section AA has failed by crushing or shearing. The most satisfactory design would call for a joint where the individual joint efficiencies were approximately equal, since the least efficient component part of the joint will determine the overall strength. To achieve this condition the forces required for failure by various methods can be equated. From this a rivet diameter could be found (providing the pitches were adjusted to fit) which would give equal joint efficiencies. However, since values of P and Z (Fig. 3-8D) are usually chosen at the beginning of the solution, it will be found that failure will occur most frequently by a combination of tearing of the plate and shearing or crushing of rivets. Therefore, it is more convenient to equate only those failure forces which involve both these mechanisms and thus solve for rivet size with fewer equations.

Example 3-3. Two plates, each $3 \times \frac{1}{2}$ in., are to be connected by four rivets, as shown in Fig. 3-8D. Using the ultimate stresses given in Example 3-1, determine the theoretical diameter of the rivet and the actual load at which the joint will fail if subjected to tension.

Solution. The strength of the solid plate, in tension, is

$$F_1 = S_t t w = 60{,}000 \times 0.50 \times 3 = 90{,}000 \text{ lbs.}$$

The shearing strength of the rivets is

$$F_2 = S_s \times 4\pi d^2/4 = 45{,}000\pi d^2 = 141{,}200\ d^2$$

The crushing strength of the rivets and the plates is

$$F_3 = S_c 4td = 90{,}000 \times 4 \times 0.5\ d = 180{,}000\ d$$

The tearing strength of the plate at section AA is

$$F_4 = S_t t(w - d) = 60{,}000 \times 0.5 \times (3 - d) = 90{,}000 - 30{,}000\ d$$

The tearing strength of the plate at section BB, plus the shearing strength of one rivet at section AA, is

$$F_5 = S_t t(w - 2d) + S_s \pi d^2/4 = 90{,}000 - 60{,}000\ d + 35{,}300\ d^2$$

The tearing strength of the plate at section BB, plus the crushing strength of one rivet at section AA, is

$$F_6 = S_t t(w - 2d) + S_c td = 90{,}000 - 60{,}000\ d + 90{,}000 \times 0.5\ d = 90{,}000 - 15{,}000\ d$$

The lowest strength will probably correspond to F_4 and F_5; therefore, equating them

$$F_4 = F_5 = 90{,}000 - 30{,}000\ d = 90{,}000 - 60{,}000\ d + 35{,}300\ d^2$$

from which d is equal to 0.85. Substituting this value in the strength equations,

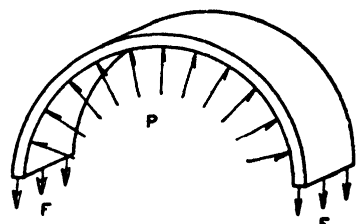
$$\begin{aligned} F_2 &= 102{,}000 \text{ lbs.} \\ F_3 &= 153{,}000 \text{ lbs.} \\ F_4 &= 64{,}500 \text{ lbs.} \\ F_5 &= 64{,}500 \text{ lbs.} \\ F_6 &= 77{,}250 \text{ lbs.} \end{aligned}$$

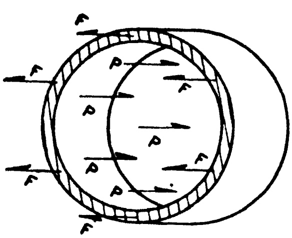
The strengths F_t and F_s limit the capacity of the joint. The efficiency of the joint is the ratio F_t/F_1 or 64,500/90,000 or 0.717, or 71.7%.

Another mode of failure of this joint, not considered in the preceding analysis, is the possibility of the plate tearing along the line of the diagonal pitch. To guard against this condition, however, it is only necessary that the distance $2\ (Z - d)$ be equal to or greater than the distance $(P - d)$. This may be accomplished by making the back pitch (distance from section AA to BB) sufficient size, say $2d$.

3-7. Rivets in Tension. Rivets were at one time considered unreliable for use as tension members, but comparatively recent tests indicate that they may be used satisfactorily in this manner and may be designed for the full working strength of the rivet material. Further discussion of these applications may be found in Chapter 6.

3-8. Stresses in Thin Walled Cylindrical Vessels. In vessels of usual proportions the stress induced in the shell by the internal pressure may be considered as uniform across the wall section, since the shell thickness is small compared to the diameter. The expression for the wall stress is found by equating the force tending to rupture the shell to the resistance offered by the wall.





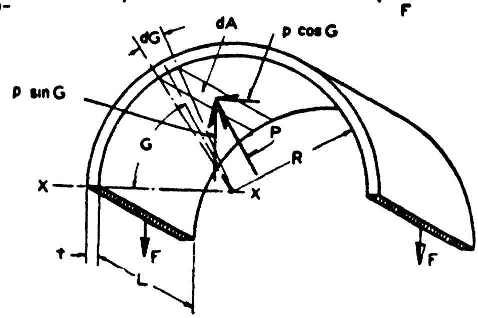


FIG. 3-9. Forces on the Circumferential Section of a Cylinder.

FIG. 3-10. Forces on the Longitudinal Section of a Cylinder.

Fig. 3-9 shows the end of a closed cylindrical vessel. Forces F, representing the wall resistance along a circumferential section, must be equal to the summation of internal pressures p. If p represents the internal unit pressure, psi., R the inner radius and t the wall thickness of the vessel in inches, and S the unit tensile stress in the vessel wall, psi., then the head area subjected to pressure is πR^2, and the total pressure against the head is $\pi R^2 p$. The wall area resisting rupture is equal to

$$\pi(R + t)^2 - \pi R^2 = \pi(2Rt + t^2)$$

When the wall thickness is small in comparison with the inner radius of the vessel, the quantity $\pi(2Rt + t^2)$ is approximately equal to $2\pi Rt$ (and approaches it more closely as t becomes smaller with respect to R). For thin walled cylinders this simplification is permissible, and the total resistance to rupture in the circumferential section can be taken as $2\pi Rts$. Equating the resistance to the total pressure,

$$2\pi RtS = \pi R^2 p$$

or

$$S = \frac{pR}{2t} \tag{3-1}$$

for the stress in a thin walled circumferential section.

Fig. 3-10 shows one half of the cylindrical portion of a vessel, in which the forces p represent the internal pressure and the forces F the wall resistance along the longitudinal sections. Consider an increment of the surface of the inner wall, subtended by angle dG, at an angle G from a reference line xx. For a cylindrical section of length L, the area dA of the increment is $RL \times dG$. The normal unit pressure p on any increment of the inner surface may be resolved into two components: $p \sin A$ perpendicular to section xx, and $p \cos A$ parallel to section xx. The total pressure perpendicular to section xx on the increment under consideration, is $(p \sin A)RL \times dG$ and the total pressure on the entire half of the cylinder, perpendicular to section xx, is $\int_0^\pi pRL \sin G\, dG$. For equilibrium this pressure must equal $2F$, the resisting forces in the cylinder walls. For a cylinder of length L the total resistance is $2tLs$, and

$$2tLs = \int_0^\pi pRL \sin G\, dG$$

Integrating and clearing,

$$S = \frac{pR}{2t}\int_0^\pi \sin G\, dG = \frac{pR}{2t}\Big[-\cos G\Big]_0^\pi$$

or

$$S = \frac{pR}{t} \tag{3-2}$$

for the stress in a longitudinal section. From a comparison with Eq. 3-1, it is seen that the unit stress in a circumferential section is one half that of the longitudinal section.

The components of the internal pressure parallel to section xx cancel each other, since the total pressure on the cylinder half parallel to section xx is

$$\int_0^\pi pRL \cos G\, dG$$

Integrating and clearing,

$$pRL \left[\sin G \right]_o^\pi = O$$

3-9. Design of Riveted Joints—ASME-UPV Code. The design of stills and other forms of pressure vessels with riveted joints is usually handled by one of two codes of design and construction practice: Section VIII, on Unfired Pressure Vessels, of the American Society of Mechanical Engineers' Boiler Construction Code, employed for various gases and liquids (referred to as the ASME-UPV Code)[12]; and the 1936 Code for Unfired Pressure Vessels for Petroleum Liquids and Gases, sponsored by the American Petroleum Institute and the American Society of Mechanical Engineers (referred to as the API-ASME Code).[6] Although these codes differ to some extent their applications to design problems are essentially the same in principle; therefore in this chapter only the first will be considered. Although a fairly comprehensive treatment is embodied in this text, reference should be made to the codes themselves for specialized details and for changes in the code specifications which may be made from time to time.

The ASME-UPV Code applies to unfired pressure vessels having an internal pressure greater than 15 psi. gage and inner diameters greater than 6 in. The required shell thickness for internal pressure may be found from the following, which is an adaptation of Eq. 3-2,

$$t = \frac{pR}{Se} \tag{3-3}$$

where p is the maximum allowable internal pressure, psi., R the inside radius of the weakest course of the shell, S the maximum allowable unit stress psi., e the efficiency of the longitudinal joint, or of ligaments between openings, and t the minimum theoretical plate thickness.

Equation 3-3 is not applicable to vessels in which the shell thickness exceeds 10% of the inside radius. For such conditions the thickness t may be found from

$$t = \sqrt{\frac{(Se + p)R^2}{(Se - p)}} - R \tag{3-4}$$

ASME-UPV Code specification numbers, corresponding allowable working stresses for plates and straps, and acceptable materials are given in part in Table 3-1. The working stresses, in temperature ranges below 650° F., are based upon the lower range of the ultimate tensile strength and a factor of safety of 5. Allowable stresses in single shear are 7600 psi. for iron rivets and 8800 psi. for steel rivets; the allowable crushing or bearing stress is 19,000 psi. for steel plate.

Table 3-1.—Materials and Allowable Working Stresses for Unfired Pressure Vessels, Adapted from ASME-UPV Code

ASME Code Spec. No.	Material Data and Description	Grade	Specified Minimum Tensile Strength 1000 psi.	Allowable Unit Tensile Stress, Thousands psi. at Various Temperatures, ° F.							
				−20 to 650	700	750	800	850	900	950	1000
S-2	Steel plates—flange and firebox quality	A	45	9.0	8.8	8.4	6.9	5.7	4.4	2.6	
		B	50	10.0	9.6	9.0	7.5	6.0	4.4	2.6	
S-1	Carbon steel for boilers		55	11.0	10.4	9.5	8.0	6.3	4.4	2.6	
S-42	Carbon-silicon steel, ordinary strength range	A		11.0	10.4	9.5	8.5	7.2	5.6	3.8	2.0
		B	60	12.0	11.4	10.4	9.1	7.4	5.6	3.8	2.0
S-44	Molybdenum steel	A	65	13.0	13.0	13.0	12.5	11.5	10.0	8.0	5.0
S-43	Low-carbon nickel steel	A		13.0	12.3	11.1	9.4	7.6	5.6	3.8	2.0
S-55	Carbon-silicon steel, high strength range, 4½" plates and under	A									
S-44		B	70	14.0	14.0	14.0	13.5	12.0	10.2	8.0	5.0
S-43		B		14.0	13.3	11.9	10.0	7.8	5.6	3.8	2.0
S-55		B		14.0	13.3	11.9	10.0	7.8	5.6	3.8	2.0
S-44		C	75	15.0	15.0	15.0	14.4	12.7	10.4	8.0	5.0
S-43		C		15.0	14.1	12.4	10.1	7.8	5.6	3.8	2.0
S-28	Chrome-manganese-silicon alloy steel	A									
		B	85								

TABLE 3-2.—PROPORTIONS AND EFFICIENCIES OF SINGLE- AND DOUBLE-RIVETED LONGITUDINAL LAP JOINTS
(Figs. 3-2 and 3-3)

Plate Thickness	Rivet Hole Diameter	Efficiencies, %, for Single- and Double-riveted Joints — Allowable Plate Stress, psi.							Dimensions			
		9000	10,000	11,000	12,000	13,000	14,000	15,000	P	R	M	B
1/4	11/16	60.5 69.5	60.5 69.5	60.7 69.5	60.5 69.5	57.3 69.5	53.2 69.5	49.7 69.5	1 3/4	2 1/4	1 1/16	1 3/4
9/32	11/16	60.7 69.5	60.7 69.5	60.3 69.5	55.3 69.5	51.1 69.5	47.4 69.5	44.2 69.5	1 3/4	2 1/4	1 1/16	1 3/4
5/16	13/16	59.4 69.1	59.4 69.1	59.4 69.1	59.3 69.1	56.1 69.1	52.1 69.1	48.6 69.1	2	2 5/8	1 1/4	1 7/8
11/32	13/16	59.4 69.1	59.4 69.1	59.4 69.1	59.3 69.1	56.1 69.1	52.1 69.1	48.6 69.1	2	2 5/8	1 1/4	1 7/8
3/8	15/16	58.3 68.9	58.3 68.9	58.3 68.9	58.1 68.9	55.4 68.9	51.4 68.9	47.9 68.9	2 1/4	3	1 7/16	2
13/32	15/16	58.3 68.9	58.3 68.9	58.3 68.9	58.1 68.9	55.4 68.9	51.4 68.9	47.9 66.4	2 1/4	3	1 7/16	2
7/16	1 1/16	57.5 68.5	57.5 68.5	57.5 68.5	57.3 68.5	55.0 68.5	51.0 68.5	47.6 65.8	2 1/2	3 3/8	1 5/8	2 1/8
15/32	1 1/16	57.5 68.5	57.5 68.5	57.5 68.5	57.3 68.5	55.0 68.5	51.0 68.5	47.6 65.8	2 1/2	3 3/8	1 5/8	2 1/8
1/2	1 1/16	57.5 68.5	57.6 68.5	56.7 68.5	52.0 68.5	55.0 68.5	44.5 66.1	41.6 61.6	2 1/2	3 3/8	1 5/8	2 1/8

TABLE 3-3.—PROPORTIONS AND EFFICIENCIES OF RIVETED BUTT JOINTS
(Figs. 3-4, 3-5, and 3-6)

Plate Thickness	Strap Thickness	Rivet Hole Diam.	Efficiencies: Double Rivet	Efficiencies: Triple Rivet	Efficiencies: Quad. Rivet	Dimensions: L	K	P	U	V	M	B	C	D
1/4, 9/32	1/4	11/16	82.8	87.5	93.8	11	5 1/2	2 3/4	4	2	1 1/16	1 3/4	2 1/8	2 1/4
5/16, 11/32	9/32	13/16	81.9			13	6 1/2	3 1/4	4 1/2	2 1/4	1 1/4	1 7/8	2 7/16	2 5/8
3/8, 13/32	5/16					14	7	3 1/2						2 3/4
7/16, 15/32	3/8	15/16	81.3	88.3	94.0				5	2 1/2	1 7/16	2	2 3/4	3 1/16
1/2, 17/32	7/16					16	8	4						3 1/8
9/16		1 1/16		86.7	93.4						1 5/8	2 1/4	3	3 5/16
19/32, 5/8, 21/32	1/2													
11/16, 23/32, 3/4		1 3/16		85.5	92.7	16 1/2	8 1/4	4 1/8			1 13/16	2 3/8	3 1/4	3 9/16
25/32, 13/16	9/16			84.6	92.3	17	8 1/2	4 1/4						3 13/16
27/32				84.1	91.8									
7/8	5/8	1 5/16			91.2	17 1/2	8 3/4	4 3/8			2	2 5/8	3 1/2	3 7/8
29/32				83.6	90.5									
15/16	11/16				90.1	18	9	4 1/2						3 15/16
31/32				83.2	89.5									
1				83.4	90.2									
1 1/32					89.6									
1 1/16					89.0	19	9 1/2	4 3/4						4 1/4
1 3/32					88.5						2 3/16	2 7/8	3 3/4	
1 1/8	3/4	1 7/16			88.0									
1 5/32					87.5									
1 3/16														
1 7/32					87.2	20	0	5						4 5/16
1 1/4					86.8									

TABLE 3-4.—PROPORTIONS OF RIVETED LAP JOINTS FOR GIRTH SEAMS OF CYLINDRICAL VESSELS WITH UNSTAYED HEADS

Plate Thickness (inclusive)	Rivet Hole Diam.	Rivet Pitch		Min. Back Pitch	Marginal Pitch
		Single-Rivet	Double-Rivet		
1/4, 9/32	11/16	1 3/4			1 1/16
5/16 to 13/32	13/16	1 7/8			1 1/8
7/16 to 1/2	15/16	2 1/8			1 7/16
17/32	15/16	2 1/16	3	1 11/16	1 7/16
9/16	1 1/16	2 3/8	3 1/4	1 7/8	1 5/8
19/32 to 21/32	1 1/16		3 1/4	1 7/8	1 5/8
11/16 to 3/4	1 3/16		3 3/4	2 1/8	1 13/16
25/32 to 31/32	1 5/16		4	2 3/8	2
1 to 1 1/4	1 7/16		4 1/2	2 5/8	2 3/16

The proportions and efficiencies of riveted lap joints, corresponding to Figs. 3-2 and 3-3, are given in Table 3-2; of riveted butt joints corresponding to Figs. 3-4, 3-5, and 3-6, in Table 3-3; and the proportion of riveted lap joints for girth seams in Table 3-4. The proportions and design are in accordance with the data of the ASME-UPV Code, although these tables are not an integral part of the code itself. The efficiencies given in Table 3-3 vary slightly for the higher strength materials, but are accurate within 0.5%.

When vessel diameters and operating pressures are of such magnitude that plate thicknesses greater than 1 1/4 in. must be employed, the proportions of the joints given in Table 3-3 cannot be used, and an analysis of the joint must be undertaken. This is usually accomplished by determining the possible types of failure and setting up strength equations similar in principle to those developed in Example 3-3. Quadruple-riveted butt joints, similar to those shown in Figs. 3-6 or 3-7, are usually employed for heavy plate thicknesses. As the plate thickness and the rivet pitch and diameter are unknown, it is good practice to assume a ratio of rivet pitch and diameter for the longitudinal joint based upon the proportions given in Table 3-3, and solve for the plate thickness and rivet diameter.

The marginal and back pitches, and the butt strap thickness, may be determined from Section U-27, ASME-UPV Code, which gives the following data for computing the distance between rows of rivets, or back pitch B:

$$\text{If } P/d \leqq 4,\ B \text{ (minimum)} = 1\tfrac{3}{4}d \qquad (3\text{-}5)$$

$$\text{If } P/d > 4,\ B \text{ (minimum)} = 1\tfrac{3}{4}d + 0.1(P - 4d) \qquad (3\text{-}6)$$

where P is the pitch of the rivets in the outer row when a rivet in the inner row comes midway between two rivets in the outer row, or where P is the pitch of the rivets in the outer row less the pitch of the rivets in the inner row when two rivets in the inner row come between two rivets in the outer row. The marginal pitch M, from Section U-28, ASME-UPV Code, should not be less than 1½ or more than 1¾ times the diameter of the rivet holes. Or,

$$1.5d < M < 1.75d \qquad (3\text{-}7)$$

The butt strap thickness, for plate thicknesses greater than 1¼ in., should not be less than ⅔ of the shell thickness.

Example 3-4. Design an outside storage tank for anhydrous ammonia in which the pressure is maintained at a maximum of 200 psi. with a maximum temperature of 125° F. The tank is to have a minimum inner diameter of 6 ft., and is to be made of rolled plates corresponding to specifications S-1, ASME-UPV Code.

Solution. From Equation 3-3,

$$te = \frac{pR}{S}$$

where S is equal to 11,000 psi. (Table 3-1), R is 3 ft. or 36 in., and p is 200 psi. Substituting,

$$TE = 200 \times \frac{36}{11{,}000} = 0.655$$

Reference to Table 3-3 indicates that the following solutions are available:

Type of Butt Joint	Plate Thickness	Efficiency	tE
Triple-riveted	25/32	84.6	0.661
Quadruple-riveted	23/32	92.7	0.666

In deciding between these alternatives the cost of the thicker shell and the cheaper joint should be compared to the cost of the thinner shell and the more expensive joint. Such cost data may not be readily procurable, but a comparative estimate can be made on the basis of about $.10 per driven rivet and approximately $.02 per cubic inch of steel plate. These data check favorably with the cost data of Bliss.[19] A comparison of the two types of joints will show that the long pitches are 8½ in. and 16½ in. for the triple and quadruple joints, respectively, and an exact equivalent length basis will be obtained by comparing 33 of the former and 17 of the latter pitches with a common length of 280.5 in. For this length the plate area is equal to $\pi 72 \times 280.5$ or 63,500 sq. in., and the thickness differential of $(25/32 - 23/32)$ or 1/16 in. gives an increase in plate thickness of $63{,}500 \times 0.0625$ or 3970 cu. in., and an increased plate cost of 3970×0.02 or $79.40 for the triple-riveted over the quadruple-riveted joint design.

For the quadruple-riveted joint, the butt-strap thickness is ½ in. and the widths are 12 in. and 25⅝ in., giving a total cross-sectional area of 18.82 in.; for the triple-riveted joint the strap thickness is 9/16 in., and the widths are 13¼ in. and 20¼ in., giving a cross-sectional area of 18.85 sq. in.; the difference between these is too small to warrant further computation. As an offset against the increased cost of the triple-riveted joint design the expense of the additional rivets in the quadruple-riveted joint must be considered. In the former there are 10 rivets per long pitch, giving a total of 330 rivets for the length under consideration; in the latter there are 22 rivets per long pitch giving a total of 374 rivets. On the basis of $.10 per rivet, the increased riveting cost for the quadruple-riveted joint will be $(374 - 330)0.10$, or $4.40, which is insufficient to offset the additional plate cost of the triple-riveted joint. The quadruple-riveted joint will, therefore, be selected on a cost basis, and will involve a 23/32-in. plate and a 1 3/16-in. diameter rivet.

The data for the girth and head, or circumferential, joints may be selected from Table 3-4. For a 23/32-in. plate, a double-riveted lap joint has a pitch of 3¾ in. and a rivet diameter of 1³⁄₁₆ in. The strength equations for the joint are:

Solid plate

$$F_1 = tPS_t$$

or

$$F_1 = 0.719 \times 3.75 \times 11{,}000 = 29{,}600 \text{ lbs.}$$

Rivet shear

$$F_2 = 2\pi d^2 S_s/4$$

or

$$F_2 = 2\pi 1.188^2 \times 8800/4 = 19{,}470 \text{ lbs.}$$

Rivet bearing

$$F_3 = 2tdS_b$$

or

$$F_3 = 2 \times 0.719 \times 1.188 \times 19{,}000 = 32{,}400 \text{ lbs.}$$

Net tension

$$F_4 = t(R-d)S_t$$

or

$$F_4 = 0.719(3.75 - 1.188)11{,}000 = 20{,}300 \text{ lbs.}$$

The efficiency of the joint is equal to F_2/F_1, which is equal to 19,470/29,600 or 66%. The efficiency of the circumferential joint need only be 50% of the efficiency of the longitudinal joint (see Eq. 3-1 and 3-2), and thus these values are satisfactory.

Example 3-5. Design longitudinal and circumferential joints for a CO_2 surge tank of ASME S-2, grade B steel plate having an ultimate tensile strength of 50,000 psi. The vessel diameter is 90 in., and the internal pressure is 400 psi.

Solution. From Table 3-1, the allowable tensile stress is 10,000 psi. From Eq. 3-3,

$$te = 400 \times \frac{45}{10{,}000} = 1.8$$

For an efficiency e of 100%, the plate thickness t will be 1.8 in., which is beyond the plate thickness range of Table 3-3. It will, therefore, be necessary to design a special joint, and it may be advisable to employ a double-shear butt joint, similar to Fig. 3-7, quadruple-riveted, with a scalloped outer strap to facilitate calking. The two halves of the joint are symmetrical about the center plane of the plate to eliminate eccentricities due to loading. For this joint let L represent the long pitch or representative section, p the short pitch, which is equal to $L/3$, d the rivet hole and the rivet diameter, N the number of rivet areas, S_t the allowable tensile stress, which is 10,000 psi., S_b the allowable bearing stress, 19,000 psi., and S_s the allowable shearing stress, 8800 psi.

The load on a representative portion of the joint, from Eq. 3-2, is

$$F = pRL = 400 \times 45\,L = 18{,}000\,L$$

The joint strengths are:

$$F_1 = \text{Solid plate} = tLS_t = tL \times 10{,}000 = 10{,}000\,tL$$

$$F_2 = \text{Rivet shear} = N\pi d^2/4S_s = 18 \times \pi d^2/4 \times 8800 = 124{,}500\,d^2$$

(based upon nine rivets, all in double shear).

$$F_3 = \text{Rivet bearing} = NtdS_b = 9\,td \times 19{,}000 = 171{,}000\;\; td$$

(based upon the projected area of nine rivets).

$$F_4 = \text{Net tension, Section } WW = t(L-d)S_t = t(L-d)10{,}000$$
$$= 10{,}000tL - 10{,}000\;td$$

(The net tension may be based upon the plate thickness instead of the butt strap thickness, because strap thicknesses for plates over 1¼ in. thick must be at least ⅔ the plate thickness.)

F_5 = Net tension, section XX and shear, section $WW = t(L - 2d)S_t + N\pi d^2/4S_s$

$= t(L - 2d)10{,}000 + (2\pi d^2/4 \times 8800) = 10{,}000\ tL - 20{,}000\ td + 13{,}840\ d^2$

(In this case failure is based upon tearing at Section XX, with one rivet in double shear at Section WW.)

F_6 = Net tension, Section XX, and bearing, Section $WW = t(L - 2d)S_t + tdS_b$

$= t(L - 2d)\ 10{,}000 + (td \times 19{,}000) = 10{,}000\ tL - 1000\ td$

(Failure is based upon tearing at Section XX, with one rivet crushing in the plate, at Section WW.)

F_7 = Net tension, Section YY, and shear, Sections WW and XX

$= t(L - 3d)S_t + N\pi d^2/4S_s = t(L - 3d)10{,}000 + (6\pi d^2/4 \times 8800)$

$= 10{,}000\ tL - 30{,}000\ td + 41{,}500\ d^2$

(Failure is based upon tearing at Section YY, with one rivet shearing at Section WW, and two rivets shearing at Section XX.)

F_8 = Net tension, Section YY, and bearing, Sections WW and XX

$= t(L - 3d)S_t + NtdS_b = t(L - 3d)10{,}000 + (3td \times 19{,}000)$

$= 10{,}000\ tL + 27{,}000\ td$

(Failure is based upon tearing at Section YY, with one rivet crushing at Section WW, and two rivets crushing at Section XX.)

F_9 = Net tension, Section ZZ, and shear or bearing at Sections WW, XX, and YY.

(This mode of failure need not be considered, because the net section in tension at ZZ is the same as at YY, but there are six rivets instead of three in the outer rows to resist shear or bearing.)

In determining the plate thickness and the rivet diameter, it is necessary to make some assumptions as to their relative proportions. In Table 3-3, the ratio of the pitch P to the rivet diameter varies from 3.47 to 3.30. Assuming a ratio of P/d equal to 3⅓, then L is equal to $3P$ or $10d$.

Because of the number of joint strength expressions listed, it is advisable to select the more compatible of the above equations thus minimizing the work of solving for the thickness t and the rivet diameter d. Joint failure is most probable in net tension, in shear, or in a combination of net tension and shear. Initiating the solution by equating the load F carried by section L of the vessel wall and the strength F_4,

$$F = F_4 = 18{,}000\,L = 10{,}000\,tL - 10{,}000\,td$$

$$18{,}000 \times 10d = (10{,}000\,t \times 10\,d) - 10{,}000\,td$$

$$180{,}000\,d = 90{,}000\,td$$

or

$$t = 2 \text{ in.}$$

Equating strengths F_4 and F_5,

$$F_4 = F_5 = 10{,}000\ tL - 10{,}000\ td = 10{,}000\ tL - 20{,}000\ td + 13{,}840\ d^2$$

Substituting known and assumed values for t and L,

$$10{,}000(2)10d - 10{,}000(2)d = 10{,}000(2)10d - 20{,}000(2)d + 13{,}840\ d^2$$

$$\therefore \quad d = \frac{20{,}000}{13{,}840} = 1.444 \text{ in., say } 1\tfrac{7}{16} \text{ in. diameter.}$$

Then $P = 3\frac{1}{3}\, d = 3\frac{1}{3} \times 1\frac{7}{16} = 4.79$ in., say $4\frac{3}{4}$ in.,

and $L = 3P = 3 \times 4\frac{3}{4} = 14\frac{1}{4}$ in.

The joint strengths may be checked by using the formulated strength equations, as follows:

$$F = 18{,}000(14\tfrac{1}{4}) = 256{,}400$$

$$F_1 = 10{,}000(2)14\tfrac{1}{4} = 285{,}000$$

$$F_2 = 124{,}500(1\tfrac{7}{16})^2 = 257{,}000$$

$$F_3 = 171{,}000(1\tfrac{7}{16})2 = 492{,}000$$

$$F_4 = 10{,}000(2)14\tfrac{1}{4} - 10{,}000(2)1\tfrac{7}{16} = 256{,}250$$

$$F_5 = 10{,}000(2)14\tfrac{1}{4} - 20{,}000(2)1\tfrac{7}{16} + 13{,}840(1\tfrac{7}{16})^2 = 256{,}100$$

$$F_6 = 10{,}000(2)14\tfrac{1}{4} - 1{,}000(2)1\tfrac{7}{16} = 282{,}125$$

$$F_7 = 10{,}000(2)14\tfrac{1}{4} - 30{,}000(2)1\tfrac{7}{16} + 41{,}500(1\tfrac{7}{16})^2 = 284{,}500$$

$$F_8 = 10{,}000(2)14\tfrac{1}{4} + 27{,}000(2)1\tfrac{7}{16} = 362{,}500$$

From these results it is seen that the probable modes of failure of the joint will be effected by: shearing the rivets, F_2; net tension in the outer row, F_4; or rivet shear in the outer row and net tension in the row adjacent to the outer row, F_5. The minimum joint efficiency will be given by F_5/F_1 or 256,100/285,000 which equals 0.90 or 90%. This joint efficiency corresponds closely to those given in Table 3-3, and in several of the strength equations, particularly F_8, the efficiency is appreciably greater than 100%. The value of 256,100 for F_5 is less than its equated value of F_4, or 256,250, because the rivet diameter and the pitch lengths are selected on a basis of commercial rather than theoretical dimensions.

The P/d ratio for the rivets in rows XX, YY, ZZ is $4\frac{3}{4}/1\frac{7}{16}$, which is less than 4, and the minimum back pitch, from Eq. 3-5, is

$$B = 1\tfrac{3}{4}\, d = 1\tfrac{3}{4} \times 1\tfrac{7}{16} = 2.52, \text{ say } 2\tfrac{5}{8} \text{ in.}$$

The P/d ratio for the rivets in row WW is $14\frac{1}{4}/1\frac{7}{16}$, which is greater than 4. Since there are two rivets in row XX between adjacent rivets in row WW, the value of P is $14\frac{1}{4} - 4\frac{3}{4}$ or $9\frac{1}{2}$.

From Eq. 3-6,

$$B = (1\tfrac{3}{4} \times 1\tfrac{7}{16}) + [0.1(9\tfrac{1}{2} - 4)1\tfrac{7}{16}] = 2.9, \text{ or } 3 \text{ in.}$$

The marginal pitch M, from Eq. 3-7, should be greater than $1.5 \times 1\frac{7}{16}$ and less than $1.75 \times 1\frac{7}{16}$. As the value of M lies between 2.16 and 2.52, a marginal pitch of $2\frac{1}{4}$ in. is satisfactory. The butt strap thickness should not be less than $\frac{2}{3}$ of the plate thickness. $\frac{2}{3} \times 2$ gives 1.33 in.; and a $1\frac{3}{8}$-in. butt strap is indicated.

The total force F_h on the girth or head joints (from Eq. 3-1) is

$$F_h = \pi R^2 p = \pi \times 45^2 \times 400 = 2{,}550{,}000 \text{ lbs.}$$

If the same size of rivet is used for the girth and longitudinal joints for convenience in fabrication, the shearing strength of one rivet in single shear is

$$F_s = S_s \pi\, d^2/4 = 8800\, \pi \times (1\tfrac{7}{16})^2/4 = 14{,}300 \text{ lbs.}$$

The bearing strength of one rivet is given by

$$F_b = S_b t d = 19{,}000 \times 2 \times 1\tfrac{7}{16} = 54{,}500 \text{ lbs.}$$

The number of rivets required in the girth or head joints will be based upon the shearing strength of the rivet, since it is the lower of the two values. The number of rivets is given by

$$F_h/F_s = \frac{2{,}550{,}000}{14{,}300} = 178$$

If a single-riveted lap joint is employed, the pitch is equal to the circumference of the shell divided by the number of rivets, or

$$2\pi \frac{R}{N} = 2\pi \frac{45}{178} = 1.59 \text{ in., approximately}$$

Because of driving clearance the rivet pitch should be equal to about three times the rivet diameter, or $3 \times 1\frac{7}{16}$, or $4\frac{5}{16}$ in. This indicates that a triple-riveted lap joint will probably be required. Assuming three parallel rows of 60 rivets each, the strength of the joint in net tension will be

$$(2\pi R - Nd)tS_t = (2\pi \times 45 - 60 \times 1\tfrac{7}{16})\, 2 \times 10{,}000 = 3{,}960{,}000$$

which indicates ample strength as compared to the load on the girth joint. The pitch of the joint will be

$$2\pi R/N = 2\pi \times 45/60 = 4.72 \text{ in.}$$

and, since P/d is equal to 4.72/1.438 or 3.28, which is less than 4, the back pitch, from Eq. 3-5, will have a minimum value of $1\frac{3}{4}\,d$, or about $2\frac{5}{8}$ in. The marginal pitch may be the same as in the axial joint, or $2\frac{1}{4}$ in.

Example 3-6. A cylindrical vessel has an inner diameter of 40 in., is subjected to a pressure of 1800 psi., and is constructed of S-1 steel with a joint efficiency slightly greater than 90%. Find the required thickness.

Solution. The allowable stress, from Table 3-1, is 11,000 psi.; from Eq. 3-3, the required thickness is

$$t = \frac{1800 \times 20}{11{,}000 \times 0.90} = 3.64 \text{ in.}$$

The ratio between the shell thickness and inner radius is 3.64/20 or over 18%, and the value of t must be computed by Eq. 3-4. Since the efficiency is slightly greater than 90%, a value of Se equal to 10,000 may be assumed. Substituting,

$$t = \sqrt{\frac{(10{,}000 + 1800) \times 20^2}{10{,}000 - 1800}} - 20 = 4.0 \text{ in.}$$

Substituting the first value in an alternate form of Eq. 3-4,

$$S = \frac{p[(R+t)^2 + R^2]}{e[(R+t)^2 - R^2]} = \frac{1800(23.64^2 + 20^2)}{0.90(23.64^2 - 20^2)} = 12{,}000 \text{ psi.}$$

which is materially greater than the permissible value of 11,000 psi. from Table 3-1.

3-10. Head Selection and Design. The heads of cylindrical pressure vessels may be of flat, dished, flanged dished, elliptical, conical, or hemispherical form. Flanged and dished heads are ordinarily used for riveted vessels and will be the only type discussed in this chapter. Other types of heads are covered in Chapter 4.

Nomenclature for flanged and dished heads is illustrated in Fig. 3-11. Seamless heads may be obtained in stock sizes from 12 to 168 in. outer diameter;

heads with one welded center seam are available in stock sizes from 180 to 216 in. outer diameter. Standard dished heads are available in gages from 3/16 to ½ in. by sixteenths, from ½ to 2 in. by eighths, and from 2¼ to 3 in. by fourths. The crown radius is usually equal to the inner diameter of the head minus 6 in., but varies with individual manufacturers and the forming dies available.

The knuckle radius, by the ASME-UPV Code, Section 38, must not be less than three times the thickness of the head, but in no case can it be less than 6% of the outer diameter of the head. The heads listed in Table 3-5 are made according to these specifications; the knuckle radius is equal to three times the head thickness, or is equal to 6% of the outer diameter (to the next largest

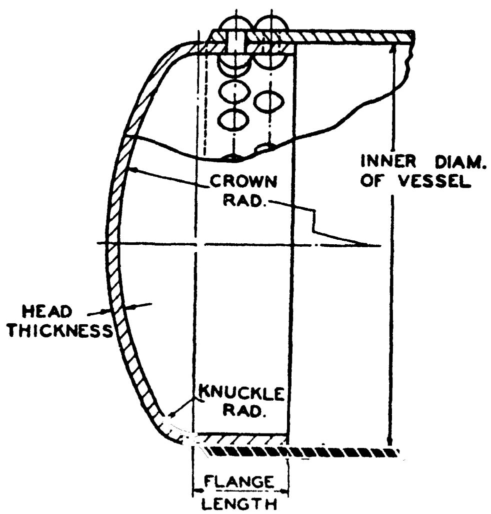


Fig. 3-11. Flanged and Dished Head Nomenclature.

⅛ in.), whichever is larger. To illustrate, the 24-in. diameter head, in gages from 3/16 to ½ in., is made with a 1½-in. knuckle radius, since this value is based upon 0.6 × 24 or 1.44. The knuckle radii for ⅝, ¾, 1, and 1½-in. gage heads are respectively 1⅞, 2¼, 3, and 4½ in., since these are based upon 3t. Standard and recommended maximum straight flanges for dished heads are indicated in Table 3-5.

The thickness of a blank unstayed head with the pressure on the concave side is calculated by

$$t = \frac{0.833pL}{Se} \tag{3-8}$$

where p is the maximum allowable internal pressure, psi., L the inner radius of the dish, in., S the maximum allowable unit stress, psi. (from Table 3-1), e

the efficiency of any joint in the head itself, exclusive of the joint with the shell, and t the minimum theoretical head thickness. (For seamless heads e is equal to 1.00.)

TABLE 3-5.—STANDARD AND RECOMMENDED MAXIMUM STRAIGHT FLANGES FOR DISHED HEADS

Gage of Head	Std. Str. Fl., All Diam.	Rec. Max.	Range of Head O. D.
3/16	2¼	2	
¼	2¼	3	
5/16	3	3½	
⅜	3	4	
7/16	3½	5	Up to 30″
		5½	32″, 34″
		6	36″ and over
½, ⅝	3½	6	
¾, ⅞, 1	4	6	Up to 36″
		8	38″ and over
1⅛ and over	4½	6	Up to 36″
		8	38″ and over

If a flanged-in manhole or access opening that exceeds 6 in. in any dimension is required in the head, the thickness as computed by Eq. 3-7 should be increased by not less than 15%, but in no case by less than ⅛ in. additional thickness over a blank head. If more than one manhole is in the same head, the minimum distance between openings shall not be less than one fourth the outer diameter of the head. Fig. 3-12 shows a standard manhole cover for an 11 × 15 in. elliptical manhole; the figure at the upper right represents the manhole opening in a dished head. The distance Y, representing the depth of "flanged-in," measured from the exterior of the vessel at the major axis of the elliptical hole, must be at least three times the head thickness for heads up to 1½-in. gage. For heavier gages the depth Y is to be made equal to the thickness plus 3 in. Flanging adds considerably to the cost of the head, and instead of flanging manhole openings they may be reinforced by a riveted manhole frame or other attachment to give the necessary strength. This type of reinforcement will be treated in Chapter 10. Very large manholes may be made with deep flanges reinforced by bands shrunk on over the external surface of the flange.

In vessels in which both heads are concave to the internal pressure, at least one manhole is required to permit a mechanic to enter the vessel and "buck-up"

the rivets while the second head is fastened in place. To permit all riveting and "bucking-up" to be handled from the exterior, one of the heads is sometimes placed with the convex side towards the internal pressure. This type of head shall only be subjected to a working pressure equal to 60 per cent of that permitted in a standard (concave side to pressure) head of the same dimensions.

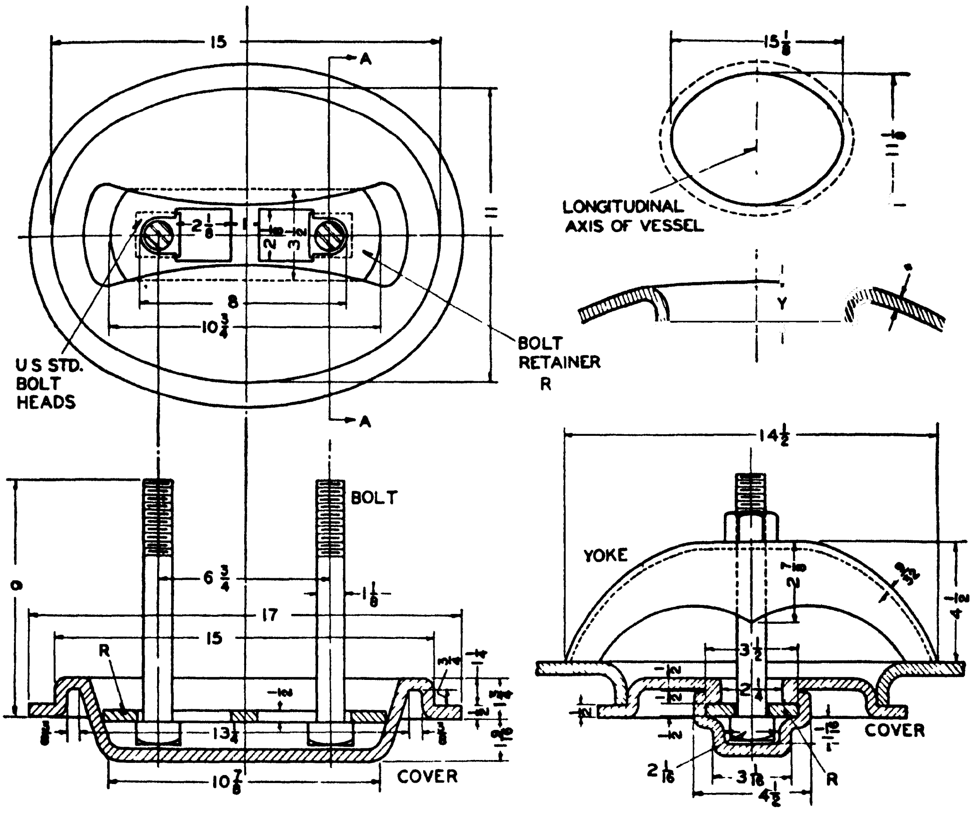


FIG. 3-12. Standard 11-in. × 15-in Manhole and Manhole Cover.

For a definite internal pressure the thickness of a head with the convex side toward the pressure can be obtained from

$$t = \frac{1.39pL}{Se} \tag{3-9}$$

The application of these data to the preceding design problems is illustrated by the following example.

Example 3-7. Select a dished head with a flanged-in manhole, with the concave side to the pressure, for the storage tank described in Example 3-4.

Solution. From section 3-10, the crown radius L of a dished head for a 72-in. diameter head is $(72 - 6)$ or 66 in. Substituting in Eq. 3-8,

$$t = \frac{0.833 \times 200 \times 66}{11{,}000 \times 1.0} = 1 \text{ in.}$$

on the assumption that the material of the head is the same as that of the shell. This thickness is for a blank head; the thickness for a head with a flanged-in manhole is 15% greater, or 1.15 in., requiring a $1\frac{1}{4}$-in. gage head.

If a head with the concave side to the pressure is used for one end of the container, and a head with the convex side to the pressure is used for the other end, the manhole in the head is not required, and the thickness of the first head is 1 in. The thickness of the head convex to the pressure is found by substitution in Eq. 3-9,

$$t = \frac{1.39 \times 200 \times 66}{11{,}000 \times 1.0} = 1.667 \text{ in.}$$

and the nearest standard gage will be $1\frac{3}{4}$ in.

This latter arrangement will probably prove more economical than the former, because the cost of flanging the $1\frac{1}{4}$-in. heads for manholes, and the manhole covers and fittings, will be higher than the cost of a 1-in. and $1\frac{3}{4}$-in. blank head.

From Fig. 3-11 it may be seen that the length of the straight flange must be equal to or greater than the sum of the back pitch and the marginal pitches. By reference to Table 3-4, the back pitch is $2\frac{1}{8}$ in., and the marginal pitch $1\frac{13}{16}$ in. The minimum flange length is therefore $[2\frac{1}{8} + 2(1\frac{13}{16})]$ or $5\frac{3}{4}$ in. Table 3-5 indicates that for gages over $\frac{3}{4}$ in. and diameters over 38 in. a flange length of 8 in. is the recommended maximum. Therefore a $5\frac{3}{4}$ in. flange is specified.

Example 3-8. Select flanged and dished heads, concave to pressure, with manholes, for the CO_2 surge tank of Example 3-5.

Solution. For a head of outer diameter 90 in., the crown radius is $(90 - 6)$ or 84 in. The head thickness, from Eq. 3-8, using the same material as the shell plate, is

$$t = \frac{0.833 \times 400 \times 84}{10{,}000 \times 1.0} = 2.8 \text{ in.}$$

Increasing the thickness by 15% to allow for the manhole,

$$t \times 1.15 = 2.8 \times 1.15 = 3.22 \text{ in.}$$

requiring a $3\frac{1}{4}$-in. head.

For the flange length, two back pitches are required (between three rows of rivets), plus two marginal pitches, or $(2 \times 2\frac{5}{8}) + (2 \times 2\frac{1}{4})$ equal to $9\frac{3}{4}$ in. Table 3-6 indicates, however, that the maximum recommended straight flange is 8 in. This design will require a special head, with a $3\frac{1}{4}$-in. gage and a 10-in. flange.

Another alternative is to make the head of a somewhat stronger material than the shell. Consider a high tensile strength carbon steel with an ultimate tensile strength range of from 65,000 to 77,000 psi., similar to S-55 Grade A, Table 3-1. This material has an allowable stress of 13,000 psi. for temperatures below 650° F. In such a case the gage of a blank head, from Eq. 3-8, will be

$$t = \frac{0.833 \times 400 \times 84}{13{,}000 \times 1.0} = 2.16 \text{ in.}$$

Increasing the thickness by 15% to allow for the manhole,

$$t \times 1.15 = 2.16 \times 1.15 = 2.48 \text{ in.}$$

indicating that a 2½-in. head will be satisfactory. The flange length on this head will still exceed the recommended maximum, although the manufacturers of heads show that a reasonable increase is quite feasible. An alternative procedure to eliminate the use of a head with a special flange length might be the substitution of a butt joint for the triple-riveted lap joint.

3-11. Corrosion. Many fluids have a corrosive effect upon the containing vessel. Vessels subject to corrosion should be installed in such a way that exterior surfaces, particularly manway and handhole covers, are readily accessible to permit proper inspection of the exterior and interior, except where the vessel is of such a size and is connected so as to permit its removal from its permanent position for inspection. The bottoms of vertical vessels should be designed to permit proper drainage.

The thickness of the shell plate, as designed for the maximum working pressure, should be increased by a uniform amount to provide for corrosive action. This is called *corrosion allowance*. If possible, the effect of the corrosive action of a particular substance on the vessel material should be obtained from the manufacturer of the product, or from published sources. When the effects of corrosion are indeterminate prior to the design of the vessel, although known to be inherent to some degree in the service for which the vessel is to be used, the best judgment of the designer must be exercised in establishing a reasonable maximum excess shell thickness. A minimum corrosion allowance of 1/16 in. must be provided for such cases, unless a protective lining is employed. The ASME-UPV Code recommends that tell-tale holes, from ¼ to ⅛ in. diameter, spaced not more than 2 ft. apart, should be drilled to a depth of 60 per cent of the thickness required for a seamless vessel of like diameter, to provide some positive indication when the vessel thickness has been reduced to a dangerous degree by corrosive action.

Corrosion-resistant metal linings may be employed as a surface layer integral with the shell plate, in deposited form as applied with a metallizing gun, or in sheet form mechanically attached. Such linings must be applied so as to preclude any possibility of contact between the corrosive agent and the shell by infiltration or seepage past the lining. Paint of any character is not considered permanent protection against corrosion.

Pressure vessels coated with glass or other enamels can be made of steel or iron not less than 3/16 or more than ⅝ in. thick, and may be welded either by oxy-acetylene or electric arc processes. The maximum allowable working pressure p is given by

$$p = 5000\,t/R \qquad (3\text{-}10)$$

where R is the inner radius, and t the thickness of the plate. The ratio R/t shall not exceed 160.

PROBLEMS—CHAPTER 3

1. A triple-riveted lap joint has a plate thickness of ¾ in., a rivet diameter of 1⅛ in., a pitch of 3¾ in., a back pitch of 2¼ in., and a marginal pitch of 1¾ in. The rivets are staggered. Determine the efficiency of the joint, listing all possible modes of failure.

2. List the possible modes of failure, and determine the efficiencies for them for a standard triple-riveted butt joint, ⅞ in. thick plate.

3. Same as Problem 2 but for a 1⅛ in. thick plate and a quadruple-riveted joint.

4. Design a CO_2 surge tank made of ASME S-1 steel. The vessel diameter is 90 in., and the internal pressure is 250 psi. at 80° F. Select the most economical longitudinal joint, and design circumferential joints to correspond.

5. Select flanged dished heads, concave to pressure for the vessel of Problem 4.

6. Select a flanged dished head, convex to pressure, for the vessel of Problem 4.

7. Determine the maximum permissible cost of a manhole cover by a comparison of the data of Problems 5 and 6.

8. Find the plate thickness for a seamless vessel subjected to an internal pressure of 300 atmospheres, if the inner diameter is 24 in., the plate material S-55 steel, and the temperature of the fluid 875° F.

9. Like Problem 8, for a pressure of 100 atmospheres.

10. Like Problem 8, for a temperature 240° F.

11. Chlorine is unloaded from a tank car into a storage vessel at a pressure of 85 psi. gage. The storage tank is equipped with a safety valve leading into a closed line, set to relieve at a pressure corresponding to a temperature of 140° F. The storage tank is made of ASME S-1 steel, has an inner diameter of 6 feet, and has riveted joints. Design the tank joint.

12. Select a suitable flanged dished head for the storage tank of Problem 11. Use an S-42 Grade B steel. Sketch the head, showing the joint details, and specifying the necessary "flange-in" for a manhole.

13. Design a cylindrical vessel, 7 ft. 6 in. in diameter, to contain at least 750 cu. ft. of compressed air at a pressure of 210 psi. The vessel is to be made of S-42 Grade A steel, and the plate width should not exceed 6 ft. 6 in. If possible, the outer courses should be 6 ft. 3 in. wide. The vessel is to be equipped with flanged, dished heads, each with a manhole, made of S-42 Grade B steel. Make a detail drawing of the vessel, showing a section of the plates and head, and an enlarged detail of the juncture of the longitudinal and circumferential riveted joints.

14. What are the maximum and minimum diameters for glass-lined pressure vessels?

15. A pressure vessel lined with enamel is made of S-2 steel ½ in. thick, and has an inner diameter of 50 in. What is the maximum allowable working pressure?

CHAPTER 4

WELDED PRESSURE VESSELS

4-1. Welding Processes. Welding is a process whereby parts are permanently united by causing their surfaces to flow together. Modern welding, as applied to metals, may be accomplished by four principal processes: electric arc welding, oxygen-gas welding, forge welding, and electrical resistance welding.[33,34] In electric arc welding, a high local temperature is generated by an arc between the surfaces to be joined and a bar or rod of filler metal, termed a welding rod. The arc melts both the base metal and the filler metal. In oxygen-gas welding, acetylene or hydrogen is burned with oxygen to produce a high temperature flame, which is directed on the parts to be joined as well as on the welding rod. Gas welding produces high temperatures over a much wider area than arc welding. In forge welding, the parts to be joined are heated in a forge or furnace, and fusion is effected by pressure or hammer blows. In electrical resistance welding, heat is generated by the flow of electricity across the joint to be welded, and pressure is applied to complete the weld after the metal becomes plastic. For lap joint resistance welding the process is termed seam welding if the weld is continuous, and spot welding if a series of discontinued welds are employed. Resistance butt welding is used for joining two coplanar plates, or for attaching a bar or rod to a plate or disk. Flash welding is another form of electrical resistance welding, in which an arc is generated between two separated surfaces that are then pressed together to complete the weld.

4-2. Brazing and Soldering. In the arc or gas process the term welding is used only when the melting point of the filler metal is within approximately 50° F. of that of the parts to be joined; when the melting points differ by a greater amount, the process is termed brazing, braze welding, or soldering. Brazing and soldering are often used when a totally dissimilar filler metal is required, and may eliminate many of the bad effects that might result from high temperature treatment of the bonded parts. Copper alloys are usually employed as filler metals for brazing, tin and other metal alloys for soldering processes. Sweating is the process of joining parts by brazing or soldering where the filler metal flows between capillary-thin spaces to make the bond. This process is referred to as electric-furnace brazing when it is used in mass-production work, where the electric furnace is employed to effect fusion.

4-3. Hard Facing. Hard facing is a welding process in which an extremely hard, abrasion-resistant overlay, such as Stellite, is deposited upon the wear surfaces of cutting tools and other parts. (Metal wear arises from three principal sources: rolling friction, sliding friction, and impact. Roll crushers are

subjected to rolling friction; the bottom of a drag-line bucket, and the edges and surfaces of cutting tools, to sliding friction; whereas the hammers of a swing hammer mill are worn chiefly by impact.) A high carbon steel welding rod is necessary as an underlay or intermediate welding material upon which the hard face overlay can be deposited. A wide variety of impact and abrasion-resistant materials are available in welding rod form, and the welds may be deposited in a succession of layers to meet the requirements of the most exacting service and specifications. Since the welds are extremely hard, any machining subsequent to welding is usually accomplished by grinding or other abrasive processes.

4-4. Fluxes. For successful fusion the surfaces to be welded must be perfectly clean and remain so during the process. Many alloys contain metals that are easily oxidized, and it is essential that some cleaning agent or flux be used to remove any oxides formed during welding. If oxides are occluded in the metal they will prevent good bonding. Fluxes may be used separately, which is usually the case for brazing and soldering, or as a coating on the welding rod. Specific fluxes have been developed for various classes of welding and brazing. Their selection is dependent largely upon the method of joining and the material of the parts. In arc welding coated rods provide a gaseous shield over the region undergoing fusion, excluding the atmosphere and preventing oxidation. For structural applications, welds made with coated rods may be subjected to higher allowable stresses than those made with a bare or uncoated rod.

4-5. Heat Treatment of Welded Joints. In arc or gas welding the high local temperatures may induce warping or distortion, or internal strain and stress in the parts joined. Thin sections subjected to localized heating are more likely to warp and buckle than heavy sections, but the latter are more subject to internal strain. Properly applied clamps or fixtures to hold the parts in position during the welding process will minimize distortion and may also serve as heat conductors. Stress relieving, which may be accomplished either by furnace annealing or by peening or hammering, may be utilized to release "locked-up" stresses. Stress relieving by annealing is mandatory for several classes of unfired welded pressure vessels. In some instances an increase of 6% in the allowable design stress is permitted if this process is used, since the vessel is considered a more uniformly elastic body.

The high temperatures employed in arc and gas welding processes have a material effect upon the alloy structure of the parts joined, and appropriate measures must be taken to compensate for such incidental heat treatment. Annealing, subsequent heat treatment to obtain a desired alloy structure, or the use of the proper filler metal is recommended. Since many alloys are available for process equipment, it is imperative that the welding method and the filler material be selected to insure a weld with as nearly the same strength and chemical resistance as the basic metal. The welding of the chemical processing equipment should be done with care and by persons experienced in welding the particular metals to be joined. No single specification can be devised for a variety of welds.

General purpose welding and brazing rods are available for experimental use or quick repair, but for exacting specifications or for special service requirements it is advisable to obtain the recommendation of reputable manufacturers of welding supplies and equipment.

4-6. Inspection of Welded Joints. In the past the determination of the quality of a welded joint has been considered more difficult or more uncertain than the inspection of a riveted or bolted joint, since visual inspection of the outer surface of the weld was the only simple method readily available. But modern welding practice, both in structural and in pressure vessel work, requires that welding operators be examined periodically to determine and reaffirm their

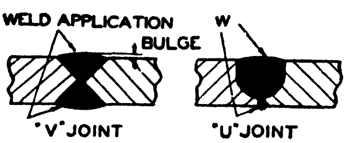


Fig. 4-1. Double-welded Butt Joints

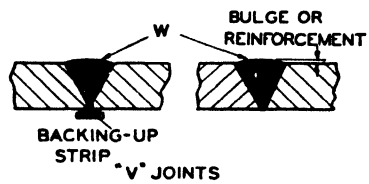


Fig. 4-2. Single-welded Butt Joints.

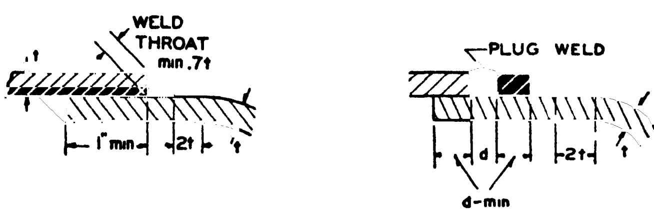


Fig. 4-3. Plug and Full-fillet Lap Welded Joints.

ability and skill. In pressure vessel and structural manufacture, test plates are welded by the operator at the same time that the joints are welded; specimens are taken from these test plates and subjected to tensile, flexural, and density determination tests in the laboratory. Radiographing is another inspection process whereby welded joints are examined by X-ray equipment sufficiently powerful to reveal excessive porosity, points of defective fusion, and other defects. Radiographing is mandatory for certain classes of pressure vessels; in some cases an increase of 12% in the allowable design stress is permitted if the main joints of the vessel have been radiographed, with subsequent repair of any defects. (The codes of practice that govern structural and pressure vessel design and construction should be consulted for detailed information regarding welded joint inspection and testing, and operator qualification.)

4-7. Types of Welded Joints. Figs. 4-1 to 4-3 show representative welded joints employed for pressure vessel fabrication. Butt joints may be welded from one or both sides of the plates, as indicated by the symbol *W*;

double-welded joints are stronger than single-welded joints, although the latter are less expensive. The edges of all butt-welded plates should be beveled or grooved by planing or burning prior to the welding process. Plate edges for double-welded vee-butt joints, shown at the left in Fig. 4-1, are machined from both sides of the plate; plate edges for double-welded U-butt joints, shown at the right in Fig. 4-1, need only be planed from one side. The single-welded butt joint with a backing-up strip, Fig. 4-2, is fabricated by holding or clamping a strip of metal along the under side of the weld and by providing sufficient weld penetration to insure satisfactory attachment of the backing-up strip. This joint is considered as strong as a double-welded joint, but is more limited in application possibilities. All welds should project or bulge slightly above the plate surface to insure a weld throat dimension at least equal to the plate thickness.

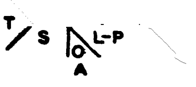
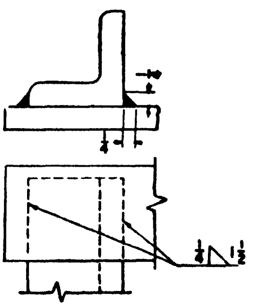


Fig. 4-4. Welding Symbol.

Double-welded lap joints, shown at the left in Fig. 4-3, and single-welded lap joints with auxiliary plug welds, shown at the right in Fig. 4-3, may be employed for circumferential seams only, and are usually subjected to definite limitations as to plate thickness, temperature, and load capacity. Some of the limitations as to size, etc., are indicated in the figure. Single-welded lap joints without plugs are not considered permissible joints for most pressure vessel applications, although they are used in structural fabrication and are sometimes employed for attaching dished heads convex to pressure. The inclined surface of a fillet weld at a lap joint must be plain or slightly convex (never concave) to insure sufficient throat depth. Plug welds are fabricated by punching or drilling holes in the outer plate and filling the hole with filler metal, which fuses with the surface of the inner plate.


Fig. 4-5. Structural Angle Fillet Welded to Plate.

4-8. Weld Representation and Specification. Some applications of structural welding are shown in Figs. 4-5, 4-7, and 4-8. Welding symbols have been standardized by the American Welding Society.[16] Fig. 4-4 shows the standard "symbol" used for fusion-welded joints. The arrowhead indicates the location of the weld. The symbol *T* is the specification reference to the type of welding rod; *S* indicates the size of the weld. The symbol adjacent to *S* is termed the shape symbol, and is descriptive of the type and shape of the weld. *O* and *A,* within the shape symbol, are the dimensions of the root opening and the included angle to which the plates are beveled (as illustrated in Fig. 4-9) ; *L* and *P* give the dimensions of length and pitch (or

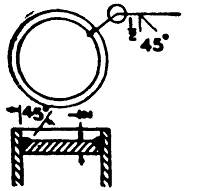


Fig. 4-6. Bevel Butt Weld for Pressure Vessel Head.

center-to-center distance) of intermittent welds. A solid circle at the juncture of the arrow and the body line of the weld symbol (Fig. 4-7) indicates that a field weld is required, i.e., one that is made in the process of assembly or erection. An outline circle at this point (Fig. 4-6) indicates an encircling or "all-around" weld. If the symbol representing the type and shape of the weld is placed below the body line, the weld is on the "near" side, or toward the observer; if the symbol is above the line, the weld should be placed on the "far" side, or away from the observer.

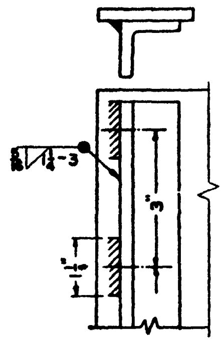


Fig. 4-7. Structural Angle Fillet Welded to Plate.

Fig. 4-5 shows a structural angle attached to the "far" side of a plate by two ¼-in. leg fillet welds 1½ in. long. The absence of a solid circle indicates that the welds are to be made in the shop before erection. Fig. 4-6 shows an "all-around" ½-in., 45°-bevel butt weld, shop welded, and applied from the "near" side. Fig. 4-7 shows a structural angle field welded to the "near" side of a plate by two 5⁄16-in. leg fillet welds each 1¼ in. long, spaced 3 in. on centers. In Fig. 4-8, the weld at the left consists of three ¼-in. diameter plug welds spaced 3 in. on centers, and a single fillet weld, both shop welded. The central weld is a 60°-single vee-butt weld between the plates, and the weld at the right is a double filler lap weld. The top view of the plates of Fig. 4-8 and the sectional and end views of the welds are indicative of approved practice in modes of weld representation, although such details are unnecessary if the welding symbols are properly executed and represented.

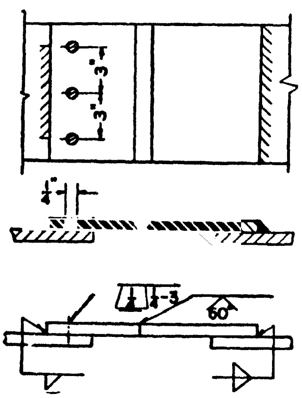


Fig. 4-8. Plate Welding.

4-9. Vessel Design, ASME-UPV Code. The design of unfired pressure vessels with fusion-welded joints is usually based upon the fusion-welded specifications of the ASME-UPV Code, which differ in some degree from those of the code for riveted vessels outlined in Chapter 3. The code for fusion welding is applicable to vessels with an inner diameter greater than 6 in. and with an internal pressure greater than 15 psi. gage. The fusion-welded portion of the ASME-UPV Code recognizes three classes of vessels which will be referred to in the following abstract as U-68, U-69, and U-70; the designation numbers are taken from those paragraphs of the code defining the application of the vessels.

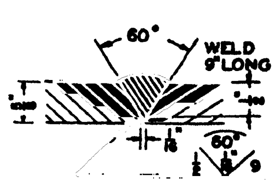


Fig. 4-9. Single-welded V Butt Joint Nomenclature.

1. Vessel use and service:
   U-68. Any service or application.
   U-69. Any application, except as containers of lethal gases or liquids. For this class of vessel, ammonia, chlorine, natural or manufactured fuel gases, propane, and butane are exempt from lethal classification.
   U-70. Any application in which the plate thickness of the vessel does not exceed 5/8 in.; vessels may not be used as containers for lethal gases or liquids.

2. Maximum operating temperatures:
   U-68. No limit.
   U-69. 700° F. for plate or multi-section head thickness less than 1½ in.; 300° F. for plate or multi-section head thickness greater than 1½ in.
   U-70. 250° F., but not to exceed materially the boiling temperature at atmospheric pressure.

3. Maximum operating pressure:
   U-68. No limit.
   U-69. 400 psi. No limit for hydraulic pressure at atmospheric temperature.
   U-70. 200 psi.

Multi-section heads are those made up of two or more sections or pieces. The head thickness limitations do not apply to heads formed of a single plate.

In general, class U-68 vessels are more expensive on account of construction details, as will be seen from the corresponding structural specifications and they must be furnished with at least one manway to permit access for interior welding. Class U-70 vessels are usually the least expensive, and may be constructed so as to require no manways.

Example 4-1. What vessel classification should be employed for a container used for storing a 26% solution of sodium cyanide at room temperature and pressure?

Solution. Because of hydrolysis, this solution produces cyanide gas, and an equilibrium vapor pressure is reached. Cyanide alone or in small quantities is lethal, therefore a class U-68 vessel is required.

Example 4-2. What vessel classification should be used for a container for storing propane at atmospheric pressure and temperature?

Solution. Although the pressure and temperature of the substance is within the permissible limits of class U-70 application, and although propane is exempted from a lethal classification in class U-69 service, it is the intention of the ASME-UPV Code that class U-70 vessels should not be used for the storage of propane and allied fluids. A class U-69 vessel is required.

Example 4-3. What vessel classification may be used as a vaporizer for carbon tetrachloride at a temperature of 270° F., and 46 psi. gage?

Solution. Since the temperature exceeds 250° F., a class U-69 vessel is indicated.

Example 4-4. If the vaporizer of Example 4-3 is operated at 226° F. and 20 psi., may a class U-70 vessel be employed?

Solution. Yes. The temperature is less than 250°, and the boiling temperature at 20 psi is higher than 226°.

Example 4-5. What vessel classification may be used for a tank containing water at atmospheric temperature and 500 psi. gage?

Solution. A class U-69 vessel may be employed, since the code specifies no limitation for hydraulic pressure at atmospheric temperature.

Example 4-6. A class U-69 vessel has a plate thickness of 1⅜ in. and is furnished with one-piece dished heads of 1¾-in. gage. What temperature limitation is the vessel subjected to?

Solution. 700° F., since the limit of 300° F. for thicknesses greater than 1½ in. does not apply to one-piece heads.

The more important structural specifications required by the ASME-UPV Code for the three classes of vessels are given in Table 4-1, which is an abstract of the ASME-UPV Code. The ASME Boiler Construction Code, Section VIII (Table 3-1) should be consulted for greater detail and for specifications regarding tests, inspection, etc., not given in Table 4-1. The codes should also be consulted for major design, to take care of any changes that may be made in the most recent edition.

4-10. Vessel Design—API-ASME Code. A code for the design of vessels for the oil industry has been jointly developed and sponsored by the American Petroleum Institute and the American Society of Mechanical Engineers. This publication, titled the API-ASME Code for the Design and Construction of Unfired Pressure Vessels for Petroleum Liquids and Gases, and denoted in this work as the API-ASME Code, differs to some extent from the ASME-UPV Code. It is established for vessels made of carbon steel, the material of common use in the oil industry. A summary of the more important features of this code follows.

Steels with ultimate tensile strengths S_u varying from 45,000 to 75,000 psi. may be used. Table 4-2 gives the value of F_s (in per cent), which is the allowable fraction of S_u that is recommended for design. (It should be noted that these allowable design stresses are larger for plates and forged steel than the values permitted by the ASME-UPV Code.) The design stress thus obtained must be multiplied by a material factor F_m, dependent upon the grade of the steel, which is equal to: 1.00 for Group A or firebox grades of forge welding and high tensile strength carbon steels; 0.97 for Group B or flange grades; and 0.92 for Group C or structural and mild steels. An ultimate tensile strength of 55,000 psi. is specified by the code for Group C steels, with the further proviso that these are not to be used for plates thicker than ⅝ in., or for metal temperatures over 450° F. All construction factors are applied to the minimum values of the tensile strength range.

Radiographing and stress relieving are mandatory for vessels made of ASTM A-150 steel, and under certain conditions for vessels made of ASTM A-149 steel. Stress relieving is mandatory for other permissible steels when the plate thickness of the shell or head at any welded joint exceeds 1¼ in., and for thinner plates when the thickness exceeds $(D + 50)/120$, where D is the shell diameter in inches. For values of D less than 20, D is assumed to be 20. Radiographing and stress relieving are credited by multiplying the allowable design stress by a radiograph factor F_a of 1.12 or a stress-relieving factor F_r of 1.06, or both. The

TABLE 4-1.—STRUCTURAL SPECIFICATIONS FOR PRESSURE VESSELS ASME-UPV CODE

	U-68	U-69	U-70
Material	Many types of steel and non-ferrous alloys satisfying the ASME Specifications for Materials Code.		Same as U-68 and U 69; cast iron may be employed under certain conditions
Minimum permissible joint efficiency	90%	80%	No minimum
Allowable working stresses, psi.	Values of stresses for operating temperature as given in Table 3-1 multiplied by efficiency of joint.		Double-welded butt joints, 8000; single-welded butt joints for girth or head seams 6500, for longitudinal seams plate thicknesses 1/4" to 3/8" 7000, plate thickness less than 1/4" 5600; double full-fillet lap joints 7000; plug or intermittent welds 5600.
Permissible longitudinal joints	Double-welded butt joint, reinforced at the center of the weld, on each side of the plate, by 1/16" for plate thickness up to 5/8", and by 1/8" for heavier plates.		Double-welded butt joint for plate thicknesses up to 5/8"; single-welded butt joint for plate thicknesses up to 1/4"; double-welded lap joint for plate thicknesses up to 3/8".

TABLE 4-1.—(*Continued*)

	U-68	U-69	U-70
Permissible circumferential joints	As for longitudinal joints.	Same except that single-welded butt joints may be used for plate thicknesses up to 5/8".	Any approved butt or lap joint.
Stress relieving	Mandatory	Mandatory whenever plate thickness exceeds 1¼"; mandatory when the plate thickness is greater than 0.58" and the shell diameter less than 20"; mandatory for all plate thicknesses and shell diameters where the diameter in inches is less than $(120t - 50)$, where t is the thickness in inches.	Not required.
Minimum thickness of shell plates, heads, and dome plates after flanging:	1/8" for shells up to 16" diameter; 3/16 for shells 16"-24" diameter; ¼" for shells 24"-42" diameter; 5/16" for shells 42"-60" diameter; 3/8" for shells over 60" diameter.		

radiograph factor can only be employed when all the main joints of the vessel are radiographed. To summarize, the allowable unit stress S is

$$S = S_u \times F_m \times F_a \times F_r \times F_s \qquad (4\text{-}1)$$

If stress relieving or radiographing is not employed, the values of F_r and F_a are taken as unity.

TABLE 4-2.—PER CENT OF MINIMUM ULTIMATE TENSILE STRENGTH FOR PLATE AND HEAD DESIGN

Metal Temperature, °F.	Plates and Forged Steel, %	Cast Steel, %
Up to 650	25.0	16.7
700	23.7	16.4
750	21.0	14.7
800	18.0	12.9
850	15.0	11.1
900	12.0	9.3
950	9.0	7.5
1000	6.2	5.7

Figs. 4-1, 4-2, and 4-3, show accepted forms of welded joints for longitudinal and circumferential seams for vessels constructed in accordance with the API-ASME Code. The double-welded butt joint, of either the vee or U type, may be used for either longitudinal or circumferential seams, and has an initial efficiency of 80%, based upon a material factor F_m of 1.00, without the inclusion of radiograph or stress-relief factors. The single-welded butt joint with a backing-up strip has the same initial efficiency, but is limited in its application to joints not over 1¼ in. thick. Single-welded butt joints without backing-up strips have an initial efficiency of 70%, and cannot be used for joints over ⅝ in. thick. Double full-fillet lap joints and single full-fillet lap joints with plug welds have an initial efficiency of 65%, but are limited in application to circumferential joints, where the plate thickness is ⅝ in. or less. Single full-fillet lap joints without plug welds have an initial efficiency of 55%, but may be used only for attaching heads convex to pressure, with plate thickness not exceeding ⅝ in. Single full-fillet lap joints with plug welds may be used for circumferential joints only for plates not over ⅝ in. thick. The working load on each plug is given by

$$L = 0.63(d - \tfrac{1}{4})^2 S \qquad (4\text{-}2)$$

where L is the allowable load, psi., d the diameter of the plug, in., and S the allowable shearing stress, psi. For plates up to 2 in. thick d must be at least equal to $t + 1/4$ in. For heavier plates d must not be smaller than $2\frac{1}{4}$ in. The minimum diameter of a plug is 1 in. Plug welds shall be considered to take not more than 20% of the total load carried. When used as a shell or nozzle, lap-welded carbon steel pipe shall have a maximum final efficiency of 80%; electric-fusion welded or electric-resistance welded pipe 85%.

The thickness of the shell is calculated by an application of Eq. 3-2, but the arithmetic mean diameter is used in place of the inside diameter. For convenience the equation may be rewritten in terms of the inside diameter and shell thickness as

$$t = \frac{pD}{2Se - p} + c \qquad (4\text{-}3)$$

where t is the thickness, c the corrosion allowance, and D the inside diameter in inches, before corrosion allowance is added; p is the maximum allowable working pressure, and S the maximum allowable tensile stress, psi.; and e is the efficiency of the longitudinal joint, expressed as a function.

Eq. 4-3 applies only when the plate thickness is less than 10% of the inner diameter of the shell. In no case shall the thickness be less than

$$t\,\text{min.} = \frac{D + 100}{1000} \qquad (4\text{-}4)$$

unless the shell is adequately reinforced by structural members.

4-11. Head Selection and Design—API-ASME Code. Vessel heads may be ellipsoidal, dished, conical, hemispherical, or flat. Flanged and dished heads described and listed in Chapter 3 are extensively employed for welded vessels. Standard ellipsoidal heads are somewhat stronger than dished heads of the same gage; the head surface has the form of an oblate ellipsoid, with a depth or elliptical semi-minor axis equal to one fourth of the inner diameter or elliptical major axis. Ellipsoidal heads are available in the same range of outer diameter and gages as dished heads.

Flanged and conical one-piece heads are obtainable in a range of sizes from 66 to 150 in. outer diameter, varying by increments of 6 in. and in gages from $5/8$ to $1\frac{1}{2}$ in., varying by increments of $1/8$ in. For diameters of 114 in. and greater, the cone angle A (Fig. 4-10) is 60°, and the diameter m of the flat spot at the center of the head is 12 in. The knuckle radius is three times the gage; the standard straight flange length is 4 in. for gages up to 1 in., and $4\frac{1}{2}$ in. for thicker heads.

One-piece hemispherical heads with flanges are available in a very limited range of sizes, but a somewhat greater variety of designs is commercially obtain-

able if segmental heads can be used. Flat heads may be straight-flanged, as illustrated in Fig. 4-11, or constructed of plate and welded in place. Blind flanges are also used for head applications, and are described in Chapter 10.

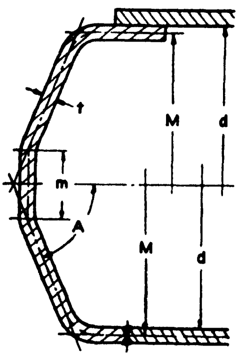


FIG. 4-10. Conical Head Nomenclature and Application.

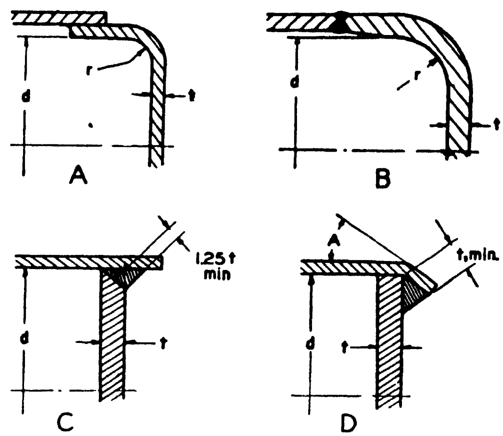


FIG. 4-11. Flat Head Nomenclature and Application.

Head thicknesses are given by the following:

Standard Ellipsoidal $t = \frac{pD}{2Se}$ (4-5)

Standard Dished $t = \frac{pLW}{2Se}$ (4-6)

Conical $t = \frac{pM}{2(\cos A)Se}$ (4-7)

Hemispherical $t = \frac{pD}{4Se}$ (4-8)

where D is the arithmetic mean diameter of the head flange and L the radius in inches of the crown measured to the centerline of the crown plate; W is a factor dependent upon the ratio of the mean knuckle radius to the mean crown radius, and is obtained from Table 4-3; M is the arithmetic mean diameter of the cone, as illustrated in Fig. 4-10; A is one half the included cone angle; S is the allowable unit tensile stress, psi.; and e is the efficiency of any joint in the head exclusive of the joint with the shell. For seamless heads e is equal to 1.0. It should be noted that Eq. 4-5 applies only to *standard* ellipsoidal heads, in which the elliptical depth is *one fourth* the inner diameter of the head. The thickness t is based upon the pressure acting on the concave side of the head; if the pressure

acts upon the convex side, the thickness obtained from Eqs. 4-5, 4-6, 4-7, and 4-8 should be multiplied by a factor of 5/3.

TABLE 4-3.—VALUES OF FACTOR W FOR DISHED HEADS

Ratio of Mean Knuckle Radius to Mean Crown Radius	W
0.06	1.80
0.07	1.70
0.08	1.65
0.09	1.60
0.10	1.55
0.11	1.50
0.12	1.47
0.13	1.44
0.14	1.41
0.15	1.40
0.16	1.38
0.17	1.37
0.18	1.35
0.19	1.32
0.20	1.30
0.25	1.25
0.50	1.12
1.00	1.00

Acceptable forms and designs of flat heads for pressure vessels, constructed in accordance with the API-ASME Code, are shown in Fig. 4-11. Type A is an arrangement designed to permit fastening by means of lap joints with or without plug welds; the required head thickness is given by

$$t = d\sqrt{\frac{0.3p}{S}} \qquad (4\text{-}9)$$

where t is the head thickness and d the inner diameter of the flanged head.

Type B, Fig. 4-11, represents a flanged head which may be attached by single or double vee- or U-butt welds. The head thickness is given by

$$t = d\sqrt{\frac{0.25p}{S}} \qquad (4\text{-}10)$$

where diameter d is taken as indicated in Fig. 4-11B.

This expression is also valid for heads forged integral with the shell or pipe. If the head thickness exceeds the shell thickness, as is usually the case, the inner periphery of the head should be machined as indicated to provide equal thickness of head and shell at the seam.

Flanged flat heads are available in outer diameters from 12 to 42 in. by 2-in. increments, and in outer diameters from 48 to 144 in. by 6-in. increments. The corner or inner knuckle radius of standard heads is equal to three times the head thickness. Standard straight flange lengths are 2 in. for 3⁄16-in., 2½ in. for ¼-in., 3 in. for 5⁄16-in. and ⅜-in., 3½ in. for 7⁄16-in., ½-in., ⅝-in., and ¾-in., 4 in. for ⅞-in. and 1-in., and 4½ in. for 1⅛-in., 1¼-in., 1⅜-in., 1½-in., 1¾-in. and 2-in. gage heads.

Flat heads cut from solid plate are less expensive than flanged heads but require a greater thickness for the same internal pressure. This thickness, for heads similar to Fig. 4-11 C and D, is found from:

$$t = d\sqrt{\frac{0.5p}{S}} \qquad (4\text{-}11)$$

The throat dimension of the weld in detail C must not be less than 1¼ times the head or shell thickness, whichever is smaller. The seam shown in Fig. 4-11D, in which the shell is crimped or rolled over to an angle A, not less than 30° nor more than 45°, is limited to a maximum inner diameter d of 18 in. Care must be taken in cold-working the shell metal to avoid injury in crimping. The throat dimension of the weld must be at least equal to the head or shell thickness, whichever is smaller.

In Eqs. 4-5 to 4-11, an allowance for corrosion should be added to the theoretical thickness obtained, if such conditions are anticipated in service. Data on representative or anticipated corrosion allowances may be found in the codes and in Chapter 10.

4-12. Representative Applications of the API-ASME Code. This section will present several examples in which the principles developed in the preceding paragraphs are applied to design practice.

Example 4-7. A stabilizer tower for use with hydrocarbons in a petroleum cracking process has a working pressure of 300 psi., at a temperature of 250° F. The tower is to have an inner diameter of 5 ft. 0 in., with a length of 40 ft. 0 in., and is to be made of a flange quality steel with an ultimate tensile strength of 60,000 psi. The tower is to have dished heads at top and bottom. Find the necessary design data if corrosion is disregarded. The ultimate strength S_u is 60,000 psi.

Solution. From Table 4-2, the permissible percentage of the ultimate strength factor F_s, for plates and rolled steel, is 25%. The material factor F_m for flange grade steel is 0.97. If radiographing or stress relieving is not required, the allowable design stress from the solid plate, from Eq. 4-1, is

$$S = 60{,}000 \times 0.25 \times 0.97 \times 1.0 \times 1.0 = 14{,}560 \text{ psi.}$$

Since any of the joints shown in Figs. 4-1 and 4-2 may be employed for the longitudinal seam, the design may be initiated by a trial assumption of the least expensive type, the

single-welded vee-butt joint, whose efficiency e is 0.70. It is advisable to include the pressure head induced by the vessel height. If the weight of the fluid within the tower is assumed as 50 lbs. per cu. ft., the total head per square foot of surface will be 50 × 40 or 2000 lbs. The unit pressure per square inch will be 2000/144, or approximately 14 psi., and the total pressure will be 314 psi. From Eq. 4-3

$$t = \frac{314 \times 5 \times 12}{(2 \times 14{,}560 \times 0.70) - 314} = 0.94 \text{ in.}$$

Since a single-welded vee joint is limited to plate thicknesses not exceeding ⅝ in., this type of joint cannot be used. Therefore a trial may be made of the next least expensive type, the single-welded vee-butt joint with a backing-up strip, which has an efficiency of 0.80, and is limited to vessel thicknesses of 1¼ in. maximum. From Eq. 4-3

$$t = \frac{314 \times 5 \times 12}{(2 \times 14{,}560 \times 0.80) - 314} = 0.819 \text{ in.}$$

as the theoretical shell thickness.

For plates smaller than 1¼ in., stress relieving is necessary whenever t exceeds $(d + 50)/120$. Substituting,

$$t = \frac{60 + 50}{120} = 0.917 \text{ in.}$$

indicating that stress relieving is not required. Had it been required, the thickness should have been recalculated by employing a design stress equal to the product of the allowable unit stress and the stress-relief factor S_r, or 14,560 × 1.06, or 15,400 psi. If radiographing had been necessary or desirable, the unit design stress should be taken as 14,560 × 1.06 × 1.12, or 17,250 psi.

The head selection is next in order. From section 3-10, flanged and dished heads with an outer diameter of 60 in. have an inner crown radius of 54 in. The knuckle radius has a minimum value of 0.06× 60, which is equal to 3.60, or 3⅝ in. for all heads less than 1¼ in. thick. (The knuckle radius for heads equal to or greater than 1¼ in. is equal to three times the gage.) Estimating the head thickness as less than 1¼ in. and greater than ½ in., the ratio of the inner knuckle radius to the inner crown radius is 3.63/54 or 0.0671. From Table 4-3, the factor W for a ratio of 0.07 between the mean knuckle and mean crown radii is 1.70. (The values of the inner radii rather than the mean radii are used because it is not feasible to make any assumption as to a plate thickness for this trial computation.) The required head thickness, from Eq. 4-6, is

$$t = \frac{314 \times 54 \times 1.70}{2 \times 14{,}560 \times 1.0} = 0.99 \text{ in.}$$

Since the head is made in one piece, the efficiency e is 1.0. The nearest available head thickness is 1 in., so the required thickness is recalculated based upon a factor W involving the ratio of the mean knuckle and crown radii, which is $(3.63 + 0.50)/(54 + 0.50)$, or 0.0756. For this ratio, W is 1.67, and the head thickness, from Eq. 4-6, is

$$t = \frac{314 \times 54.5 \times 1\ 67}{2 \times 14{,}560 \times 1.0} = 0.979 \text{ in.}$$

Since the head thickness is greater than ⅝ in., a single-welded type vee-butt joint with a backing-up strip is the minimum requirement. A computation of the strength of the joint is not necessary, because the stress in a circumferential joint is only one half that of a longitudinal joint.

The computation for the shell thickness was based upon an inner diameter of 60 in., which is the required figure if the head is to fit inside the shell. Since the requirements dictate a butt-welded joint, the outer diameter of the shell and head should be alike. Assuming an outer diameter for the shell of 60 in., and a plate thickness of about 13/16 in.

(based upon the theoretical thickness of 0.819 in.), the inner diameter will be [60 — 2(0.813)], or 58.375 in. From Eq. 4-3 the shell thickness will then be

$$t = \frac{314 \times 58.375}{(2 \times 14{,}560 \times 0.80) - 314} = 0.796 \text{ in.}$$

This figure will require a plate thickness of $^{13}\!/_{16}$ in.

Example 4-8. Redesign the stabilizer tower of Example 4-7 for an operating temperature of 900° F., using ellipsoidal heads.

Solution. From Table 4-2, the ultimate strength for design for a working temperature of 900° F. must not exceed 12% of that at room temperature. From Eq. 4-1, S_t is 0.12, and the allowable design stress for the solid plate, from Eq. 4-1, is 60,000 × 0.12 × 0.97, or 6990 psi. Since the shell thickness is greater than ⅝ in. in Example 4-7, it may be assumed that the longitudinal joint for this design must be either a double-welded butt joint or a single-welded butt joint with a backing-up strip. The joint efficiency for either type is 80%. The shell thickness, from Eq. 4-3, is

$$t = \frac{314 \times 60}{(2 \times 6990 \times 0.80) - 314} = 1.73 \text{ in.}$$

Since this thickness is greater than 1¼ in., stress relieving is mandatory. The allowable design stress should be multiplied by the stress-relief factor F_r to give 6900 × 1.06, or 7400 psi., as the allowable design stress. Estimating the thickness of the shell at 1⅝ in., the inner diameter of the shell will be [60 — (2 × 1.625)], or 56.75 in. The thickness is then found from Eq. 4-3 to be

$$t = \frac{314 \times 56.75}{(2 \times 7400 \times 0.80) - 314} = 1.55 \text{ in.}$$

A shell thickness of 1$^{9}\!/_{16}$ in. will satisfy the code requirements. Since the single-welded butt joint with a backing-up strip is limited to plate thicknesses not exceeding 1¼ in., a double-welded vee- or U-butt joint must be employed for the longitudinal seam.

The head thickness is obtained from Eq. 4-5, and is

$$t = \frac{314 \times 60}{2 \times 7400 \times 1.0} = 1.27 \text{ in.}$$

based upon the outer instead of the mean diameter of the head. A 1⅜-in. ellipsoidal head will prove aceptable theoretically, but a 1⅝-in. head will be required because the flange of the head must have a thickness at least equal to the shell thickness.

Example 4-9. Select flat or conical heads for the stabilizer tower of Example 4-7.

Solution. As a trial assumption, consider the mean diameter of a flanged flat head equal to the outer diameter. The circumferential joint will probably be a butt-welded joint, so the construction of Fig. 4-11B will be used. From Eq. 4-10, the head thickness will be

$$t = 60 \sqrt{\frac{0.25 \times 314}{14{,}560}} = 4.42 \text{ in.}$$

Since the maximum thickness of a stock head is 2 in., a flanged head cannot be used. Employing a flat head similar to construction C, Fig. 4-11, the inner diameter of the vessel is [60 — (2 × 13/16)] or 58.375 in., and the head thickness, from Eq. 4-11, is

$$t = 58.375 \sqrt{\frac{0.5 \times 314}{15{,}400}} = 5.9 \text{ in.}$$

The stress of 15,400 psi., which is based upon stress relief, is used because the calculations for the flanged head indicated that the head thickness would be greater than 1¼ in. This design will require a head thickness of at least 6 in., and is obviously not practicable.

If dished or ellipsoidal heads are not available, a conical head may be employed, made up of flat plate and cut and welded as indicated in Fig. 4-13. If the angle A is 60°, the radial seam in the head itself is a double-welded butt joint with an efficiency of 80%. The diameter M may be assumed as 60 in. and the necessary thickness, from Eq. 4-7 will be

$$t = \frac{314 \times 60}{2 \cos 60° \times 14{,}560 \times 0.80} = 1.62 \text{ in.}$$

Stress relieving will be necessary, since the thickness is over 1¼ in. The revised thickness is

$$t = \frac{314 \times 60}{2 \cos 60° \times 15{,}400 \times 0.80} = 1.53 \text{ in.}$$

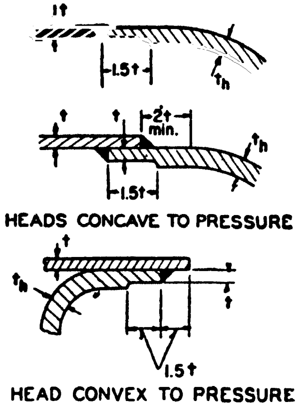


FIG. 4-12. Welded Head Joint Details.

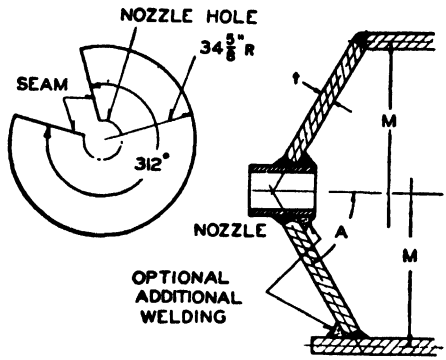


FIG. 4-13. Layout of Conical Head.

Since the preceding computations are based upon a mean diameter M of 60 in., it may be advisable to recalculate the thickness on the basis of the actual mean diameter. If a head with a thickness of 1½ in. is assumed, the mean diameter M, based upon an internal diameter of 60 in., is 61.5 in. The required thickness is

$$t = \frac{314 \times 61.5}{2 \cos 60° \times 15{,}400 \times 0.80} = 1.565 \text{ in.}$$

A head with a thickness of 1⁹⁄₁₆ in. will probably be satisfactory, although a 1⅝-in. head will fully satisfy code requirements. The necessary layout or pattern for the head is shown in Fig. 4-13.

4-13. The design and attachment of heads for pressure vessels constructed in accordance with the ASME-UPV Code are essentially similar to the data given in section 4-11. Representative methods of head attachment, and weld specifications and limitations are given in Fig. 4-12.

4-14. Volumes and Surface Areas of Vessel Heads. The volume and surface areas of dished and other types of heads are often required in order to

calculate vessel weights. In the following expressions, V represents the volume of the head in cubic inches, exclusive of any cylindrical volume in the flange; A represents the surface area of the curved portion of the head in square inches, exclusive of the flange; h is the depth of the head in inches, exclusive of the flange; and D is the inner diameter in inches of vessel.

For standard ellipsoidal head, whose depth is equal to one fourth the inner diameter:

$$V = \pi D^3/24 \tag{4-12}$$

$$A = 1.09D^2 \tag{4-13}$$

For a dished head, with a crown radius L,

$$h = L - \sqrt{L^2 - D^2/4} \tag{4-14}$$

$$V = 1.05h^2(3L - h) \tag{4-15}$$

$$A = 6.28Lh \tag{4-16}$$

For a conical head, with a flat spot diameter m, and an included half-cone angle A (Fig. 4-10),

$$h = \frac{\tan A(D - m)}{2} \tag{4-17}$$

$$V = 0.262h(D^2 + Dm + m^2) \tag{4-18}$$

$$A = 0.785(D + m)\sqrt{4h^2 + (D - m)} + 0.785d^2 \tag{4-19}$$

PROBLEMS—CHAPTER 4

1. What vessel classification may be used for a storage tank containing water at a pressure of 10 psi. gage and a temperature of 245° F.?

2. Like Problem 1 for a temperature of 300° F.

3. A vessel with dished heads has an inner diameter of 16 ft., and is used for storing ammonia at a pressure of 300 psi. The heads are made of S-1 steel. What vessel classification is required? Explain.

4. What vessel classification is suitable for domestic hot-water boilers?

5. Like Problem 3, for a pressure of 50 psi. gage.

6. Design a hot-water heater boiler for industrial service. The pipe line is equipped with a safety valve set to relieve at one pound gage pressure; the vessel diameter is 18 in., and the capacity is 50 gallons. ASME S-2 Grade B Steel, with welded joints, and dished heads, concave and convex to pressure are to be employed. Make a dimensioned sketch of the vessel.

7. A 5-foot diameter vessel made of flange grade steel with welded joints is to be used for petroleum storage at a pressure of 100 psi. gage. Both heads are flanged and dished, concave to pressure. Sketch the vessel, indicate the type of joints, and give the head and shell thickness.

8. Like Problem 7, for a pressure of 20 psi. gage.

9. Select flanged flat heads for the vessel of Problem 7.

10. Select heads cut from flat plate for the vessel of Problem 7, and give the theoretical thickness.

11. Select ellipsoidal heads for the vessel of Problem 7.

12. Find the theoretical thickness of hemispherical heads for the vessel of Problem 7.

13. Design a conical head for the vessel of Problem 7.

14. A tank 48 in. in diameter is used for chlorine storage at a pressure of 150 psi. gage and a temperature of 850° F. If ⅛-in. allowance is made for corrosion, determine the plate thickness and the vessel classification if S-2 Grade A steel is used, with welded joints.

15. Find the theoretical thickness, excluding corrosion, of a flanged dished head concave to pressure, for the vessel of Problem 14. S-2 Grade B steel.

16. Like Problem 15 for a head convex to pressure.

17. A vertical tower 24 ft. high and 3 ft. in diameter is filled with water at atmospheric temperature and pressure. Design the tower and select a dished bottom.

18. Design a vertical cylindrical vessel approximately 10 in. in diameter to hold one ton of metallic mercury at atmospheric temperature and pressure. The tank is to be made of S-2 Grade A steel, with welded joints, and a bottom cut from flat plate.

CHAPTER 5

MECHANICS

5-1. Applied Mechanics treats of the effects of forces on the motion of rigid bodies as applied to problems in engineering. Statics is that part of the subject that treats of bodies at rest, or moving at a constant velocity, the state or rest being considered a limiting condition of motion.

5-2. Forces and Force Systems. Any force has position, magnitude, and direction and may therefore be represented vectorially. Coplanar forces are those lying in the same plane; concurrent forces are those passing through a common point. Any body, if acted upon by one or more forces, tends to move under the action of those forces; in Fig. 5-1, for example, the body tends to move towards the top and towards the right under the action of the concurrent, coplanar forces A and B. The resultant of a system of forces is a single force that will produce the same effect that the entire system will induce. Any system of concurrent, coplanar forces may be replaced by a single resultant, although two or more resultants of a system are frequently encountered. Fig. 5-2 shows space and force diagrams for the force system of Fig. 5-1. The space diagram shows the position and direction of forces A and B; the force diagram shows the direction and magnitude of forces A and B by vectorial representation. The resultant of A and B is force R and is represented by the dotted line. The position and

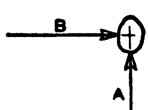


FIG. 5-1. Force System.

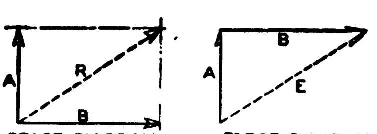


FIG. 5-2. Composition of Concurrent Forces.

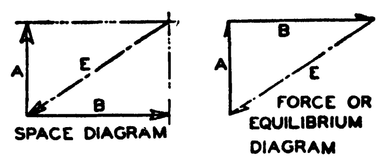


FIG. 5-3. Equilibrium of Concurrent Forces.

direction of R may be ascertained from the space diagram, Fig. 5-2; the magnitude may be scaled from the force diagram.

The equilibrant of a system of coplanar, concurrent forces is a single force that balances or neutralizes the entire system, although two or more equilibrants are sometimes used. The equilibrant has the same magnitude and position as the resultant, but acts in the opposite direction. Fig. 5-3 shows space and force diagrams for the system of Fig. 5-1, with the equilibrant E indicated by a broken

line. The space diagram gives the position and direction of the equilibrant; the magnitude of the equilibrant may be scaled from the force diagram.

In the system of Fig. 5-1, the direction of the equilibrant may be ascertained by inspection, but if a series of forces having widely different directions and magnitudes are acting at a point, the method of analysis shown in Fig. 5-4 may be used. The figure at the left is a space diagram of a force system, with four forces at a point O. The forces are designated by boundary areas; the 3-lb. force, for example, is designated as force AB, the 5-lb. force as BC, and so forth. The central figure is a force polygon in which the forces are vectorially represented by using the appended scale. The forces are laid off in regular order, beginning with AB, parallel to the force lines in the space diagram. The beginning of the vector representing force BC coincides with the end of the vector representing AB; the beginning of the vector CD coincides with the end of the vector representing BC, and so forth. The equilibrant is drawn between points

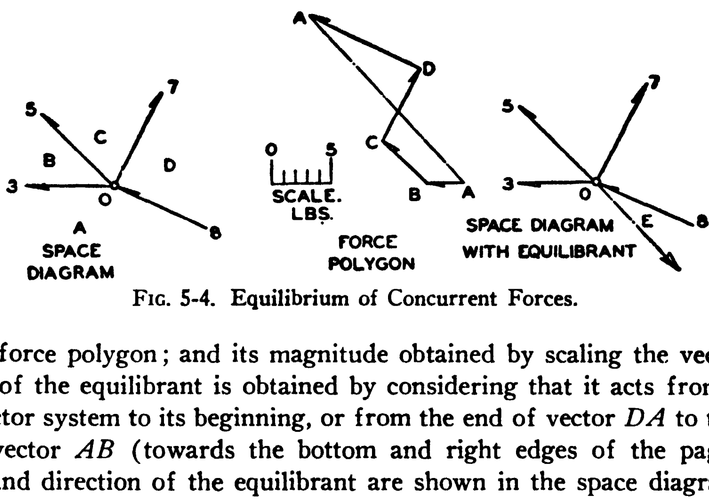


FIG. 5-4. Equilibrium of Concurrent Forces.

A in the force polygon; and its magnitude obtained by scaling the vector. The direction of the equilibrant is obtained by considering that it acts from the end of the vector system to its beginning, or from the end of vector DA to the beginning of vector AB (towards the bottom and right edges of the page). The position and direction of the equilibrant are shown in the space diagram at the right, which is a replica of that at the left with the equilibrant E added. The position of E is obtained by drawing it parallel to its representation in the force polygon.

Any force may be replaced by two or more forces termed the components of the original force. For example, in Fig. 5-2, A and B may be considered the components of R. Horizontal and vertical components are the most generally used.

5-3. Moment of a Force. The term "moment" is used to express or describe a tendency to induce rotation. This rotational tendency varies directly with the magnitude of the force and its distance from the axis of rotation. The moment of a force with respect to a point is the product of the force and the perpendicular distance from its line of action to the point, and is usually expressed in inch-pounds. In Fig. 5-6, the moment of F about point A is Fb.

The resultant of a system of parallel forces has a magnitude equal to the algebraic sum of the forces, a direction parallel to them, and a moment, with respect to any point in the system, equal to the algebraic sum of the moments of the forces about that point. In Fig. 5-5, the magnitude of the resultant is $5 + 6 + 7 + 2$ or 20 lbs. and its position is obtained by taking moments about the 5-lb. force, as follows:

$$5 \times 0 + 6 \times 3 + 7 \times 7 + 2 \times 9 = 20d$$

or

$$d = 85/20 = 4.25 \text{ in.}$$

5-4. Couples. Two parallel equal forces, opposite in direction, constitute a couple. The perpendicular distance between them is termed the arm of the couple. Rotative tendency or moment is the only effect of a couple, and is equal to the product of the arm and one of the forces. The position of the axis of rotation has no effect on the moment of the couple. To illustrate, consider two 5-lb. vertical forces 6 in. apart, with the left-hand force acting downward and

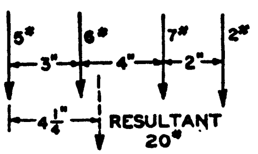


FIG. 5-5. Composition of Parallel Forces.

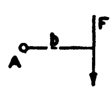


FIG. 5-6. Moment of a Force about an Axis.

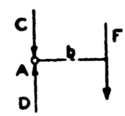


FIG. 5-7. Resolution of a Force about an Axis.

the right-hand force acting upward, then the moment of the couple is equal to 5×6 or 30 in.-lbs., this being the product of the arm and one of the forces. If the moment is calculated with respect to an axis of rotation 2 in. to the left of the right-hand force, the summation of the two moments will be $5 \times 4 + 5 \times 2$ or 30 in.-lbs., a result identical with the simpler method. If an axis of rotation 3 in. to the left of the left-hand force is assumed, the moment of the forces is $5 \times 9 - 5 \times 3$ or 30 in. lbs.

Any force at some distance from a point of reference may be resolved into a couple and a force through that point. The combination of a force and a couple to replace a single force is necessitated by the fact that the force F in Fig. 5-6 has a tendency towards downward motion as well as a moment effect with respect to point A. In Fig. 5-6, the force F, which acts at a distance b from point A, may be replaced by a couple Fb and a force F at A. This principle may be explained by reference to Fig. 5-7, which is a replica of the force system of Fig. 5-6, except that two forces C and D, each equal in magnitude to F, have been introduced at point A. Since these forces act at A, they have no moment effect on the system; since they are equal, coincident, and opposite, they have no

force effect on the system. The force system of Fig. 5-7 may be considered, however, to consist of a couple composed of forces F and D, and a force C at A. Since D is equal to F, the moment of the couple is Fb; since C is equal to F, the latter may be considered to act at A.

5-5. Resultants. Both analytical and graphical methods may be used to determine the resultant or the equilibrant of a system of non-parallel, non-concurrent forces. In the analytical determination, each force is resolved into a couple and a force through a point of reference. The algebraic summation of the moments of all the couples gives the moment effect of the entire force system. Each force is resolved into horizontal and vertical components, from which the corresponding components of the resultant are obtained by algebraic summation. The magnitude, position, and direction of the resultant are obtained from these components. The line of action of the resultant must then be located at such a distance from the point of reference that its moment will be equivalent to the moment of the entire system. The equilibrant of the system will have the same magnitude and position as the resultant, but will be opposite in direction.

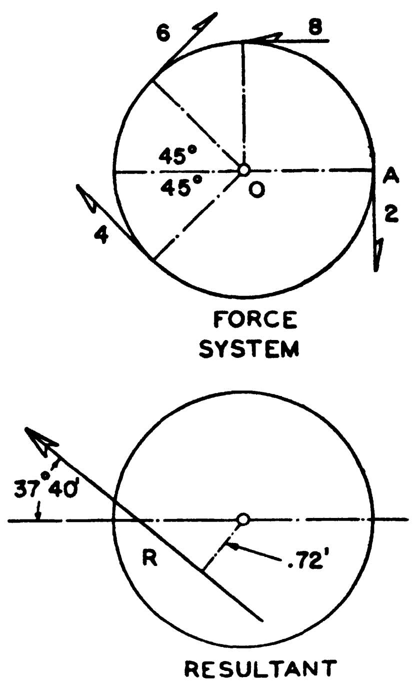


Fig. 5-8. Resolution of Non-parallel, Non-concurrent Forces.

Example 5-1. Find the resultant, with respect to the shaft axis O, of the four forces acting on the periphery of the 3-ft. diameter pulley shown in Fig. 5-8.

Solution. Horizontal components acting towards the right, vertical components acting upward, and clockwise rotation are considered positive. The resolution of the forces into components, the moments of the forces, and the summations are tabulated as follows:

Force	Moment	Horiz. Comp.	Vert. Comp.
8	−12	−8	0
2	+ 3	0	−2
4	+ 6	−2.83	+2.83
6	+ 9	+4.24	+4.24
Summation	+ 6	−6.59	+5.07

The magnitude of the resultant force is $\sqrt{6.59^2 + 5.07^2}$ or 8.32 lbs., acting towards the upper left quadrant. The angle of the resultant with the horizontal axis is one whose tangent is 5.07/6.59 or 37° 40'. The moment arm of the resultant is 6/8.32 or 0.72 ft. The actual location of the resultant is shown in the lower part of Fig. 5-8.

The graphical determination of a system of non-parallel, non-concurrent forces may be obtained as illustrated in Figs. 5-9, 5-10, and 5-11. In Fig. 5-9, at the left, a space diagram of three forces—*AB, BC,* and *CD*—is shown, the forces being designated by the boundary areas. The diagram at the right is a vectorial representation of forces *AB, BC,* and *CD.* The magnitude and direction of the resultant *AD* can be obtained directly from the diagram at the right, by drawing the vector *AD* and scaling its magnitude, but an additional construction is required to determine its position. In Fig. 5-10, if a point *P* is selected at random in the diagram at the right, the force *AB* may be resolved into two components *AP* and *PB,* termed rays. Rays *AP* and *PB,* as components of vector *AB,* will replace *AB* in the system. If ray *PC* is then drawn, the rays *BP* and *PC* will be the components of *BC* and will replace it in the system. As the direction of *BP* as a component of *BC* is opposite to that of *PB* as a component of *AB,* the two cancel each other. Similarly, in Fig. 5-11, rays *CP* and *PD* are components of and will replace *CD;* and *CP* as the component of *CD* will cancel *PC* as the component of *BC.* This leaves the system with two forces *AP* and *PD,* the components of the resultant *AD.* The position of *AD* is found in the diagrams at the left. Selecting a point 1 at random on the force line *AB,* Fig. 5-10, the lines 2-1 and 1-3, termed strings, are drawn parallel to components *PB* and *AP.*

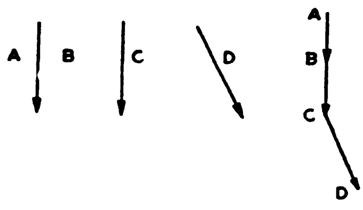


Fig. 5-9. Composition of Non-parallel, Non-concurrent Forces.

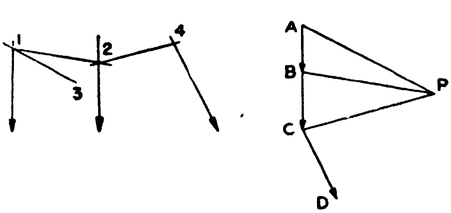


Fig. 5-10. Composition of Non parallel Non-concurrent Forces.

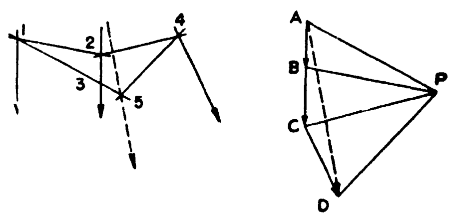


Fig. 5-11. Composition of Non-parallel, Non-concurrent Forces.

Strings 2-1 and 1-3 may be considered as replacements for force *AB.* String 4-2, drawn parallel to *PC,* and string 1-2 may be considered as replacements for force *BC.* As strings 2-1 and 1-2 cancel each other, strings 1-3 and 4-2 are replacements for forces *AB* and *BC.* Similarly, in Fig. 5-11, string 4-5, drawn parallel to *PD,* and string 2-4 may be considered replacements for force *CD,* and as strings 4-2 and 2-4 cancel each other, strings 1-3 and 4-5 may be considered replacements for all three forces *AB, BC,* and *CD.* If 4-5 and 1-3 be extended to intersect at 5, a single force through point 5 will represent the resultant of

4-5 and 1-3 and consequently of forces AB, BC, and CD. The direction of this resultant is obtained by drawing it parallel to AD. The diagram at the left in Fig. 5-11 is termed a space or funicular diagram; that at the right a force polygon.

5-6. Equilibrium. When a rigid body is subjected to the action of a system of forces, the body is said to be in equilibrium if the summation of all the forces is zero, and if the summation of all the moments about any point in the body is zero. This condition is usually expressed as follows:

Summation of horizontal components of all forces

$$\Sigma H = 0 \tag{5-1}$$

Summation of vertical components of all forces

$$\Sigma V = 0 \tag{5-2}$$

Summation of moments of all forces

$$\Sigma M = 0 \tag{5-3}$$

The above are termed the equations of equilibrium. It follows that since equilibrium denotes a constant state of motion, each part of a body must be in equilibrium if the body as a whole is in equilibrium. Further amplification of this principle will be considered in section 5-14.

Moments of Areas

5-7. In applying mechanics to engineering, the concepts of centroid, moment of inertia, and radius of gyration are useful and of extreme importance. The centroid of a mass (or area) is that point at which the mass could be concentrated to give the same moment effect as the distributed mass. The centroid of a circle is its center; the centroid of a rectangle is the point of intersection of the lines parallel to and midway between the sides; the centroid of a triangle is on a line parallel to the base and one third of the perpendicular distance from the base to the vertex: the location of one axis of these centroids is shown in Fig. 5-12 as the axis gg.

The moment of an area with respect to a given axis is termed the first moment of the area. In Fig. 5-13, the first moment of the differential area dA with respect to axis xx is ydA; the first moment of all the differential areas comprising a total area is given by

$$\int ydA = hA \tag{5-4}$$

where dA is a differential area, y is the distance from the given axis to the differential area, A is the total area, and h the distance from the given axis to the centroid of the total area. This expression may be employed to find the centroid of compound areas, as follows:

Example 5-2. Find the centroid of the tee shown in Fig. 5-14.

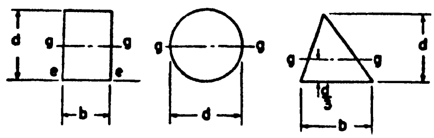
Solution. The horizontal axis of the centroid is axis xx, because it is symmetrical with respect to the two rectangles into which the area may be divided. The vertical axes of the centroids of the two separate rectangles are zz and yy, for the same reason. Taking moments about axis yy, the moment summation, from Eq. 5-4, is:

$$(3 \times 1 \times 2\tfrac{1}{2}) + (4 \times 1\tfrac{1}{2} \times 0) = h[(3 \times 1) + (4 \times 1\tfrac{1}{2})]$$

Solving,

$$h = 0.833 \text{ in.}$$

The center of gravity of the area is therefore located on axis xx, at a point 0.833 in. to the left of axis yy.



SYMB.	QUANTITY	RECTANGLE	CIRCLE	TRIANGLE
A	AREA	bd	$\pi\frac{d^2}{4}$	$\frac{bd}{2}$
I_{gg}	MOMENT OF INERTIA ABOUT CENTROIDAL AXIX gg	$\frac{bd^3}{12}$	$\frac{\pi d^4}{64}$	$\frac{bd^3}{36}$
I_{ee}	MOMENT OF INERTIA ABOUT EDGE AXIS ee	$\frac{bd^3}{3}$		$\frac{bd^3}{12}$
K	RADIUS OF GYRATION WITH RESPECT TO AXIS gg	$\frac{d}{\sqrt{12}}$	$\frac{d}{4}$	$\frac{d}{\sqrt{18}}$

Fig. 5-12. Properties of Common Sections.

5-8. Moment of Inertia. The moment of inertia of an area (also referred to as the plane moment of inertia of an area and as the second moment of an area) with respect to a given axis is the sum of the products of each elementary area and the square of its distance from the axis, and is given by:

$$I = \int y^2 dA \tag{5-5}$$

The moments of inertia of commonly used areas, with respect to the centroidal axis gg, are given in Fig. 5-12. If the moment of inertia of an area with respect to an axis other than the centroidal axis is required, it may be obtained by the following: The moment of inertia of an area with respect to any axis in its plane is equal to the sum of its moment of inertia about a parallel centroidal axis and the product of the area and the square of the distance between the axes. Referring to Fig. 5-15,

$$I_{ff} = I_{gg} + Ah^2 \tag{5-6}$$

where ff and gg are the reference and centroidal axes, h the distance between them, and A the area.

The moment of inertia, with respect to an axis *ee* for rectangular and triangular areas, is given in Fig. 5-12.

Example 5-3. Find the moment of inertia, with respect to the horizontal and vertical centroidal axes, of the tee-shaped area of Fig. 5-14.

Solution. By inspection, the line *xx* is the horizontal centroidal axis. From Fig. 5-12, for the left rectangle

$$I = 1 \times 3^3/12 = 2.25 \text{ in.}^4$$

For the right rectangle

$$I = 4 \times 1.5^3/12 = 1.125 \text{ in.}^4$$

The moment of inertia for the entire area, with respect to the *xx* axis, is

$$I_{xx} = 2.25 + 1.125 = 3.375 \text{ in.}^4$$

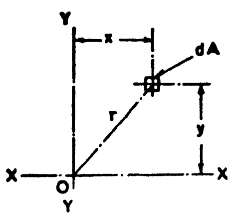


Fig. 5-13. Moment of an Area.

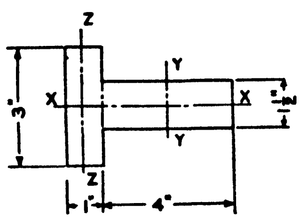


Fig. 5-14. Tee Section.

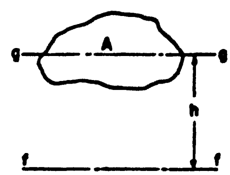


Fig. 5-15. Moment of an Area about a Noncentroidal Axis.

From Example 5-2, the vertical centroidal axis *ff* is at a distance 0.833 in. to the left of axis *yy*, and consequently 1.667 in. to the right of axis *zz*. From Fig. 5-12, for the left rectangle

$$I_{zz} = 3 \times 1^3/12 = 0.25 \text{ in.}^4$$

For the right rectangle

$$I_{yy} = 1.5 \times 4^3/12 = 8 \text{ in.}^4$$

From Eq. 5-6, the moment of inertia of the left rectangle, with respect to axis *ff*, is

$$I_{ff} = I_{zz} + Ah^2 = 0.25 + (3 \times 1 \times 1.667^2) = 8.583 \text{ in.}^4$$

The moment of inertia of the right rectangle, with respect to axis *ff*, is

$$I_{ff} = I_{yy} + Ah^2 = 8 + (1.5 \times 4 \times 0.833^2) = 12.167 \text{ in.}^4$$

The moment of inertia for the entire area, with respect to the vertical centroidal axis *ff*, is

$$I_{ff} = 8.583 + 12.167 = 20.75 \text{ in.}^4$$

5-9. Radius of Gyration. It is often convenient to express the moment of inertia of an area in terms of the total area and the square of a distance:

$$I = \int y^2 dA = k^2 A \qquad (5\text{-}7)$$

The quantity k is termed the radius of gyration, and is the distance from the axis at which the area could be considered concentrated and still have the same moment of inertia effect. A transposition of this equation gives

$$k = \sqrt{\frac{I}{A}} \tag{5-8}$$

the form in which it is commonly used.

5-10. Section Modulus. In stress analysis, particularly in beam of uniform section, a quantity equal to the moment of inertia of the area divided by the distance from the centroid to the extreme edge of the area is frequently encountered. This quantity is termed the section modulus Z, and is given by

$$Z = \frac{I}{c} \tag{5-9}$$

where c is the distance from the centroid to the extreme edge of the area.

Example 5-4. Find the section modulus, with respect to the vertical centroidal axis, and the least radius of gyration of the tee-shaped area of Fig. 5-14.

Solution. The distance from the vertical centroidal axis ff to the left edge of the area is 2.167 in., and to the right edge is 2.833 in.; the latter is the greater distance. From Ex. 5-3, I_{ff} is equal to 12.167, and the section modulus, from Eq. 5-9, is

$$Z = \frac{12.167}{2.167} = 5.61 \text{ in.}^3$$

The least radius of gyration depends upon the minimum value of I, and the area of the figure (9 sq. in.). From Ex. 5-3, the value of I_{xx} is 3.375, and the least radius of gyration, from Eq. 5-8, is

$$k = \sqrt{\frac{3.375}{9}} = 0.612 \text{ in.}$$

5-11. Product and Polar Moment of Inertia. In Fig. 5-13, if the differential area dA is multiplied by the product of its coordinates with respect to point O, the resulting quantity is termed the product of inertia of the differential area. The product of inertia of a total area with respect to a pair of coordinate axes is, therefore,

$$P = \int xy\,dA \tag{5-10}$$

The moment of inertia of a plane area about an axis perpendicular to the plane is called the polar moment of inertia, and from Fig. 5-15 is given by

$$J = \int r^2 dA \tag{5-11}$$

From the figure,

$$I_{xx} = \int y^2 dA$$

and

$$I_{yy} = \int x^2 dA$$

but

$$r^2 = x^2 + y^2$$

therefore

$$J = \int y^2 dA + \int x^2 dA = I_{xx} + I_{yy}$$

From Fig. 5-12, for a circle

$$I_{xx} = I_{yy} = I_{gg} = \frac{\pi d^4}{64}$$

$$J = I_{xx} + I_{yy} = \frac{\pi d^4}{64} + \frac{\pi d^4}{64} = \frac{\pi d^4}{32}$$

The polar section modulus is equal to the polar moment of inertia divided by the extreme edge distance. For a circle, the polar section modulus is

$$\frac{J}{d/2} = \frac{2\pi d^4}{32d} = \frac{\pi d^3}{16}$$

Beams

5-12. A beam is a comparatively rigid, horizontal member, subjected to parallel transverse forces which may be either coplanar or in intersecting planes. Beams may be statically determinate or indeterminate. Determinate beams are those in which it is possible to compute the magnitude, position, and direction of the reacting or supporting forces by the method of statics; while indeterminate beams are those in which it is necessary to consider the deflection or deformation of the beam in the force analysis. Fig. 5-16 illustrates several important types of statically determinate beams for various loadings. The beam at A is a simply supported or simple beam with a single concentrated load. B shows a single-overhung beam with uniformly distributed loads over portions of its length. C shows a double-overhung beam with uniformly varying loads at both ends. D shows a cantilever, or single-support beam with a concentrated load at its free end; this type of beam is statically determinate only for the beam extension, and not for the portion built into the wall or support. A concentrated load in actuality is never applied at a point; it must extend over an appreciable area, no matter how small, but is considered concentrated for convenience in analysis and computation.

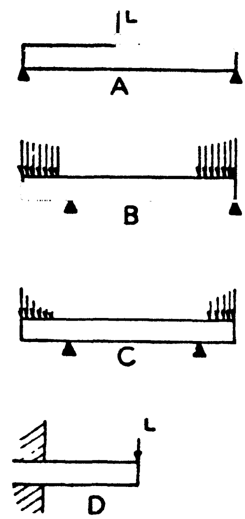


Fig. 5-16. Types of Beams and Beam Loading.

5-13. Determination of Beam Reactions. A reaction is defined as an equilibrating or opposing force, and is effected by the pressure between a particle and the rigid body with which it is in contact. Two smooth surfaces in contact are assumed to induce a reaction perpendicular to the surfaces; in the case of a beam, the reactions are perpendicular to the horizontal surface of the beam and are consequently parallel to the vertical forces acting on the beam. In deter-

minate beams, the supporting forces, or beam reactions, are computed by applying the equations of equilibrium, Eqs. 5-1, 5-2, and 5-3.

Example 5-5. Find the reactions for the beam of Fig. 5-17.

Solution. As there are no horizontal forces

$$\Sigma H = 0$$

In the summation of the vertical forces, those acting downward are considered negative, and those acting upward positive. The weight of the beam is equal to the product of the weight per foot of length and the length, and is 10×12 or 120 lbs.

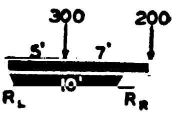


FIG. 5-17. Single-overhung Beam.

$$\Sigma V = -300 - 200 - 120 \pm R_L \pm R_R = 0$$

Both positive and negative signs are given for the reactions R_L and R_R; although it is known that the sum of the reactions must be positive, it is not always possible to determine the character of an individual reaction by inspection.

In considering the moment summation, there are two reactions, each unknown in both direction and magnitude. To eliminate one of the unknown reactions, the origin of moments is taken at R_L. Clockwise direction of rotation is considered positive, counterclockwise rotation negative. The beam weight, 120 lbs., is considered concentrated at the center of gravity of the beam or 6 ft. from either end.

$$\Sigma M \text{ about } R_L = \pm(R_L \times 0) + (300 \times 5) + (120 \times 6) \pm (R_R \times 10) + (200 \times 12) = 0$$

Solving,

$$\Sigma M = 4620 \pm 10R_R = 0$$

From the above, the moment induced by R_R must be negative, or counterclockwise. The right reaction must consequently act upward, and has a magnitude of 462 lbs.

Taking moments about R_R,

$$\Sigma M \text{ about } R_R = \pm(R_L \times 10) - (300 \times 5) - (120 \times 4) \pm (R_R \times 0) + (200 \times 2) = 0$$

Solving,

$$\Sigma M = \pm 10R_L - 1580 = 0$$

The moment induced by R_l must be positive or clockwise. The left reaction will act upward, and has a magnitude of 158 lbs.

Restating the vertical summation,

$$\Sigma V = -300 - 200 - 120 + 158 + 462 = 0 \qquad \text{(check)}$$

5-14. Stresses in Beams. Fig. 5-18 shows the front view of a portion of a beam. If the beam as a whole is in equilibrium, then any section, such as the portion to the left of plane *cc*, must be in equilibrium. Section *cc* is apparently in horizontal equilibrium, because no horizontal forces are indicated; but it is acted upon by a vertical force R, apparently producing both an unbalanced vertical summation and an unbalanced moment of magnitude Ra. Obviously, for this section of the beam to remain in equilibrium, the section *cc* of the beam itself must provide a balancing vertical force, and a balancing moment.

Fig. 5-19 represents section *cc* of the beam with a ball-and-socket arrangement at the center, a small strut at the top, and a chain at the bottom. The ball-and-socket arrangement furnishes a vertical force F_3, balancing the upward

vertical force R, thus producing vertical equilibrium. The rotative effect of R is balanced by the strut giving the force F_1 at distance b from the center of rotation, and by the chain providing the force F_2 at distance b from the center of rotation. For Fig. 5-19, the equations of equilibrium are,

$$\Sigma H = -F_1 + F_2 = 0$$

$$\Sigma V = -F_3 + R = 0$$

$$\Sigma M = +Ra - F_1 b - F_2 b = 0$$

In the actual beam section, Fig. 5-20, it is seen that the vertical force F_3 is supplied by the shearing resistance of the section, and that the horizontal forces

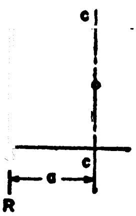


Fig. 5-18. Free-body of Beam End.

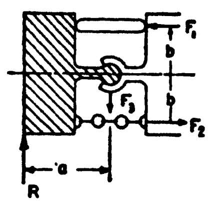


Fig. 5-19. Symbolic Representation of Beam Section Under Stress.

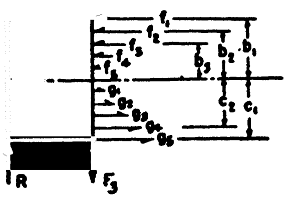


Fig. 5-20. Internal Resisting Forces in a Beam Section.

F_1 and F_2 are replaced by varying stresses along the beam section. In this figure, the equations of equilibrium are:

$$\Sigma H = -f_1 - f_2 - f_3 \ldots + g_1 + g_2 + g_3 \ldots = 0$$

$$\Sigma V = +R - F_3 = 0$$

$$\Sigma M = -(f_1 \times b_1) - (f_2 \times b_2) - (f_3 \times b_3) \ldots$$
$$+ (g_1 \times c_1) + (g_2 \times c_2) + (g_3 \times c_3) \ldots = 0$$

The important problem in beam design is to determine the magnitude of the summation f_1, f_2, f_3, g_1, g_2, g_3, . . . etc. In this determination, the following basic assumptions are made:

a. The tensile and compressive stress values of the beam material are essentially the same in magnitude.

b. The proportional limit of the material is not exceeded or, stated in another manner, the deformations vary directly as the stresses.

c. The elastic limit of the material is not exceeded, or the material will not take a permanent "set."

d. The entire transverse section of the beam, originally plane, remains plane and normal to the longitudinal beam fibers.

Fig. 5-21 represents a beam section of general form, perpendicular to the span of the beam. The line zz is a reference axis somewhere between the upper and lower edges of the section. The differential area dA, equal to xdy, is located a distance y from zz. Dimension c is the distance from zz to the extreme edge or fiber of the beam section. If S represents the unit tensile or compressive stress at the extreme fiber, then S/c will represent the unit stress at a unit distance from the reference axis zz, and Sy/c the unit stress at a distance y from axis zz. The resisting force of the differential area is equal to the product of the unit stress and the area, or $(Sy/c) \times xdy$, or $SydA/c$. The resisting moment of this resisting force is equal to the product of the force and the distance, which is equal to $(SydA/c) \times y$, or Sy^2dA/c.

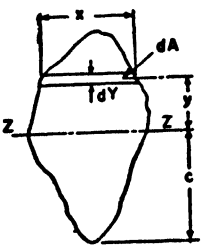


FIG. 5-21. Section of a Beam Perpendicular to Its Length.

For equilibrium, the summation of the resisting moments of all the differential areas in the beam section must equal the external bending moment. The external bending moment M is equal to $S/c\int y^2dA$. As the moment of inertia about axis zz is I_{zz} or $\int y^2dA$, M is equal to SI/c, or

$$S = \frac{Mc}{I} \qquad (5\text{-}12)$$

From the above, it may be seen that the unit stress s at distance y is My/I.

Equation 5-12 is often written in the form

$$M = SZ \qquad (5\text{-}13)$$

where Z, the section modulus, is employed because it serves as a direct measure of the strength of a beam.

The horizontal force on area dA is $SydA/c$. The horizontal summation over the entire section is $\int SydA/c$. From Eq. 5-4, $\int ydA$ is equal to hA. For equilibrium, the summation $\int SydA/c$ must equal zero, and ShA/c is therefore zero. As S/c and A are finite quantities, h must be equal to zero. It follows that the axis of reference zz is therefore at the centroid of the section. The axis zz is termed the Neutral Axis of the beam section since the stress is zero at this line.

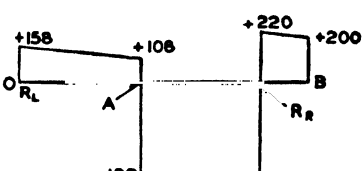


FIG. 5-22. Shear Diagram with Loads and Reactions Concentrated at Points.

5-15. Shear Diagrams. Fig. 5-22 shows a graph of the vertical forces on the beam of Example 5-1, Fig. 5-17, and is called a shear diagram. The horizontal line OB represents the beam length, and the ordinates the magnitude of the shearing forces at any point. A casual inspection of Fig. 5-22 would indicate that the shear-

ing stress at point A is apparently multi-valued, or (+108), (0), or (−192) pounds. This apparent inconsistency is explained in Fig. 5-23, in which the loads and reactions affecting the beam are shown as they are actually applied—over finite lengths. A reference to the actual or true shear diagram in this figure shows that no ambiguity exists; the shearing stress at point A is zero; the stresses immediately to the left and right of A are (+108) and (−192) pounds. The diagram of Fig. 5-22 is always used, however, for the sake of simplicity.

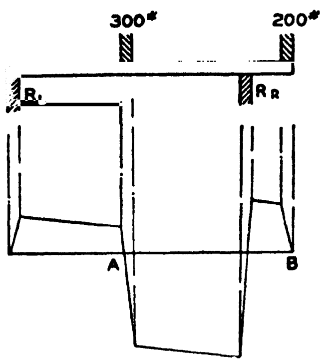


Fig. 5-23. Shear Diagram with Distributed Loads and Reactions.

Fig. 5-24 shows a portion of a beam with a distributed and several concentrated loads. For equilibrium, the moment at any point must equal zero and is equivalent to the summation of all the moments about that point. The moment at section zz may be written as

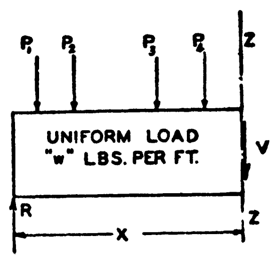


Fig. 5-24. Free-body of General Beam.

$$M = Rx - P_1(x-a) - P_2(x-a-b) - P_3(x-a-b-c)\ldots - wx^2/2$$

or, $M = Rx - P_1x - P_2x - P_3x \ldots - wx^2/2 + P_1a + P_2a + P_3a \ldots$

Taking the first derivative of this moment:

$$\frac{dM}{dx} = R - P_1 - P_2 - P_3 \ldots - wx$$

But $(R - P_1 - P_2 - P_3 \ldots - wx)$ is equal to the algebraic summation of the vertical forces to the left of section zz, which must be equal to the vertical shear V on the section. From the calculus, it is known that if the first derivative of an expression is set equal to zero, the resulting values of the variable give the maxima and minima of the original expression. As the vertical shear is equal to the first derivative of the moment, and the shear V at any section is zero, the moment M at that section must be either a maximum or a minimum. In Fig. 5-22, for example, V is equal to zero at R_L, A, R_R, and B, and the bending moments at these points are therefore either maxima or minima.

The area of the shear diagram at any section is $\int V dx$. If

$$\frac{dM}{dx} = V$$

then $$\int dM = \int V dx = M$$

that is, the area of the shear diagram either to the right or to the left of any point gives the magnitude of the bending moment at that point. (The algebraic sum of the total area of the shear diagram must of course equal zero.) As applied to the beam of Fig. 5-17, for which the shear diagram is shown in Fig. 5-22, the moments are

$$\begin{aligned}
M \text{ at } R_L \text{ (area to left)} &= 0 \text{ (min.)} \\
M \text{ at } A \text{ (area to left)} &= 5(158 + 108)/2 = 665 \text{ lbs. (max.)} \\
M \text{ at } A \text{ (area to right)} &= 5(192 + 242)/2 - 2(220 + 200)/2 \\
&= 665 \text{ lbs. (max.)} \\
M \text{ at } R_R \text{ (area to right)} &= 2(220 + 200)/2 = 420 \text{ lbs. (max.)} \\
M \text{ at } B \text{ (area to right)} &= 0 \text{ (min.)}
\end{aligned}$$

If this beam is of uniform section throughout, it should be designed for the maximum moment, which exists at *A*. By such use of the shear diagram, both the position and magnitude of the maximum moment can be found.

5-16. Graphical Determination of Beam Moments. Beam reactions and moments may be obtained by graphical construction. Fig. 5-25 shows the load, funicular and force diagrams for a simple beam in which the uniform load caused by the beam weight is neglected. The force diagram is constructed by laying out the 600- and 800-lb. loads *AB* and *BC* in order, selecting a pole point *P* at random, and drawing rays *AP, BP,* and *CP*. In the funicular diagram *V*, strings *KM, MN,* and *NQ* are drawn parallel to *AP, BP,* and *CP,* intersecting the 600- and 800-lb. force position lines at *M* and *N*, and the lines of the reactions R_L and R_R at *K* and *Q*. In diagram *W, AP* and *PC* may be considered the components of the resultant of the loads *AB* and *BC*, and are also the equilibrants of force *CA*, which is consequently the equilibrant of *AB* and *BC*. Diagrams *X* and *Y* are replicas of *V* and *W*. String *KQ* is drawn in diagram *X*. If *PS* in diagram *Y* is drawn parallel to *QK*, then *AP* and *PS* are the equilibrants of *SA*, the left reaction R_L, and *SP* and *PC* are equilibrants for *CS*, the right reaction R_R. The magnitudes of R_L and R_R may be scaled from diagram *Y*.

The bending moment at any point in a beam may be obtained from the diagrams that give the reactions. In Fig. 5-26, portions of the funicular and force diagrams for a beam are shown; strings *FD* and *CF* are parallel respectively to rays *GP* and *PK*. Axis *xx* indicates the section in the beam length at which the moment is to be found. From the funicular diagram, it may be seen that the

moment of force AB with respect to axis xx is $y \times AB$. In the two diagrams, triangles FCD and PKG are similar, consequently,

$$y : CD = JP : GK$$

or

$$CD \times JP = y \times GK = y \times AB$$

as GK represents force AB laid out to scale. The product $CD \times JP$ represents the bending moment at axis CD. JP is termed the pole distance of the force diagram and is the perpendicular distance from the origin or pole to the line of the forces. The moment at any point in a beam is the product of the intercept between the strings and the pole distance. All measurements in the funicular diagram are linear, either inches or feet; all measurements in the force diagram are in pounds; the moment has dimensions of either inch-pounds or foot-pounds.

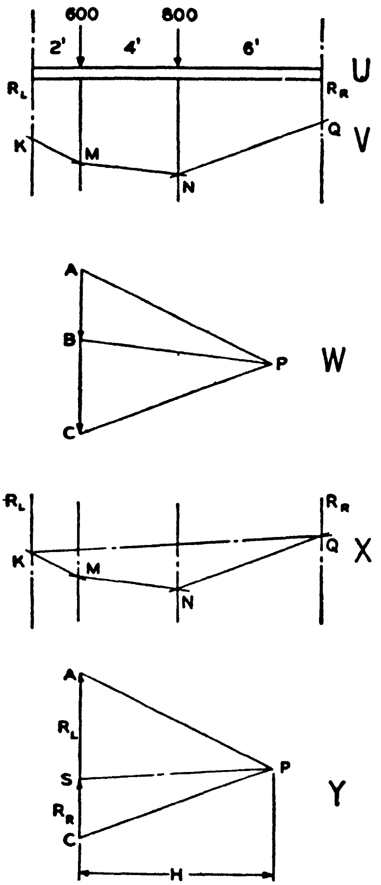


FIG. 5-25. Graphical Method of Finding Reactions and Bending Moments for a Beam.

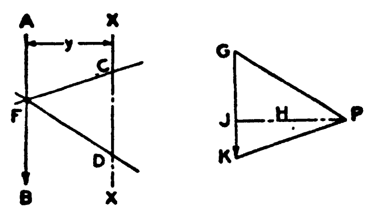


FIG. 5-26. Beam Moment Obtained from Force and Space Diagrams.

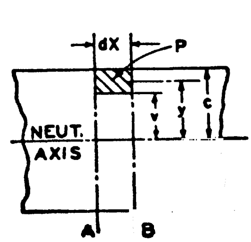


FIG. 5-27. Portion of Beam Subjected to Longitudinal Shear.

5-17. Longitudinal Shear. In any beam, there is usually some variation in the magnitude of the bending moment along the beam span. In a uniformly loaded, simply supported beam, for example, the bending moment varies from zero at either reaction to a maximum at the center of the span. Fig. 5-27 shows

a portion of the front view of a beam of uniform cross section, with two vertical sections A and B, a distance dx apart.

A small prism or block, P, with an upper edge coinciding with the upper surface of the beam, and a lower surface at a distance v from the neutral axis, is shown in the figure. The dimension y is the distance from the neutral axis to the centroid of the block P. If the positive bending moments at sections A and B are M_A and M_B, the average unit compressive stresses at the sections are $M_A y/I$ and $M_B y/I$. The total compression on the left and right ends of the small block bounded by these sections is:

$$\text{Left end } \frac{M_A}{I}\int_v^c y\,dA$$

$$\text{Right end } \frac{M_B}{I}\int_v^c y\,dA$$

The resultant force on the block in the direction of the beam length is the difference of these integrals, or

$$\frac{M_B - M_A}{I}\int_v^c y\,dA$$

This force must be balanced by a horizontal shearing force at the bottom of the block. If the breadth of the block and beam perpendicular to the plane of the paper is b, the total area in shear is bdx. If the horizontal unit shearing stress is S_h, the total shearing force is $S_h bdx$.

Equating the forces, $$S_h bdx = \frac{M_B - M_A}{I}\int_v^c y\,dA$$

or $$S_h = \frac{M_B - M_A}{Ibdx}\int_v^c y\,dA$$

Since $(M_B - M_A)$ is equal to dM, then $(M_B - M_A)/dx$ will equal dm/dx or V, the vertical shear in the beam. Substituting, S_h will equal

$$\frac{V}{Ib}\int_v^c y\,dA$$

From Eq. 5-4, $\int_v^c y\,dA$ is equal to hA, where h is the average distance from the neutral axis, and therefore

$$S_h = \frac{VhA}{Ib} \qquad (5\text{-}14)$$

where S_h is the unit horizontal shearing stress, V is the total vertical shear, I is the moment of inertia, b the width, and A the area outside the shear plane of the beam section.

The unit horizontal shearing stress at the neutral surface of a rectangular beam is one and one-half times as great as the average unit shear across the vertical section of the beam. To illustrate, Fig. 5-28 represents the end view or vertical section of a rectangular beam. The average unit vertical shearing stress is V/bd. From Eq. 5-14, in which h equals $d/4$, and I equals $bd^3/12$,

$$S_h = \frac{V}{(bd^3/12)(b)} \times \frac{d}{4} \times \frac{bd}{2} = \frac{3V}{2bd} \quad (5\text{-}15)$$

which is one and one-half times as great as the average unit vertical shear.

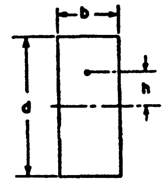


Fig. 5-28. Beam Section Undergoing Longitudinal Shear.

The longitudinal shear is of considerable importance in the design of wooden beams of comparatively heavy loads and short spans, because the shear, rather than the flexure, may be the controlling stress.

Example 5-6. Find the maximum concentrated load that may be carried at the center of the span of a 6 × 8 in. beam made of southern long-leaf pine, which has allowable flexural and shearing stresses of 1400 psi. and 100 psi.

Solution.

Case a: 24-ft. span.

For a load W, the end reactions are $W/2$, and the moment at the center is $(W/2)12 \times 12$ or $72W$ in.-lbs. The moment of inertia I is $bd^3/12$ or $6 \times 8^3/12$ or 256 in.^{4} From Eq. 5-12

$$72W = \frac{1400 \times 256}{4}$$

and $$W = 1240 \text{ lbs.}$$

The average unit vertical shear is $620/6 \times 8$ or 12.9 psi. The longitudinal shear is therefore 12.9×1.5 or 19.4 psi., a very low value compared to the allowable shear of 100 psi.

Case b: 3-ft. span.

For a load W, the moment at the center is $(W/2)1\frac{1}{2} \times 12$ or $9W$ in. lbs. From Eq. 5-12

$$9W = \frac{1400 \times 256}{4}$$

and $$W = 9920 \text{ lbs.}$$

The average unit longitudinal shear is $1\frac{1}{2} \times 4960/6 \times 8$ or 155 psi. Since this is greater than the allowable unit longitudinal shear of 100 psi., the permissible reaction V is found from Eq. 5-15 to be

$$V = \frac{2 \times 6 \times 8 \times 100}{3} = 3200 \text{ lbs.}$$

The allowable load W at the center of the span is equal to twice the reaction or vertical shear, and is 6400 lbs.

Eccentric Loads and Columns

5-18. Both machine and structural members are often subjected to a combination of flexural stresses and direct tensile or compressive stresses.

Fig. 5-29 shows a short block subjected to an eccentric load P, which (from section 5-4) may be replaced by a load P at the neutral axis and a couple whose moment is Pe. The central figure represents the distribution of the compressive stress S_c induced by the direct load, the tensile and compressive stresses S'_t and S'_c induced by the couple, and the resultant tensile and compressive stresses S_t and S_r. The effect of the stresses induced by the couple is to reduce the compressive stresses at one edge of the block and increase them at the other. Fig. 5-30 shows an example of large eccentricity and the tendency towards decrease and increase in eccentricity caused by tensile and compressive loads; it is obvious that an increase in tensile loading may actually reduce the unit stress in the vertical section of the C-shaped object by reducing the effective eccentricity from e_1 to e_2. On the other hand, an increase in compressive loading usually results in a stress increase because of the change in eccentricity from e_1 to e_3.

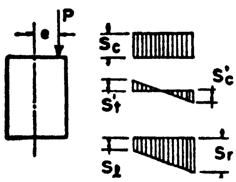


FIG. 5-29. Eccentrically-loaded Member.

In Fig. 5-29, the direct stress caused by the load is equal to P/A, where A is the cross-sectional area subjected to stress. The moment M of the couple is $P \times e$, and the resultant flexural stress is equal to M/Z or Pe/Z. The resultant stress S_r is therefore

$$S_r = P/A \pm Pe/Z \qquad (5\text{-}16)$$

If the resultant stress S_r is negative, the character of the stress at one edge of the section is opposite to that at the other.

In most instances, the permissible magnitude of stress S_r is based upon either the allowable flexural, tensile, or compressive stress, whichever is smaller. In columns, however, if the induced flexural stress is small compared to the induced compressive stress, the utilization of the allowable compressive stress for design results in an unnecessarily large member. If the allowable axial and flexural unit stresses are represented by S_a and S_x, then

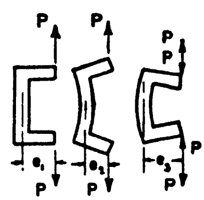


FIG. 5-30. Effect of Eccentric Load.

$$S_x = Pe/Z = Pec/I = Pec/Ak^2$$

and

$$S_a = P/A$$

The necessary area will be

$$A = \frac{P}{S_a} + \frac{Pec}{S_x k^2}$$

Substituting values of S_c and S_f,

$$A = \frac{S_c A}{S_a} + \frac{S_f A}{S_x}$$

or,

$$\frac{S_c}{S_a} + \frac{S_f}{S_x} \leqq 1 \qquad (5\text{-}17)$$

The above expression leads to the conclusion that if the sum of the quotients of the actual over allowable stresses in compression (or tension) and flexure are equal to or less than unity, the member design is satisfactory.

In some materials where the tensile resistance is negligible or small compared to the compressive resistance, such as brick, concrete, and cast iron, any type of loading that may superimpose a tensile stress on a compressive stress, and thereby give a resultant net tension, is likely to be dangerous. The condition should be considered in design; unreinforced concrete bases and piers subjected to eccentric loads, for example, are usually constructed so that under no condition of load application can any portion of the section be subjected to tensile stresses.

5-19. Columns and Struts. Machine and structural members subjected to compressive forces are termed columns or struts. Column failure may occur by pure compression, as in a cubical block; by buckling, as illustrated by a yardstick subjected to axial loads; or by a combination of these phenomena. Such column action is dependent upon the slenderness ratio of the body, which is generally expressed as L/k, where L is the column length in inches and k is the least radius of gyration of the section about a centroidal axis. If we consider a column whose length is great compared to its least cross-sectional dimension, with a value of L/k in excess of 200, failure usually occurs by buckling, and the magnitude of the critical unit load at which failure is imminent is given by

$$P/A = \pi^2 E/(L/k)^2 \quad (5\text{-}18)$$

Here P is the total load, A the gross cross-sectional area, E the modulus of elasticity, psi., and L/k the slenderness ratio.

Eq. 5-18, known as Euler's equation for long columns, does not include any effect produced by direct compression, and is valid only for columns with round ends which are free to turn at the supports. The derivation of Euler's equation is based upon the differential equation for the moment in a beam, or $EI(d^2y/dx^2)$. (See Eq. 5-27.) Fig. 5-34, curve A, shows the theoretical relationship between the critical unit load P/A and the slenderness ratio L/k for ordinary commercial steels having a modulus E of 29×10^6. Experimental data indicate that the theoretical values of the critical unit load given by this curve are reasonably accurate for slenderness ratios greater than 200, but are far too high for the lower ranges of L/k because of the complications caused by the effect of direct stresses.

Fig. 5-31 illustrates theoretical end conditions which may be present in structural and machine columns. The contact ends of round-end columns, Fig. 5-31A, are laterally guided, so that they remain in vertical alignment, but the ends are not perfectly frictionless, and free turning cannot be realized in practice. Even test columns that are supported in spherically-seated bearings develop enough friction at the ends to prevent free turning. Fig. 5-31B shows a fixed-end column in which the effective length L_B is equal to one half the actual

length L_A. Theoretically this column will resist four times the buckling load that can be carried by a round-end column having the same length L_A. Fig. 5-31C

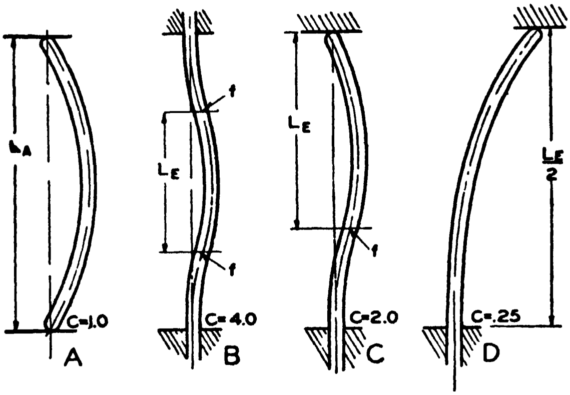


FIG. 5-31. End Conditions in Columns.

shows a column fixed at one end and laterally guided at the other so that vertical alignment is maintained; in theory, the effective length L_E is equal to $L_A/\sqrt{2}$. Fig. 5-31D shows a column with one end fixed and the other free to move laterally. The theoretical effective length L_E is equal to $2L_A$, and thus this column has only one fourth the resistance to buckling as a round-end column of the same length.

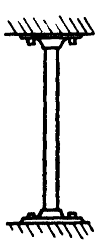


FIG. 5-32. Pipe Column.

To allow for the effect of column end conditions Euler's equation may be written:

$$P/A = C\pi^2E/(L/k)^2 \qquad (5\text{-}19)$$

where C is a factor dependent upon end conditions, theoretical values of which are given in Fig. 5-31. In practice, however, it is seldom desirable to employ these theoretical values of C. As an illustration consider the pipe column shown in Fig. 5-32, in which the ends of the column are screwed into flanges bolted in place. This end condition resembles a fixed-end connection and may act as such, but there is usually sufficient elasticity in the bolts and flanges, as well as some misalignment of the seating surfaces, to make it quite dangerous to employ a value of C equal to 4. Fig. 5-33 shows a compression link employed in a forging machine. The column ends are pin-connected and are very carefully fitted into the forks. The radius of gyration about axis xx is less than about yy, but the end condition with respect to axis xx is analogous to that of Fig. 5-31B, while

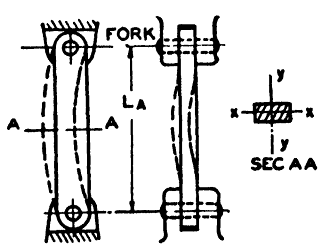


FIG. 5-33. Compression Link of Forging Machine.

the end condition with respect to axis *yy* is pin-connected and similar to that of Fig. 5-31A. Actually, the column should be investigated for failure about both axes, using a value of unity for factor C with respect to the *yy* axis, and a value of from 2 to 3 for C with respect to the *xx* axis, depending upon the type of fit between the pin and the hole. For bolted or riveted flat end columns, or for columns that are welded at both ends (such as tubular structures in airplane frames) a value of C equal to 2 is often employed.

5-20. Machine Columns. For machine parts, columns with an L/k ratio less than 40 are usually designed on the basis of the prismoidal equation for direct compression,

$$S_c = \frac{P}{A} \tag{5-20}$$

For L/k ratios between 40 and 120 failure usually occurs by a combination of buckling and direct compression, and the following empirical equation is employed to determine the critical stress:

$$\frac{P}{A} = S\left[1 - \frac{S_y(L/k)^2}{4\pi^2 CE}\right] \tag{5-21}$$

S is the equivalent or significant stress and S_y is the yield point of the material. If S is made equal to S_y, P/A is equivalent to the critical unit load at which failure is imminent. In practice, S is usually selected as from one-half to one-third S_y, corresponding to an apparent factor of safety of from 4 to 6. The use of proper values of C must be considered in the same way as discussed with relation to Eq. 5-19.

For machine columns with an L/k ratio greater than 120, Eq. 5-19 is usually modified to

$$P/A = C\pi^2 Ef/(L/k)^2 \tag{5-22}$$

The factor f is introduced so that the unit load P/A is equated to the safe unit load P/A from Eq. 5-21 at a value of L/k of 120. To illustrate, assume a round-end column made of SAE 1020 steel, with an ultimate strength of 60,000 psi. and a yield point of 30,000 psi. The unit load at L/k equal to 120, on the basis of a design stress S equal to $S_y/2$, by Eq. 5-21, is

$$\frac{P}{A} = 15{,}000\left[1 - \frac{30{,}000(120)^2}{4 \times 1 \times \pi^2 \times 29 \times 10^6}\right] = 9300 \text{ psi.}$$

Equating this value to Eq. 5-22,

$$9300 = \frac{1 \times \pi^2 \times 29 \times 10^6 \times f}{120^2}$$

and

$$f = 0.469$$

The factor f is introduced so that approximately the same degree of safety will be present in the higher ranges of the slenderness ratio as in the slenderness

ratio range between 40 and 120, where the design stress S is some proportion (in this case one half) of the yield point of the material.

The allowable unit loads calculated by Eq. 5-21, for a design stress S equal to 15,000 psi., and L/k ratios between 40 and 120, are shown by curve D, Fig. 5-34. Curve F shows the continuation of these values, calculated by Eq. 5-22 for L/k ratios greater than 120, using a value of factor f of 0.469. Curves E and G are similar to D and F, but are based upon design stress of one-third S_y or 10,000 psi., and the corresponding value of f in Eq. 5-22. It may also be seen that the maximum value of P/A in Eq. 5-21 is used as the unit stress in Eq. 5-20 for L/k ratios less than 40, where direct compression controls.

5-21. Structural Columns. Structural steel column design is based upon two empirical equations:

For L/k ratios less than 120,

$$\frac{P}{A} = 17{,}000 - 0.485(L/k)^2 \tag{5-23}$$

and for L/k ratios greater than 120,

$$\frac{P}{A} = \frac{18{,}000}{1 + \dfrac{(L/k)^2}{18{,}000}} \tag{5-24}$$

The above equations are part of the specifications of the American Institute of Steel Construction (AISC), and are represented by curve B, Fig. 5-34.

Another class of equations frequently used in structural design are the "straight-line" column equations, so called because the allowable unit loads vary inversely as the slenderness ratio. One of the straight-line equations, adopted by the American Railway Engineering Association (AREA), is

$$\frac{P}{A} = 16{,}000 - 70(L/k) \tag{5-25}$$

This expression has a maximum value of 14,000 psi. and is limited to a maximum slenderness ratio of 150. It is represented by curve C, Fig. 5-34.

Cast iron columns are usually designed on the basis of

$$\frac{P}{A} = 9000 - 40(L/k) \tag{5-20}$$

The slenderness ratio should not exceed 70. It is represented by curve H, Fig. 5-34.

Deflection of Beams

5-22. Since all structural materials are elastic and therefore subject to some deformation, any moment applied to a beam will cause it to bend and deflect

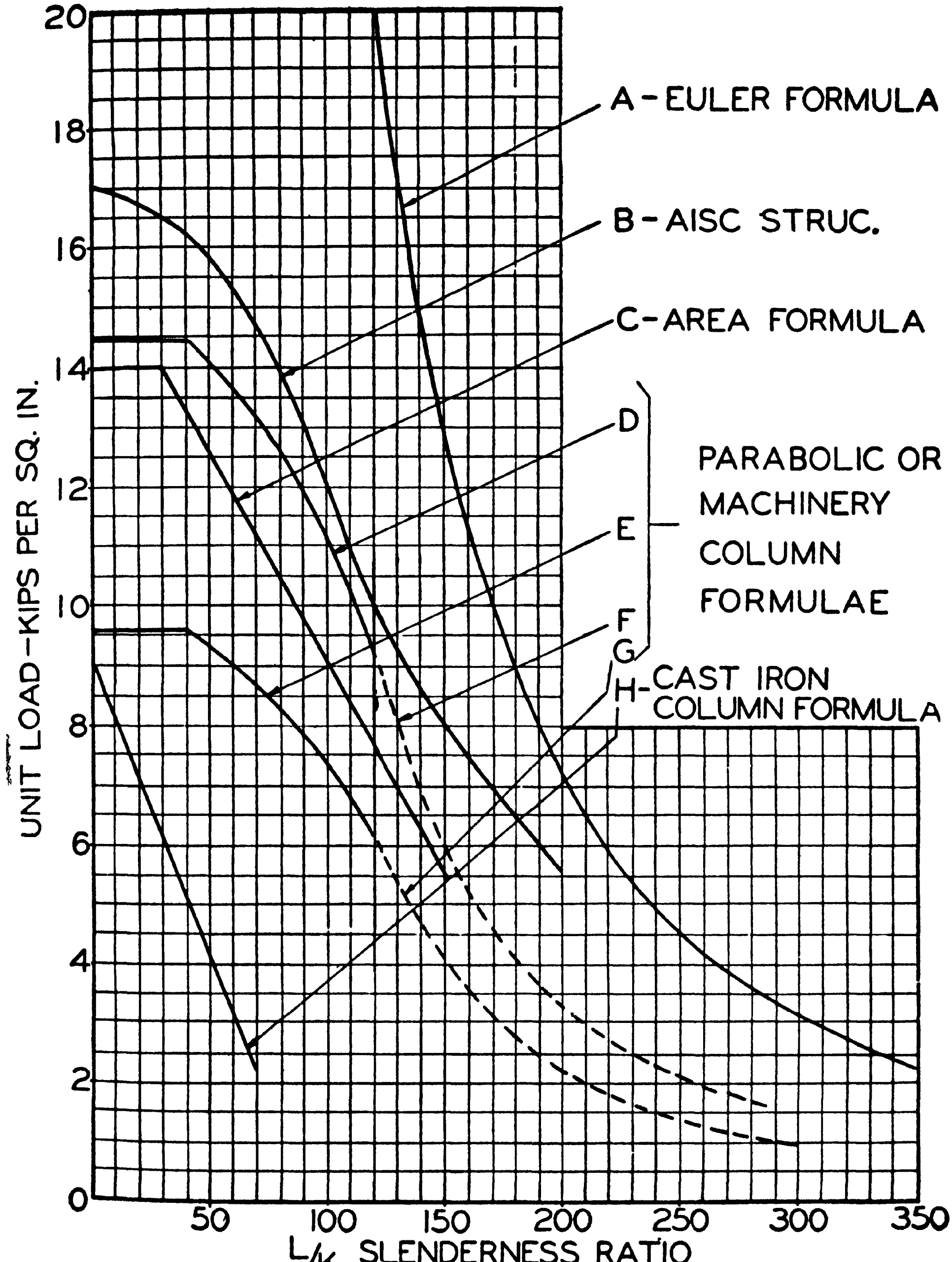


Fig. 5-34. Comparisons of Unit Loads and Slenderness Ratios of Columns

to some extent. The analysis of beam deflections is based upon the same assumptions employed for analyzing beam stresses (section 5-14).

The radius of curvature R of a beam of uniform section is given by

$$R = \frac{EI}{M} \tag{5-26}$$

and indicates that with a constant moment of inertia I and modulus of elasticity E, the radius of curvature varies inversely as the bending moment M. From this

BEAM DEFLECTIONS

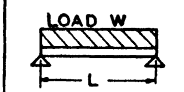
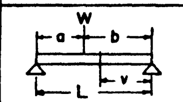
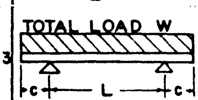
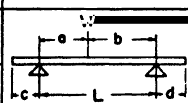
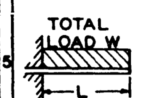
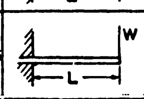
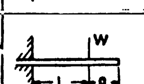
	TYPE OF BEAM	MAXIMUM DEFLECTION y	POSITION OF MAXIMUM DEFL.
1		$\frac{5WL^3}{384EI}$	AT CENTER
2		$\frac{Wav^3}{3EIL}$	WITHIN LONGER SEGMENT, AT $v = b\sqrt{\frac{1}{3} + \frac{2a}{3b}}$
3		$\frac{Wc}{24EIL}\left[3c^2(c+2L)-L^3\right]$	AT ENDS
		$\frac{WL^2}{384EIL}(5L^2-24c^2)$	AT CENTER
4		LIKE NO. 2	BETWEEN SUPPORTS
		$-\frac{Wabc}{6EIL}(L+b)$	AT LEFT END
		$-\frac{Wabd}{6EIL}(L+a)$	AT RIGHT END
5		$\frac{WL^3}{8EI}$	AT END
6		$\frac{WL^3}{3EI}$	AT END
7		$\frac{WL^2}{6EI}(2L+3a)$	AT END

FIG. 5-35. Beam Deflections.

expression, a relationship termed the differential equation for the moment in a beam may be obtained, as follows

$$M = \frac{EI \times d^2y}{dx^2} \tag{5-27}$$

This equation is applied to problems in beam deflection by expressing the moment M in terms of the length x of the beam, and integrating the resulting equation twice to determine the vertical displacement or deflection y. (The derivation of Eq. 5-26 and 5-27 may be obtained from any standard text in

Strength of Materials.) For design purposes, however, analysis and computation of beam deflections by this method are usually unnecessary, since reference may be had to tables such as Fig. 5-35 for these data. Further data on beam deflections are available in handbooks.

If the forces acting on a beam are not coplanar, they may be resolved into components along the principal axes of inertia of the beam section, and the resulting deflections computed and added vectorially to obtain the magnitude and direction of the resultant deflection.

5-23. Beams resting on more than two supports, or those in which both ends are built in or fixed to the supporting walls, are not statically determinate.

BEAM MOMENTS AND DEFLECTIONS

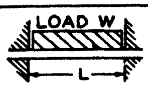
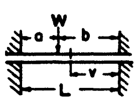
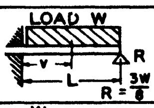
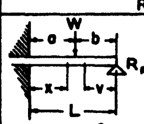
CASE	TYPE OF BEAM	MAXIMUM MOMENT	POSITION OF MAXIMUM MOMENT	MAXIMUM DEFLECTION y	POSITION OF MAXIMUM DEFLECTION
1		$\frac{WL}{12}$	AT SUPPORT	$\frac{WL^3}{384EI}$	AT CENTER
		$\frac{WL}{24}$	AT CENTER		
2		$\frac{Wab^2}{L^2}$	AT LEFT SUPPORT	$\frac{Wa^3b^3}{3EIL^3}$	UNDER LOAD
		$\frac{Wba^2}{L^2}$	AT RIGHT SUPPORT		
		$\frac{2Wa^2b^2}{L^3}$	UNDER LOAD	$\frac{2Wa^2b^3}{3EI(L+2b)^2}$	AT $v=\frac{2bL}{L+2b}$
3		$\frac{WL}{8}$	AT LEFT SUPPORT	$\frac{WL^3}{185EI}$	AT $X = .5785L$
		$\frac{9WL}{128}$	AT $v=\frac{5}{8}L$	$\frac{WL^3}{187EI}$	AT $v=\frac{5}{8}L$
4	 $R_s = W\left[1-\frac{a^2}{2L^3}(3L-a)\right]$ $R_R = \frac{Wa^2(3L-a)}{2L^3}$	$\frac{Wab}{2L^2}(L+b)$	AT LEFT SUPPORT	$\frac{Wa^3b^2}{12EIL^3}(3L+b)$	UNDER LOAD
				$\frac{Wa^2b}{6EI}\sqrt{\frac{b}{2L+b}}$	AT $v=L\sqrt{\frac{b}{2L+b}}$
		$\frac{Wa^2b}{2L^3}(3L-a)$	UNDER LOAD	$\frac{WL^3}{102EI}$	AT LOAD, IF $a = .586L$
		$\frac{WL}{5.83}$	AT EITHER, WHEN $a = .586L$	$\frac{Wb(aL^2+abL)^3}{3EIL^3(L^2+aL+bL+ab)^2}$	AT $x=\frac{2aL(L+b)}{L^2+aL+bL+ab}$

FIG. 5-36. Beam Moments and Deflection.

The fixed ends of built-in beams develop a counter-moment that affects the moment produced by the loads, and the application of the deflection theory described in section 5-22 is necessary for the determination of the moments as well as the deflection. Fig. 5-36 gives relevant data for design use for built-in beams; further information can be obtained from standard texts in Strength of Materials.

The determination of moments and stresses in beams with more than two supports may be effected by reference to the literature, or by using Figs. 5-37 and 5-38, and Tables 5-1 and 5-2. Table 5-1 gives the reaction, shear, and moment coefficients for continuous beams (Fig. 5-37) of equal span L, uniform section, supports at the same level, and having a uniformly-distributed load of

w pounds per unit of length. The reactions and the vertical shear at the critical sections are obtained by multiplying the reaction or shear coefficients by the quantity wL; the critical moments are obtained by multiplying the moment coefficients by the quantity wL^2.

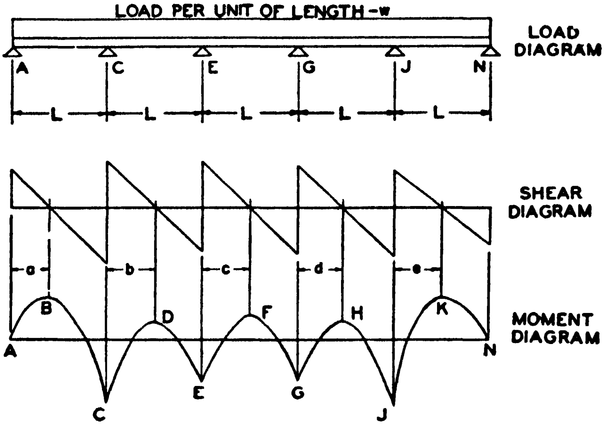


FIG. 5-37. Reactions, Shears, and Moments for Multiple-span Beams with Uniform Load.

Table 5-2 gives similar coefficients for continuous beams (Fig. 5-38) with either one concentrated load at the center of each span, or two concentrated loads at the third points of each span. The vertical shears or the reactions are

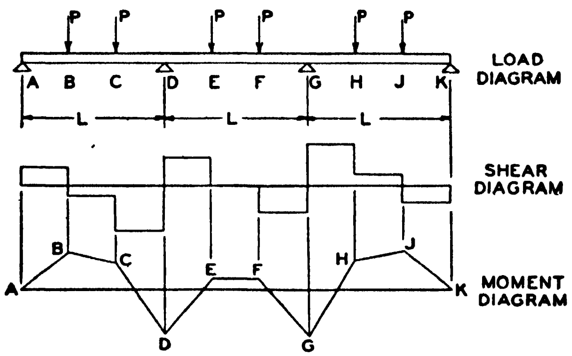


FIG. 5-38. Reactions, Shears, and Moments for Multiple-span Beams with Uniform Load.

obtained by multiplying the shear or reaction coefficients by load P; the moments are obtained by multiplying the moment coefficients by the quantity PL.

Great care must be exercised in the application of these data, since they are based upon careful horizontal alignment of the supports. When structures are supported on bases or piers resting on soft soils, the foundation may settle after

TABLE 5-1.—REACTION, SHEAR, AND MOMENT COEFFICIENTS FOR CONTINUOUS BEAMS OF UNIFORM SECTION AND EQUAL SPANS, UNIFORMLY LOADED

(Fig. 5-37)

Number of Spans	Position	Reaction	Shear Coefficient		Moment Coefficient	
			+	−	+	−
2	*A*	3/8	3/8		0	
	B				9/128	
	C	10/8	5/8	5/8		16/128
	D				9/128	
	N	3/8		3/8		0
3	*A*	4/10	4/10		0	
	B				16/200	
	C	11/10	5/10	6/10		20/200
	D				5/200	
	E	11/10	6/10	5/10		20/200
	F				16/200	
	N	4/10		4/10		0
4	*A*	11/28	11/28		0	
	B				121/1568	
	C	32/28	15/28	17/28		168/1568
	D				57/1568	
	E	26/28	13/28	13/28		112/1568
	F				57/1568	
	G	32/28	17/28	15/28		168/1568
	H				121/1568	
	N	11/28		11/28		0
5	*A*	15/38	15/38		0	
	B				225/2888	
	C	43/38	20/38	23/38		304/2888
	D				96/2888	
	E	37/38	19/38	18/38		228/2888
	F				133/2888	
	G	37/38	18/38	19/38		228/2888
	H				96/2888	
	J	43/38	23/38	20/38		304/2888
	K				225/2888	
	N	15/38		15/38		0

Shear and reaction coefficients based upon wL.
Moment coefficients based upon wL^2.
Distances a, b, etc., in Fig. 5-37, are equal to the product of L and the shear coefficient at the origin of the distance.

some period of time and cause misalignment of the structure. Such misalignment will have little effect upon the stresses and reactions in cantilevers and simply supported beams, but may have an appreciable effect on built-in beams, or on those with more than two supports. A power transmission shaft supported by two bearings is usually considered analogous to a simply supported beam, and a small degree of unavoidable misalignment of the bearings is of no great consequence, but misalignment in shafting supported by three or more bearings may introduce stresses not considered in the original design. It is evident that

TABLE 5-2.—REACTION, SHEAR, AND MOMENT COEFFICIENTS FOR CONTINUOUS BEAMS OF UNIFORM SECTION AND EQUAL SPANS, WITH ONE OR TWO CONCENTRATED LOADS IN EACH SPAN

(Fig. 5-38)

Number of Spans and Loads	Position	Reaction	Shear		Moment	
			+	−	+	−
2 Spans, one concentrated load at center of each	*A*	5/16	5/16		0	
	B		5/16	11/16	5/32	
	D	22/16	11/16	11/16		6/32
	E		11/16	5/16	5/32	
	K	5/16		5/16		0
3 Spans, one concentrated load at center of each	*A*	7/20	7/20		0	
	B		7/20	13/20	7/40	
	D	23/20	10/20	13/20		6/40
	E		10/20	10/20	4/40	
	G	23/20	13/20	10/20		6/40
	H		13/20	7/20	7/40	
	K	7/20		7/20		0
2 Spans, two concentrated loads at third points	*A*	2/3	2/3		0	
	B		2/3	1/3	2/9	
	C			4/3	1/9	
	D	8/3	4/3	4/3		3/9
	E		4/3		1/9	
	F		1/3	2/3	2/9	
	K	2/3		2/3		0
3 Spans, two concentrated loads at third points	*A*	11/15	11/15		0	
	B		11/15	4/15	11/45	
	C			19/15	7/45	
	D	34/15	15/15	19/15		12/45
	E		15/15		3/45	
	F			15/15	3/45	
	G	34/15	19/15	15/15		12/45
	H		19/15		7/45	
	J		4/15	11/15	11/45	
	K	11/15		11/15		0

Shear and reaction coefficients based upon one concentrated load P.
Moment coefficients based upon PL.

theoretical analyses of indeterminate beams may be considerably in error unless all possibilities of misalignment of supports are carefully considered. For a high degree of safety, the selection of a beam or shaft section should be based upon the most severe condition that can be anticipated.

PROBLEMS—CHAPTER 5

1. Find the resultant of the following coplanar concurrent force system:
   a. 10 lbs. horizontal, to the right
   b. 15 lbs. vertical, upward
   c. 12 lbs. 45° from the horizontal, downward to the right
   d. 25 lbs. 60° from the horizontal, upward to the right

2. Find the equilibrant of the force system of Problem 1.

3. A bell crank has arms 90° apart and 6 in. and 8 in. long. A force of 30 lbs. is applied to the extremity of the 8 in. arm, perpendicular to it, and acting in a direction towards the 6 in. arm. Find the equilibrating force on the latter, and find the direction and magnitude of the reaction of the fulcrum.

4. A concrete beam has a tee-shaped section; the stem of the tee is 2 in. wide and 12 in. high; the cross-bar of the tee is 1½ in. high and 8 in. wide. Find the centroid of the section.

5. Find the moment of inertia of the tee section of Problem 4 with respect to: a. The lower edge of the section; b. The horizontal centroid of the section.

6. Find the section modulus and least radius of gyration of the section of Problem 4, with respect to the centroidal axis found in that problem.

7. A simply-supported beam with a span of 14 feet has loads of 1,000, 2,000, and 3,000 lbs. located 3, 7, and 12 feet respectively from the left reaction. Determine the reactions graphically, and determine and locate the maximum bending moment.

8. Find the reactions analytically, draw the shear diagram, and check the moment determination of Problem 7.

9. A beam is supported at the right end and 5 feet from the left end, and has a total length of 15 feet. The beam weighs 100 lbs. per foot, and carries loads of 10,000 lbs. at the left and 2,000 lbs. 6 feet from the right end and 1,000 lbs. at the right end. Determine the reactions graphically, and determine and locate the maximum bending moment.

10. Like Problem 8, for the data of Problem 9.

11. A wooden beam is of rectangular section, and is subjected to a moment of 20,000 in. lbs. If the allowable stress is 1,000 psi., determine the dimensions of the section if the height is three times the width.

12. What is the unit horizontal shear in the beam of Problem 11?

13. Select a circular beam section for the beam of Problem 7, if the allowable stress is 12,000 psi.

14. Find the dimensions of a rectangular steel section for the beam of Problem 9, if the width of the section is ¼ the depth, and the allowable unit stress is 10,000 psi.

15. A cylindrical rod 1 in. in diameter serves as a column. What is the allowable load if the length is: a. 5 in.; b. 20 in.; c. 55 in.; d. 125 in.

16. A compression link similar to Fig. 5-33 has a length between pin axes of 30 in., and is made of steel with ultimate tensile and compressive strengths of 100,000 psi., and a yield point of 60,000 psi. What is the ratio between the critical and working loads if the section of the link is rectangular, and 1⅛ in. × 1¾ in.?

17. Like Problem 16, for a section 1½ in. × 3 in. Which of the two are most suitable?

18. Determine the permissible axial load for a standard 2-in. pipe used as a structural column with the following lengths: a. 3 feet; b. 6 feet; c. 12 feet.

19. A 4-in. Class B cast iron pipe is subjected to a unit column load of 7,000 psi. What is the maximum length?

20. A fixed end steel beam of rectangular cross-section, 2 in. wide and 3 in. deep, has a span of 15 feet, and is subjected to a uniform load of 150 lbs. per foot of length. What is the deflection?

21. A beam is fixed at one end and supported at the other, and carries a concentrated load of 1,200 lbs. at the middle of the 8-foot span. What must the diameter of a cold rolled steel rod be in order that the deflection does not exceed 0.01 in., and that the stress does not exceed 8,000 psi. Disregard the beam weight.

22. A continuous beam of four equal spans each 10 feet long carries a uniform load of 600 lbs. per foot of length. Determine the diameter of a circular beam if the allowable stress is 10,000 psi.

23. A continuous beam has three equal spans each 8 feet long. Each span carries concentrated loads of 1200 lbs. at the third points. Determine the size of a beam of square section if the allowable stress is 12,000 psi.

24. Like Problem 23, except that a uniform load of 200 lbs. per foot of length is superimposed on the loads given.

CHAPTER 6

THREADED FASTENERS AND COMBINED STRESSES

6-1. Screws, pins, keys, and nails may be classified as removable fasteners. A screw is a cylindrical part with ridges or threads of helicoidal form on its outer surface that fit corresponding grooves or threads in the hole into which it is inserted. There are two important varieties of fastening screws: those which cut their own mating thread in the hole, and those which fit in a hole independently threaded or tapped. The first type of thread is employed for wood and for self-tapping metal screws; the second, for most metal fastening purposes. Wood screws and nails are described in Chapter 11; keys and pins furnish limited restraint and are described in Chapter 16.

6-2. Pitch and Lead. Screw thread nomenclature is illustrated in Fig. 6-1. The pitch of a screw thread is the distance between adjacent crests; the lead is the distance the nut will advance axially for one turn of the screw. The lead and pitch are alike in single-threaded screws; the lead is twice the pitch in double-threaded and three times the pitch in triple-threaded screws. Multiple-threaded screws are employed when a comparatively large axial movement is required without much reduction of the area at the roots of the threads. This feature is illustrated in the two center illustrations in Fig. 6-1, where the double-threaded screw has an appreciably greater root area than the single-threaded screw, although both have the same load.

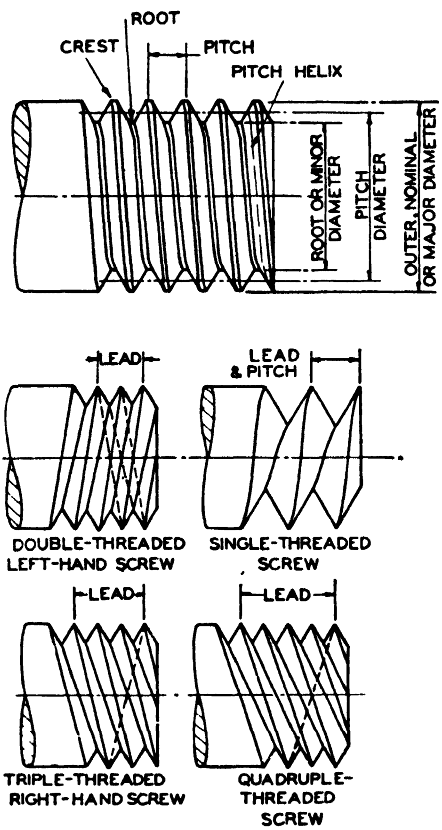


Fig. 6-1. Screw Thread Nomenclature.

6-3. Thread Types. A number of screw thread profiles for metal fastenings, illustrated in Figs. 6-2, 6-3, and 6-4, have been standardized and adopted

by the American Standards Association. The sharp "V" thread, Fig. 6-2, is the oldest form, but it is very little used today because of the difficulty of measuring to the sharp crests and the likelihood of stress concentration at the sharp roots. It has been replaced to a large extent by the American Standard form, which has the same included angle, 60°, but is slightly flattened at the crest and root. There are several types of the American Standard form: the Coarse-thread Series, which is recommended for general use; the Fine-thread Series, which has a smaller pitch and is employed where excessive vibration

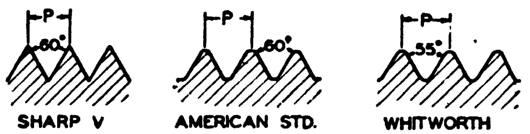


FIG. 6-2. Screw Thread Profiles for Metal Fastenings.

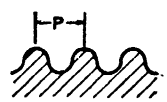


FIG. 6-3. Knuckle Thread Profile.

requires a fine-pitch thread; the special varieties such as the 8-pitch and 16-pitch Thread Series, which are available in various diameters, with pitches of ⅛ or 1/16 in. respectively. Table 6-1 gives proportions of Coarse- and Fine-thread American Standard screws. The Whitworth thread form has rounded crests and roots and is less subject to severe stress concentrations than the American Standard form; it is used in Great Britain. The Knuckle thread, Fig. 6-3, is employed principally on screws whose threads are rolled instead of cut and is used for carriage and stove bolts. The Dardelet thread, Fig. 6-4, is a self-

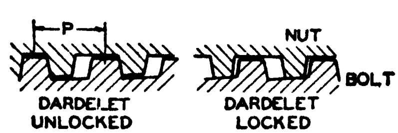


FIG. 6-4. Self-locking Screw Thread.

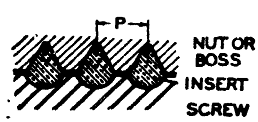


FIG. 6-5. Aero-thread Profile.

locking thread having the roots of the external, and the crests of the internal, threads at an angle of 6° to the axis. The nut may easily be screwed on the bolt, but the final tightening causes the conical surfaces to lock in position and considerable effort is necessary to unscrew the nut, thus preventing accidental loosening caused by vibration. The Aero-thread system, Fig. 6-5, employs an insert, similar to a compression spring, between the nut or tapped hole and the screw. It may be used when high-strength steel bolts are to be fastened in soft alloy parts, since it protects the tapped hole from wear caused by inserting and removing the bolt. It also compensates for the difference in expansion of the steel bolt and the light alloy member, and high stress concentration under varying temperatures is, therefore, eliminated. The insert is screwed in with a

TABLE 6-1.—PROPORTIONS OF AMERICAN STANDARD SCREW THREADS

Size*	Major Diam. In.	Coarse (NC)			Fine (NF)			8 Thread Series	Minimum Bolt Spacing In.
		Thds. per In.	Minor Diam. In.	Root Area Sq. In.	Thds. per In.	Minor Diam. In.	Root Area Sq. In.	Root Area Sq. In.	
0					80	.0438			
1	.0730	64	.0527		72	.0550			
2	.0860	56	.0628		64	.0657			
3	.0990	48	.0719		56	.0758			
4	.1120	40	.0795		48	.0849			
5	.1250	40	.0925		44	.0955			
6	.1380	32	.0974		40	.1055			
8	.1640	32	.1234	.0119	36	.1279	.0128		
10	.1900	24	.1359	.0145	32	.1494	.0175		
12	.2160	24	.1619	.0205	28	.1696	.0225		
1/4	.2500	20	.1850	.0270	28	.2036	.0325		
5/16	.3125	18	.2403	.0450	24	.2584	.0524		
3/8	.3750	16	.2938	.0680	24	.3209	.0809		
7/16	.4375	14	.3447	.0930	20	.3725	.1090		
1/2	.5000	13	.4001	.1260	20	.4350	.1485		1 1/4
9/16	.5625	12	.4542	.1620	18	.4903	.1888		
5/8	.6250	11	.5069	.2020	18	.5528	.2400		1 1/2
3/4	.7500	10	.6201	.3020	16	.6688	.351		1 3/4
7/8	.8750	9	.7307	.4200	14	.7822	.480		2 1/16
1	1.0000	8	.8376	.5500	14	.9072	.647	.551	2 1/4
1 1/8	1.1250	7	.9394	.6948	12	1.0167	.812	.728	2 1/2
1 1/4	1.2500	7	1.0644	.8930	12	1.1417	1.021	.929	2 13/16
1 1/2	1.5000	6	1.2835	1.2950	12	1.3917	1.520	1.405	3 1/4
1 3/4	1.7500	5	1.4902	1.7460				1.980	3 3/4
2	2.0000	4.5	1.7113	2.3020				2.652	4 1/4
2 1/4	2.2500	4.5	1.9613	3.0230				3.423	4 3/4
2 1/2	2.5000	4	2.1753	3.7190				4.292	5 1/4
2 3/4	2.7500	4	2.4252	4.6200				5.259	5 3/4
3	3.0000	4	2.6752	5.6200				6.324	6 1/4

* Sizes 0 through 12 are smaller than 1/4 in. Sizes 1/4 through 3 are in inches.

special tool after the hole is threaded. For all practical purposes the insert becomes an integral part of the tapped hole and can only be removed by means of a special extracting tool.

In addition to serving as fasteners, screws are used for making adjustments, for the transmission of power, as in lathe lead screws or jack screws, and for precision measurements, as in calipers and micrometers. Fig. 6-6 illustrates three thread forms used for transmitting power. The Square thread will trans-

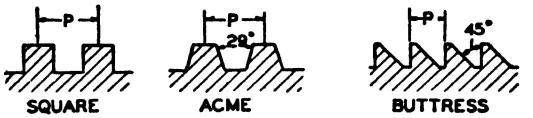


FIG. 6-6. Screw Thread Profiles for Power Transmission.

mit power without any side thrust but is difficult to cut and cannot be used conveniently with split or half-nuts on account of the difficulty of disengagement. The Acme thread is easier to cut, is stronger than the Square thread, and can be used readily with split nuts. The Buttress form has the power transmission qualities of the Square thread and strength comparable to that of the American Standard; it is employed in jack-screws and for gun breech-locks where power is transmitted in one direction only.

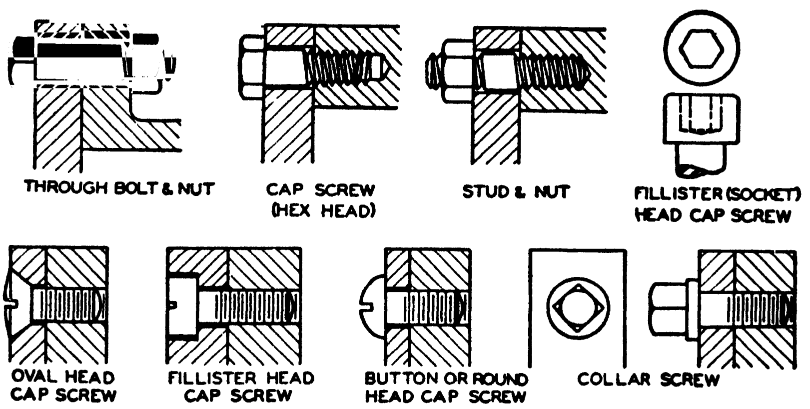


FIG. 6-7. Representative Bolts and Screws.

6-4. Types of Threaded Fasteners. A variety of fastening screws is shown in Fig. 6-7. Through-bolts and nuts are extensively employed as removable fasteners where the bolt has an appreciable amount of clearance, usually $\frac{1}{32}$ or $\frac{1}{16}$ in., in the bolt hole. Turned or carefully fitted bolts are frequently used in reamed holes when the bolt is required to resist shearing as well as tensile forces. Bolts applied to unfinished castings or forgings are usually provided with "spot-faced" bolt head and nut seats. Cap and machine screws are used to join parts when one part has an internally threaded hole. These screws

are preferable to bolts because they are easier to handle in installations where access to one end of the element is either difficult or impossible. Cap screws are available commercially in sizes from ¼ in. diameter up. Oval and fillister head cap screws, Fig. 6-7, are often preferred to hexagonal head cap screws, since the head may be recessed to avoid interference or to facilitate cleaning the part held by the screw. Hexagonal head cap screws may be fastened more tightly than screws with screw-driver slots. The fillister head cap screw with a socket head combines the advantages of the hexagonal head and the slotted fillister screw. This type is fastened by using a special wrench made of hexagonal bar stock. When a screw must be removed frequently, it is often ad-

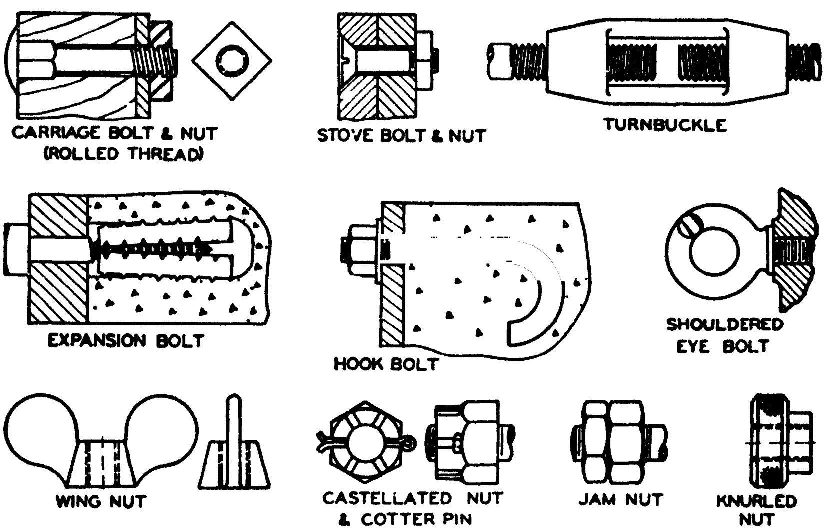


FIG 6-8. Bolts, Nuts, and Other Fasteners.

visable to substitute a stud that may be inserted into the threaded hole and jammed against the bottom so that it is only necessary to remove the nut, thus avoiding wear on the threads in the hole. (In aluminum alloy castings, the Aero thread may be employed instead of using a stud.)

Machine screws are similar in appearance to cap screws but have heads of somewhat smaller proportions. The major diameters vary from 0.073 to 0.375 in. A No. 10-24 machine screw, for example, has a diameter of 0.190 in. Machine screws have American Standard thread forms in both Coarse-thread and Fine-thread Series.

Miscellaneous threaded fasteners are shown in Fig. 6-8. Stove bolts are employed for assemblies where precision is of no great importance. They are made with either flat or round heads and the screw threads are generally rolled. The square nuts used with them are stamped from common steel. Carriage bolts

have a squared portion directly under the head to prevent rotation when the nut is tightened and are used for fastening wooden parts together or for fastening metal parts to wood. Expansion and hook bolts are used in semi-permanent fastenings in concrete and masonry. Electric motors and other machinery are usually equipped with one or more eye-bolts so that they may be lifted readily and moved with an overhead crane. A turnbuckle is a nut that has a right-hand and left-hand thread, and is used to adjust the length of tie rods and similar devices. The turnbuckle is one of the few devices in which a left-hand thread is employed as a fastener.

TABLE 6-2.—PROPORTIONS OF SCREWS, BOLTS, AND NUTS IN TERMS OF THE NOMINAL DIAMETER D

Type	Diam. of Head, or Wrench Diam.	Height of Head or Nut
Hexagonal nut	$1.5\,D + \frac{1}{8}$ in.	D
Hexagonal head cap screw ..	$1.5\,D$	$0.75\,D$
Filister head cap screw	$1.5\,D$	$0.65\,D$
Socket head cap screw	$1.5\,D$	D

Proportions of representative screws, bolts, and nuts are given in Table 6-2 and may be used for layout or design work. Socket wrench dimensions and bolt head clearances are shown in Fig. 6-9. For actual detail dimensions, reference should be made to the ASA Standards or to engineering handbooks.

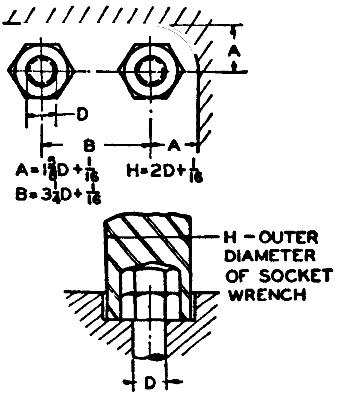


FIG. 6-9. Bolt Head Clearances.

6-5. Washers and Nuts. Plain washers, Fig. 6-10, are placed under the heads of hexagonal head screws and under square and hexagonal nuts to assist in seating the nut or head, or to distribute the pressure exerted. Collar screws, Fig. 6-7, are square head cap screws with integral washers. Rough washers are punched from common steel; finished washers may be machined from steel bar stock. Lock washers are used to prevent accidental unscrewing of bolts and nuts, either by exerting additional tension on the threads or by biting into the surfaces in contact. It is possible to obtain button and flat head cap and machine screws with assembled lock washers that cannot drop off, a feature that will be appreciated by anyone who has ever tried to insert a screw with a loose washer in a comparatively inaccessible place.

Castellated and jam nuts, Fig. 6-8, are representative examples of parts for locking and fixing nuts in place. The castellated nut is held by a cotter pin and has six locking positions per turn; the jam nut holds the regular nut in position by being screwed against it. Wing and knurled nuts are designed for

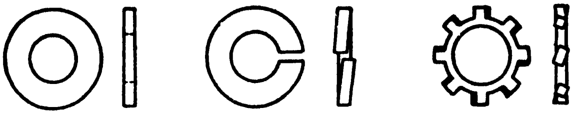


FIG. 6-10. Washers.

hand operation. Some forms of fillister head screws are supplied with knurled heads so that they may be screwed into place easily by hand, although the final tightening must be done with a screw-driver or wrench.

6-6. Self-tapping Screws. Wood screws cut their own thread as they are inserted in material. Wood screw thread profiles, Fig. 6-11, are used in most types of wood and self-tapping screws. In these screws the thread area of the screw profile is reduced to permit more strength to be obtained in the internal threads of the wood. Self-tapping screws, Fig. 6-12, are similar to wood screws. The round head screw to the left is shown holding two sheet-metal plates together. A pilot hole slightly larger than the nominal size of the screw is punched or drilled in one of the plates, and an anchor hole of the same size as the root diameter of the screw is formed in the other plate. The oval head screw at the right is used to fasten a steel plate to a part made of soft metal, such as an aluminum or copper alloy. Self-tapping screws find use where the screw is removed infrequently and where it is desired to save the cost of threading the hole by a separate operation.



FIG. 6-11. Wood Screw Thread Profile.

6-7. Stresses in Screwed Fastenings. The load applied to a bolt or screw generally tends to rupture the bolt in an axial direction. For tensile loads the section at the thread root is subjected to the maximum stress, which is

$$S_t = \frac{P}{A_r} \qquad (6\text{-}1)$$

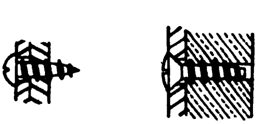


FIG. 6-12. Self-tapping Screws.

where P is the total external load, and A_r is the root area, available from Table 6-1. In some instances bolts are employed to resist motion in a direction perpendicular to the bolt axis, resulting in a shearing stress in the bolt. For

shear loads the body of the bolt should fit the hole to eliminate localization and concentration of stress. The unit stress is given by

$$S_s = \frac{4P}{\pi D^2} \qquad (6\text{-}2)$$

where D is the nominal or body diameter of the bolt.

In many cases, bolts and similar elements are subjected to a combination of axial tension and transverse shear. In Fig. 6-13, for example, the bracket is

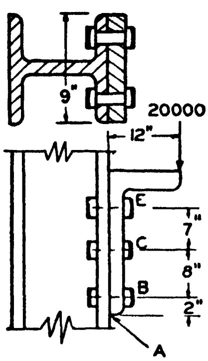


FIG. 6-13. Bolted Bracket.

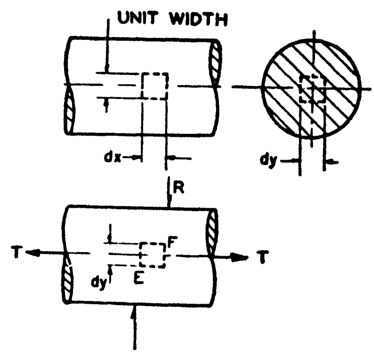


FIG. 6-15. Combined Stress Analysis.

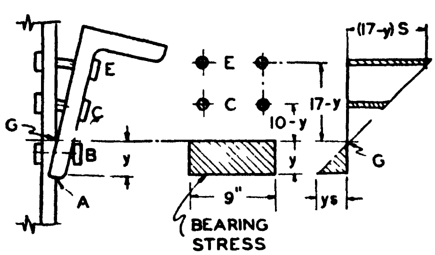


FIG. 6-14. Stress Distribution in Bolted Bracket.

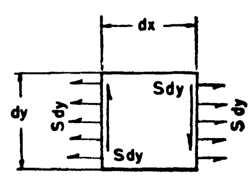


FIG. 6-16. Combined Stress Analysis.

attached to the H-column by six carefully fitted bolts, and the 20,000-lb. force tends to move the bracket downward and to rotate it about an axis in the face of the column, as shown in Fig. 6-14. This action induces both shearing and tensile stresses (at right angles to each other) within the bolts.

6-8. Combined Stress Analysis. An enlarged section of a bolt, with the external shearing forces R and tensile forces T acting upon it, is shown in Fig. 6-15. For convenience in analysis we may consider these forces acting upon an elementary section within the bolt of height dy, length dx, and of unit width. If S represents the unit tensile stress and s the unit shearing stress on

the bolt, then the total tensile force on one side of the section will be the product of S, dy, and the unit length, or Sdy; and the total shearing force on one side of the section will be the product of s, dy, and the unit length, or sdy. The section and these forces are represented on an enlarged scale in Fig. 6-16. In this figure the tensile forces Sdy are in horizontal equilibrium, and the shearing forces sdy are in vertical equilibrium, but the latter induce an unbalanced moment which is equilibrated by a pair of horizontal shearing forces whose magnitude is equal to the product of s, dx, and the unit length, or sdx, as shown in Fig. 6-17.

If s' represents the unit shearing stress parallel to the diagonal plane EF, and dz is the length of the diagonal plane, the total shearing force along this plane is equal to the product of s', dz, and the unit length, or $s'dz$. If the right

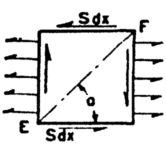


FIG. 6-17. Combined Stress Analysis.

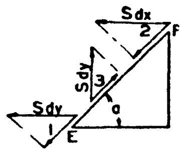


FIG. 6-18. Combined Stress Analysis

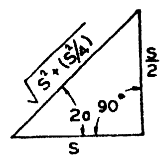


FIG. 6-19. Combined Stress Analysis

diagonal half of the elementary section is considered a free body, as in Fig. 6-18, the components of the three forces parallel to plane EF are:

$$\text{Component 1 of } Sdy = +(Sdy \times \cos a)$$
$$\text{Component 2 of } sdx = +(sdx \times \cos a)$$
$$\text{Component 3 of } sdy = -(sdy \times \sin a)$$

Since component 3 acts in a direction opposite to that of components 1 and 2, it receives a negative sign. The summation of these components must be equivalent to the total shearing force along the plane, or

$$S \cos a\, dy + s \cos a\, dx - s \sin a\, dy = s'dz$$

Dividing through by dz

$$\frac{S \cos a\, dy}{dz} + \frac{s \cos a\, dx}{dz} - \frac{s \sin a\, dy}{dz} = s'$$

and substituting $\sin a$ for dy/dz, and $\cos a$ for dx/dz,

$$S \cos a \sin a + s(\cos^2 a - \sin^2 a) = s'$$

The following trigometric indentities may be substituted:

$$2 \sin a \cos a = \sin 2a$$

and

$$\cos^2 a - \sin^2 a = \cos 2a$$

to give

$$\left(\frac{S}{2} \times \sin 2a\right) + (s \times \cos 2a) = s'$$

Differentiating this expression with respect to a

$$\frac{d}{da} \text{ of } \sin 2a = 2 \cos 2a$$

and

$$\frac{d}{da} \text{ of } \cos 2a = -2 \sin 2a$$

or

$$S \cos 2a - 2s \sin 2a = \frac{ds'}{da}$$

Equating the above expression to zero, to obtain maximum values of the variable a,

$$S \cos 2a - 2s \sin 2a = 0$$

or

$$\frac{S}{2s} = \frac{\sin 2a}{\cos 2a} = \tan 2a$$

If a right triangle is drawn as in Fig. 6-19, in which

$$\tan 2a = \frac{S}{2s}$$

then the hypotenuse of the triangle is $\sqrt{s^2 + S^2/4}$, and

$$\cos 2a = \frac{s}{\sqrt{s^2 + S^2/4}}$$

and

$$\sin 2a = \frac{S/2}{\sqrt{s^2 + S^2/4}}$$

Substituting these in the expression for s',

$$s' = \frac{S^2/4}{\sqrt{s^2 + S^2/4}} + \frac{s^2}{\sqrt{s^2 + S^2/4}} = \sqrt{s^2 + S^2/4}$$

giving the maximum value of the resultant shearing stress induced by shearing and tensile forces. This expression is usually written

$$S_m = \sqrt{S_s^2 + S_t^2/4} \qquad (6\text{-}3)$$

where S_m is the maximum resultant shearing unit stress, and S_s and S_t represent the direct unit shearing and tensile (or compressive) stresses. This equation indicates that failure will occur by shear. Such a prediction has been experimentally verified for most steels and other ductile materials.

Another theory of failure postulates that inelastic action begins when the maximum normal tensile stress on a principal plane exceeds the stress at the elastic limit as determined by a simple tensile test. This is particularly applicable

to brittle materials, such as cast iron. The maximum resultant unit stress S_n is a tensile stress and is

$$S_n = \frac{S_t}{2} + \sqrt{S_s^2 + S_t^2/4} \quad (6\text{-}4)$$

(The derivation of this formula is not given, since it is essentially similar to that of Eq. 6-3 and may be found in any text on Strength of Materials.)

Example 6-1. Find the maximum stress in the 1-in. diameter steel bolts of Fig. 6-13.

Solution. The 20,000-lb. force may be replaced by a like force at the juncture of the bracket and the column, and a clockwise couple whose moment is 20,000 × 12, or 240,000 in.-lbs. The direct shearing stress is equal to the vertical force divided by the cross-sectional area of the six bolts. From Eq. 6-2,

$$S = \frac{4 \times 20{,}000}{6\pi} = 4240 \text{ psi.}$$

The external moment is resisted not only by the tensile force exerted by the bolts, but also by a portion of the column face bearing against the lower end of the bracket; see Fig. 6-14. The bracket may be considered to rotate about some axis G, and the external moment is balanced by the moments of the tensile resistance of the bolts and the bearing resistance of the lower end of the bracket. It is first necessary to locate the neutral axis G, which may be accomplished by equating the moments of the bolt areas above G and the bracket area below G. It is reasonable to assume, for the proportions given, that G will lie somewhere between bolt lines B and C (with other proportions, of course, G may lie below B, or between C and E).

Assume that G is at a distance y from the lower edge of the bracket, then the bearing area is $9y$ sq. in. The moment arm of this area is $y/2$, the area of a bolt is $\pi(0.50)^2$, or $\pi/4$ sq. in., and the moment arms of bolts C and E are $10 - y$ and $17 - y$. Equating the bearing and tensile area moments,

$$\frac{9y^2}{2} = 2\frac{\pi}{4}[(10 - y) + (17 - y)]$$

or

$$y^2 + 0.7y - 9.4 = 0$$

and

$$y = 2.7 \text{ in. (approximately)}$$

From this, the moment arm of bolt C is $10 - 2.7$, or 7.3 in., and of bolt E is $17 - 2.7$, or 14.3 in. If s represents the unit stress at a unit distance, then the unit stresses in C and E are $7.3s$ and $14.3s$. (Since bolts B are below the axis G, they undergo no tensile stress and are of assistance only in resisting the direct shear.) The total resisting moment of bolts C and E is given by

$$2\frac{\pi}{4}[(7.3)^2 s + (14.3)^2 s] = 405s$$

The maximum unit bearing stress is at point A and is $2.7s$. The total resisting force in bearing is $(2.7s \times 2.7 \times 9)/2$, or $32.8s$ The moment arm of this force is the distance from the axis G to its centroid, which is equal to $2 \times 2.7/3$, or 1.8. The resisting moment in bearing is, therefore, $32.8s \times 1.8$, or $59s$.

The summation of the resisting moments must be equal to the external moment,

$$12 \times 20{,}000 = 405s + 59s = 464s$$

The unit stress at a unit distance is

$$s = \frac{240{,}000}{464} = 518 \text{ psi.}$$

Then the maximum compressive or bearing stress is 2.7 × 518, or 1400 psi., at the lower edge A of the bracket.

The tensile stress is a maximum in bolts E, and is 14.3 × 518, or 7410 psi., which must be combined with the unit shearing stress of 4240 psi., by Eq. 6-3, to obtain the maximum resultant shearing stress, as:

$$S_m = \sqrt{4240^2 + \frac{7410^2}{4}} = 5640 \text{ psi.}$$

The possibility of failure by direct tension at the root of the threads should also be considered. From Table 6-1 and Eq. 6-1, the stress in the upper bolts, by moments, is

$$S_t = \frac{7410}{0.55} = 13{,}480 \text{ psi.}$$

This value is greater than the resultant maximum shearing stress, and as it is considered safe to use a shearing stress up to 75% of a working tensile stress, the 13,480 psi. tensile stress is the critical one and failure may be expected due to it rather than to resultant shear.

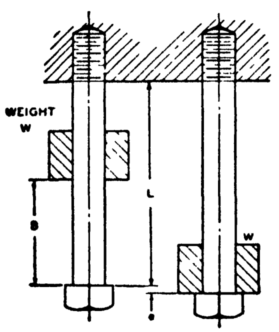


Fig. 6-20. Impact Action on Bolts.

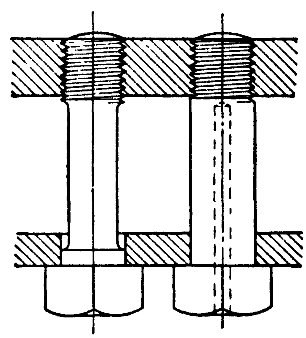


Fig. 6-21. Bolt Designs for Impact Loading.

It is of interest to note that for equilibrium the horizontal summation of the resisting forces should be equal to zero. The resisting force in bolts C and E is $2(3780 + 7410)\,\frac{\pi}{4}$ or 17,600 lbs. The average unit bearing stress is 1400/2, or 700 psi., and the total bearing resistance, considering an area of 2.7 × 9, or 24.3 sq. in., is 700 × 24.3 or 17,000 lbs. In an exact solution the bearing resistance and bolt tensile resistance should be equal; the values just computed differ somewhat because the approximate value (2.7 in.) was used for y throughout the solution. Further refinement is unnecessary when values are in such substantial agreement.

6-9. Shock and Impact Load Effects. Tie rods and bolts are sometimes subjected to shock or impact loads and should in such cases be designed to absorb impact energy as well as to resist rupture. Fig. 6-20 shows a bolt subjected to an impact load produced by the weight W falling through a distance B. When the weight strikes the head of the bolt, the body of the bolt yields and elongates a distance e during the process of absorbing the energy of the falling body and bringing it to rest. This procedure assumes that the entire impact energy is absorbed by the body of the bolt, which is not true theoretically, since the support, bolt head, and weight all deform to some extent. For bolts of usual

head proportions and fairly great length, however, the deformation of the other elements are relatively small and may be disregarded.

The total energy absorbed by the body of the bolt is equal to the product of the weight and the distance through which it moves, or $W(B+e)$. The resistance of the bolt is zero at the first instant of impact and reaches its maximum when the bolt attains its maximum elongation e. If S represents the maximum tensile stress in the body of the bolt, then the maximum resisting force at the conclusion of impact is SA, where A is the area of the body of the bolt. The average resistance during the cycle of action is $(SA+0)/2$, and the internal work within the bolt is $SAe/2$. Equating the internal work and the external energy

$$\frac{SAe}{2} = W(B+e)$$

or

$$S = \frac{2W}{A}\left(\frac{B}{e}+1\right) \qquad (6\text{-}5)$$

From this it follows that the unit tensile stress S varies inversely with the elongation e. The elongation may be increased by lengthening the bolt or by decreasing the area of the shank. Since the tensile strength of a bolt depends upon the root area of the threaded portion, a reduction in the shank area corresponding to the thread root area will result in a design with uniform stress and maximum elongation for a given length and stress. Two types of bolts applicable to live or shock loads are shown in Fig. 6-21. The bolt with the axial hole is often used to resist transverse shear but is more expensive to manufacture than a bolt with a shank of reduced diameter.

Example 6-2. A 1-in.-8-USS bolt has an effective length L of 30 in. and is subjected to a load of 150 lbs. acting through a free distance B of ½ in. Find the unit stress in a standard bolt and in live-load bolts similar to those in Fig. 6-21.

Solution. The unit elongation is equal to e/L. Since the unit stress S is equal to the product of the unit elongation and the modulus of elasticity E, the total elongation e caused by a stress S is SL/E. If E is assumed as 30×10^6 psi., and the value of L is substituted as 30 in.,

$$\frac{S \times 30}{30 \times 10^6} = e = \frac{S}{10^6}$$

The body area of a 1-in. standard bolt is $D^2/4$, or 0.7854 sq. in. Substituting known values in Eq. 6-5,

$$S = \frac{2 \times 150}{0.7854}\left(\frac{0.5}{e}+1\right) = 382\left[\frac{(0.5 \times 10^6)}{S}+1\right]$$

Transposing and solving, S is found to equal approximately 14,000 psi. in the body of the bolt. The root area of a 1-in.-8-USS bolt, from Table 6-1, is 0.550 sq. in., and the resultant stress at the thread root is $(14{,}000 \times 0.7854)/0.550$, or 20,000 psi.

If one of the bolt types shown in Fig. 6-2 is employed, with a bolt shank area equal to the root area of the thread, both the shank and root area stress is

$$S = \frac{2 \times 150}{0.550}\left(\frac{0.5}{e}+1\right) = 544\left[\frac{(0.5 \times 10^6)}{S}+1\right]$$

S is equal to approximately 16,800 psi. This stress is greater than the 14,000 psi. for the standard 1-in. bolt, but the root area of the standard bolt is subjected to a 20,000 psi. stress. For the reduced-shank bolt of Fig. 6-2 the diameter of the shank should be equal to the thread root diameter, or 0.838 in.; for the bolt with an axial hole, the hole diameter should be equal to

$$d = \sqrt{\frac{4(0.7854 - 0.551)}{\pi}} = 0.546, \text{ or } {}^{35}\!/_{64} \text{ in.}$$

6-10. Initial or Tightening Stresses in Threaded Fastenings. When a screw is inserted in a threaded hole, or when a nut is screwed on a bolt, the body of the bolt is subjected to direct tension, and shearing and compressive stresses are induced in the bolt threads and nut threads. Proportions of standard fastening screws are such that if the length of the thread engagement between a bolt and a steel nut is equal to the diameter of the bolt, or if this length between a steel screw and a hole tapped in cast iron is equal to one and one-half times the screw diameter, the fastener will fail by tension at the thread root rather than by thread shear or compression. In addition to the stresses resulting from the load carried by the bolt, a considerable degree of torsional shear is induced in the body of the bolt by the twisting force necessary to "set up" the bolt.

Observations have indicated that the tensile stress induced in a bolt or screw by an experienced mechanic when tightening it with a wrench of ordinary proportions is

$$F = 16{,}000\,D \tag{6-6}$$

where F is the total initial load resulting from tightening the bolt, and D is the nominal diameter of the bolt. Applying this relation to a ½-in. bolt, the initial load is 16,000 × 0.5, or 8000 lbs., and the unit tensile stress at the root of the threads, from Eq. 6-1 and Table 6-1, is 8000/0.126, or 63,500 psi. This stress is in excess of the ultimate tensile strength of mild steel, and unless care is exercised this bolt may fail in tightening. Considering a 1-in. bolt, the initial load is 16,000 lbs. and the unit tensile stress is 16,000/0.8376, or 19,100 psi., which is somewhat greater than 50% of the yield point of mild steel. For usual machine and structural applications, these figures indicate that bolt failure under load need not be expected if the fastening element is able to withstand the stresses induced by tightening.

In bolted pressure-tight joints some comparatively elastic medium or gasket is usually employed as a seal. This member often yields excessively, and if the bolts are tightened by inexperienced or careless mechanics, very high initial stresses may be induced in these elements. In some instances design of the bolts for initial stress is desirable; in others, design for a combination of initial and load stresses is indicated.

Fig. 6-22 shows a U-shaped steel block clamped by a long eye-bolt. The horizontal or beam portion of the block has a section 2 in. wide and ¼ in. thick; the section is enlarged at the region where the bolt passes through, so

that the entire beam section has a uniform moment of inertia. If the bolt is screwed up until the deflection of the horizontal or beam portion of the block is 0.1365 in., then the resultant load on the bolt necessary to produce a deflection of this magnitude, considering the horizontal section as a simple beam with a concentrated load in the center, from Fig. 5-35, case 2, will be

$$W = \frac{48EIy}{L^3} = \frac{48 \times 30 \times 10^6 \times 2 \times 0.25^3 \times 0.1365}{8^3 \times 12} = 1000 \text{ lbs.}$$

The bolt elongation e corresponding to this load will be

$$e = \frac{LW}{AE} = \frac{6 \times 1000}{0.7854 \times 30 \times 10^6} = 0.000254 \text{ in.}$$

If an additional load W' of 500 lbs. is applied to the bolt (Fig. 6-22), the increase in the bolt elongation will be 0.000127 in., which will reduce the beam deflection

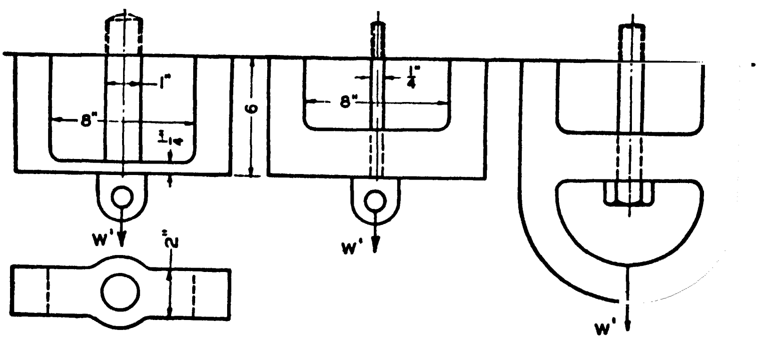


FIG. 6-22. Initial Stress Analysis. FIG. 6-23. Initial Stress Analysis. FIG. 6-24. Initial Stress Analysis.

to 0.1365 − 0.00127, or 0.13523. This deflection will correspond to a load of (0.13523/0.1365) 1000, or 990 lbs. The load on the bolt, therefore, will be equal to 990 + 500, or 1450 lbs., which is practically the same as the sum of the initial and external loads W and W'. Accordingly, this bolt would be designed for the combination of initial and applied loads.

The block shown in Fig. 6-23 is similar to that shown in Fig. 6-22, but the horizontal section is much stiffer and the bolt much smaller. An initial load of 1000 lbs. will give a deflection y in the horizontal section,

$$y = \frac{1000 \times 8^3 \times 12}{48 \times 30 \times 10^6 \times 2 \times 2^3} = 0.000266 \text{ in.}$$

and the bolt elongation corresponding to this load will be

$$e = \frac{6 \times 1000}{0.0491 \times 30 \times 10^6} = 0.00406 \text{ in.}$$

The application of an external load W' of 500 lbs. would cause a further elongation in the bolt of 0.00203 in. The block separates from the supporting

frame the instant this additional elongation takes place, which is immediately counteracted by a shortening of the initial elongation of the bolt and by a decrease in the deflection of the beam section. The elongation caused by the external load is absorbed by these two actions, in the ratio of the initial extension of the bolt and the initial deflection of the beam. The shortening of the bolt is

$$\frac{0.00203 \times 0.00406}{0.00406 + 0.000266} = 0.0019$$

and the actual bolt elongation is, therefore, $0.00406 + 0.00203 - 0.0019$, or 0.00419 in. This elongation corresponds to a load of 1030 lbs., indicating that the initial load W on the bolt has decreased to 530 lbs., while the external load W' remains at 500 lbs. The bolt in this case would be designed primarily to withstand the initial stress.

If an external load W' of 1500 lbs., instead of 500, be applied (Fig. 6-23), the bolt elongation would be 0.00609 in., which is greater than the sum of the beam deflection and original elongation, and would, therefore, completely relieve the initial load, and the bolt stress would be based only upon the external load. The load conditions shown in Fig. 6-24 are essentially the same as those of Fig. 6-23.

The preceding analyses indicate that the actual stress in a bolt may vary between values obtained by considering the external load only or by considering the sum of the initial and external loads. This latter case is common when soft, comparatively elastic gaskets are employed and the bolt design requires careful consideration of the relative yield of the gasket and bolts.

6-11. Miscellaneous Stress Analyses. Allowable loads for bolts and screws employed in machine applications are usually based upon the external load only and may be found from an empirical expression, as follows:

$$S - N\sqrt{A_r} \qquad (6\text{-}7)$$

where S is the allowable unit tensile stress, A_r the root area of the bolt or screw, and N a constant. This constant is 1000 for bronze and one twelfth of the ultimate strength for carbon and alloy steel bolts, except that it must never be greater than 15,000. Bolts 2 in. in diameter and larger are usually designed for a stress of 8000 psi. for carbon steel and up to 20,000 psi. for alloy steels, the initial stress being disregarded.

Bolt design for pressure-tight joints should be handled in accordance with the specifications of the ASME-UPV Code, which are described in Chapter 10. Bolt design for wooden structures is treated in Chapter 11; applications of set screws in Chapter 16.

When bolts are used for bracing heads in pressure vessels (staybolts), or for fastenings to confine live steam or lethal gases, it is often desirable to have some preliminary indication of bolt failure before loss of life or property occurs. Staybolts are sometimes furnished with tell-tale holes to give some indication

of leakage caused by bolt failure; the drilled bolt of Fig. 6-21 may be considered representative. If live steam or lethal gases are confined in the space between the plates, a crack or flaw in the bolt will permit the vapor to emerge from the drilled hole, and precautionary measures can be taken before serious failure occurs. Tell-tale holes are usually of comparatively small diameter, 1/8 or 1/4 in., since an opening of any size is sufficient to give warning of impending failure.

The load-carrying capacity of expansion bolts in concrete may be estimated from the following, in which D is the nominal diameter of the bolt, and F the safe load:

$$F = 200(12D - 1) \tag{6-8}$$

This expression is based upon manufacturers' data for the safe load-carrying capacity, and serves not only for the type of expansion bolt shown in Fig. 6-8, but also for the type in which a hard lead-alloy anchor is expanded by means of a tapered or conical bolt head and a conical sleeve on the bolt. Failure in expansion bolts usually occurs by pulling or breaking out of the concrete, so the bond strength of the masonry is usually the determining factor. Eq. 6-8, therefore, has a maximum value of 1600 lbs., regardless of the bolt size.

PROBLEMS—CHAPTER 6

1. A 5/8-in. diameter eyebolt made of SAE 1025 steel is used for lifting and handling a motorized speed reducer weighing 1050 lbs. What is the unit stress at the thread root, and how does it compare with the permissible stress?

2. Find the nominal diameter of an eyebolt made of SAE 1020 steel for lifting a load of 2100 lbs. Due to the possibility of sudden application of the lifting medium, there is some likelihood of shock.

3. An 8×4 in. structural angle 6 in. long is attached to a building column by four 5/8-in. bolts located at the vertices of a 3-in. square in the 8-in. leg. The upper bolts are 3 in. from the 4-in. horizontal leg, which carries a load of 1400 lbs. located 3½ in. from the face of the column. Determine the actual stresses in the bolts and compare them with the permissible.

4. An 8×4-in. structural angle, similar to that of Problem 3, carries a load of 12,000 lbs. located 3 in. from the column face. The bolt centers are staggered. They have a minimum pitch of 3 in., a minimum edge distance of 1½ in., and are located on the rivet gage lines of the 8-in. leg. How many ¾-in. bolts will be required and what will be the length of the bracket if the permissible resultant shearing stress is 8000 psi.?

5. A bracket similar to Fig. 6-13 is 6 in. wide and is attached to a brick wall by six 5/8-in. diameter expansion bolts. One bolt is located 3 in., two are located 7 in., and three are located 12.-in. from the lower edge of the bracket. If the maximum compressive stress in the masonry wall is 250 psi., what load may be carried if applied 10 in. from the vertical face of the bracket?

6. Like Problem 5, except that the two center bolts are located 9 in. from the lower edge of the bracket.

7. A gear transmission weighing 530 lbs. is bolted to the end of a machine tool. The transmission case is made of cast steel and has a 14-in. square vertical base. SAE 1025 steel cap screws are located at the four corners of the base 1 in. from the edges. Find the size of the screws if the weight of the transmission is considered concentrated 9 in. from the base.

8. Like Problem 7, but with six cap screws, three on each side.

CHAPTER 7

STRUCTURAL ANALYSIS

7-1. Structural design, at one time the exclusive province of the civil and structural engineer, is of considerable importance to the chemical and industrial engineer. The chemical engineer, however, is not concerned primarily with the original design of plate girders and roof trusses, but more particularly with the design of attachments and brackets for pipe, supports for heat exchangers and stills, and with the analysis of existing structures, such as beams, joints, and trusses, to determine whether additional loads, such as those caused by pipe lines and overhead tanks, can be carried safely.

7-2. Construction Codes. Present-day structural analysis and design are usually based upon the specifications of the American Institute of Steel Construction [58] and the American Welding Society,[16] New York, referred to as AISC Code and AWS Code. Recently (Sept. 10, 1942) the AISC has adopted a revised code termed "National Emergency Specifications for the Design, Fabrication, and Erection of Structural Steel for Buildings," issued by the War Production Board, Washington, D. C., in which maximum unit stresses somewhat higher than those permitted by the AISC Code are used. These specifications, herein termed NES Code, are planned for maximum material conservation and are applicable to structures of a temporary or emergency character, designed and constructed by experienced engineers, although such buildings will lend themselves to long-time service if designed so that reinforcement may be added to critical elements in the future.

Table 7-1 summarizes the maximum allowable stresses and other pertinent information for both the AISC and the NES Codes for structural design. The former code states that stress values greater than those given may not be used; the latter code further states that lower unit stresses than those specified shall not be used. (For the duration of the war NES values are mandatory for major design.)

7-3. Structural Sections. Several of the structural sections in common use are shown in Fig. 7-1. Structural angles are used singly or in pairs as tension or compression members in trusses and brackets. Channel, *WF* or wide flange, and American Standard or *I*-beam sections are employed as floor beams, joists, and columns. Built-up sections, constructed of angles and plates or channels and plates riveted or welded to produce an integral member, are often used for long columns or long-span, heavily loaded beams

A representative selection of structural sections is shown in Tables 7-2 to 7-6, with dimensions as indicated in Fig. 7-1. In the channel, *WF*, and *I*-beam

TABLE 7-1.

	Allowable Unit Stresses, psi. AISC	Allowable Unit Stresses, psi. NES
Tension		
Structural steel, net section	20,000	24,000
Rivets, nominal diam.	15,000	15,000
Bolts and threaded parts, root area	13,000	13,000
Butt welds, section through throat	13,000	15,000
Compression		
Columns, gross section, axially loaded		
$L/k \leqq 120$	$17{,}000 - 0.485\left(\frac{L}{k}\right)^2$	$17{,}000 - 0.485\left(\frac{L}{k}\right)^2$
$L/k > 120$	$\dfrac{18{,}000}{1+\frac{1}{18{,}000}\left(\frac{L}{k}\right)^2}$	$\dfrac{18{,}000}{1+\frac{1}{18{,}000}\left(\frac{L}{k}\right)^2}$
Webs of rolled sections at toe of fillet	24,000	24,000
Butt welds, section through throat	18,000	24,000
Shear		
Rivets, pins, and turned bolts	15,000	17,000
Unfinished bolts	10,000	12,000
Webs of beams	13,000	14,000
Butt weld throat	11,300	14,000
Fillet weld throat	11,300	15,000
Bearing		
Rivets and turned bolts, single shear	32,000	32,000
double shear	40,000	40,000
Pins	32,000	32,000
Unfinished bolts	25,000	25,000
Milled contact surfaces	30,000	30,000
Bending		
Tension in extreme fiber of rolled beams and built-up members	20,000	24,000
Compression in extreme fibers of rolled beams and built-up members	$\dfrac{22{,}500}{1+\frac{1}{1800}\left(\frac{L}{B}\right)^2}$	$\dfrac{22{,}500}{1+\frac{1}{1800}\left(\frac{L}{B}\right)^2}$
Pin flexure	30,000	30,000

TABLE 7-2.—BEAMS; AMERICAN STANDARD
(INCH UNITS)

Nominal Size	Area of Section	Depth of Section	Width of Flange	Web Thickness	Axis $x-x$		Axis $y-y$		Distances				Grip	Max. Flange Rivet
					I	k	I	k	n	m	G	g		
24 × 7	29.25	24.00	7.247	.747	2371.8	9.05	48.4	1.29	7/8	1 5/8	3	4	7/8	1
20 × 7	29.20	20.00	7.273	.873	1648.3	7.51	52.4	1.34	15/16	1 3/4	3 1/4	4	15/16	1
	26.26	20.00	7.126	.726	1550.3	7.68	48.7	1.36	15/16	1 3/4	3 1/4	4	15/16	1
	23.74	20.00	7.000	.600	1466.3	7.86	45.8	1.39	15/16	1 3/4	3 1/4	4	7/8	1
20 × 6 1/4	21.90	20.00	6.391	.641	1263.5	7.60	30 1	1.17	13/16	1 9/16	3	3 1/2	13/16	7/8
18 × 6	20.46	18.00	6.251	.711	917.5	6.70	24.5	1.09	11/16	1 3/8	2 3/4	3 1/2	11/16	7/8
	17.50	18.00	6.087	.547	837.8	6.92	22.3	1.13	11/16	1 3/8	2 3/4	3 1/2	11/16	7/8
15 × 6	21.85	15.00	6.278	.868	687.2	5.61	30.6	1.18	13/16	1 5/8	3	3 1/2	13/16	7/8
	18.91	15.00	6.082	.672	632.1	5.78	27.2	1.20	13/16	1 5/8	3	3 1/2	13/16	7/8
	17.68	15.00	6.000	.590	609.0	5.87	26.0	1.21	13/16	1 5/8	3	3 1/2	13/16	7/8
15 × 5 1/2	16.06	15.00	5.738	.648	508.7	5.63	17.0	1.03	5/8	1 1/4	2 3/4	3 1/2	5/8	3/4
	13.12	15.00	5.542	.452	453.6	5.88	15.0	1.07	5/8	1 1/4	2 3/4	3 1/2	9/16	3/4
12 × 5 1/4	14.57	12.00	5.477	.687	301.6	4.55	16.0	1.05	11/16	1 5/16	2 3/4	3	11/16	3/4
	11.84	12.00	5.250	.460	268.9	4.77	13.8	1.08	11/16	1 5/16	2 3/4	3	5/8	3/4
12 × 5	10.20	12.00	5.078	.428	227.0	4.72	10.0	.99	9/16	1 1/8	2 1/2	3	1/2	3/4
10 × 4 3/4	11.69	10.00	5.091	.741	158.0	3.68	9.4	.90	1/2	1	2 1/2	2 3/4	1/2	3/4
	8.75	10.00	4.797	.447	133.5	3.91	7.6	.93	1/2	1	2 1/2	2 3/4	1/2	3/4
	7.38	10.00	4.660	.310	122.1	4.07	6.9	.97	1/2	1	2 1/2	2 3/4	1/2	3/4
8 × 4	7.43	8.00	4.262	.532	68.1	3.03	4.7	.80	7/16	7/8	2 1/4	2 1/4	7/16	3/4
	5.97	8.00	4.079	.349	60.2	3.18	4.0	.82	7/16	7/8	2 1/4	2 1/4	7/16	3/4
	5.34	8.00	4.000	.270	56.9	3.26	3.8	.84	7/16	7/8	2 1/4	2 1/4	7/16	3/4
7 × 3 3/4	5.83	7.00	3.860	.450	41.9	2.68	3.1	.74	3/8	13/16	2	2 1/4	3/8	5/8
	4.43	7.00	3.660	.250	36.2	2.86	2.7	.78	3/8	13/16	2	2 1/4	3/8	5/8
6 × 3 3/8	4.29	6.00	3.443	.343	23.8	2.36	2.1	.69	3/8	3/4	2	2	3/8	5/8
	3.61	6.00	3.330	.230	21.8	2.46	1.8	.72	3/8	3/4	2	2	5/16	5/8
5 × 3	2.87	5.00	3.000	.210	12.1	2.05	1.2	.65	5/16	11/16	2	1 3/4	5/16	1/2

TABLE 7-3.—*WF* SECTIONS; COLUMNS AND BEAMS
(INCH UNITS)

Nominal Size	Area of Section	Depth of Section	Flange Width	Flange Thickness	Web Thickness	Axis $x - x$ I	Axis $x - x$ k	Axis $y - y$ I	Axis $y - y$ k	Distance m	Distance G	Distance g	Max. Flange Rivet
WF Sections and Light Columns													
6 × 6	7.35	6.19	6.050	.471	.300	50.9	2.63	17.4	1.54	13/16	2	3½	¾
	5.89	6.00	6.000	.375	.250	39.2	2.58	13.5	1.51	11/16	2	3½	¾
	4.57	5.79	5.990	.270	.240	28.1	2.48	9.7	1.46	9/16	1¾	3½	¾
5 × 5	4.70	5.00	5.000	.360	.240	21.3	2.13	7.51	1.26	⅝	2	2¾	¾
	3.98	4.86	4.990	.292	.230	17.1	2.07	6.05	1.23	⅝	2	2¾	¾
4 × 4	2.93	4.00	4.000	.265	.220	8.31	1.68	2.74	.97	9/16	1¾	2¼	⅝
	2.22	3.87	3.950	.200	.170	6.06	1.65	1.96	.94	½	1¾	2¼	⅝
Light Beams													
12 × 4	6.47	12.31	4.030	.424	.260	155.7	4.91	4.55	.84	¾	2	2¼	¾
	4.86	12.00	4.000	.269	.230	105.3	4.65	2.79	.76	⅝	1¾	2¼	¾
10 × 4	5.61	10.25	4.020	.394	.250	96.2	4.14	4.19	.86	11/16	2	2¼	¾
	4.40	10.00	4.000	.269	.230	68.8	3.95	2.79	.80	9/16	1¾	2¼	¾
8 × 4	4.43	8.12	4.015	.314	.245	48.0	3.29	3.30	.86	⅝	2	2¼	¾
	3.83	8.00	4.000	.254	.230	39.5	3.21	2.62	.83	9/16	1¾	2¼	¾
6 × 4	4.72	6.25	4.030	.404	.260	31.7	2.59	4.32	.96	11/16	2	2¼	¾
	3.53	6.00	4.000	.279	.230	21.7	2.48	2.89	.90	9/16	1¾	2¼	¾
Joists													
12 × 4	4.14	11.91	3.970	.224	.200	88.2	4.61	2.25	.74	9/16	1¾	2¼	⅝
10 × 4	3.39	9.87	3.950	.204	.180	51.9	3.92	2.01	.77	½	1¾	2¼	⅝
8 × 4	2.95	7.90	3.940	.204	.170	30.8	3.23	1.99	.82	½	1¾	2¼	⅝
6 × 4	2.50	5.83	3.940	.194	.170	14.8	2.43	1.89	.87	7/16	1¾	2¼	⅝

TABLE 7-4.—CHANNELS; AMERICAN STANDARD
(INCH UNITS)

Nominal Size	Area of Section	Depth of Section	Width of Flange	Web Thickness	Axis $x-x$ I	Axis $x-x$ k	Axis $y-y$ I	Axis $y-y$ k	Distance m	Distance G	Distance n	Max. Flange Rivet	Axis $y-y$ x
15 × 3½	14.64	15.00	3.716	.716	401.4	5.24	11.2	.87	1 5/16	2¾	⅝	1	.80
	11.70	15.00	3.520	.520	346.3	5.44	9.3	.89	1 5/16	2¾	⅝	1	.78
	9.90	15.00	3.400	.400	312.6	5.62	8.2	.91	1 5/16	2¾	⅝	1	.79
12 × 3	11.73	12.00	3.415	.755	196.5	4.09	6.6	.75	1 1/16	2½	½	⅞	.72
	8.79	12.00	3.170	.510	161.2	4.28	5.2	.77	1 1/16	2½	½	⅞	.63
	6.03	12.00	2.940	.280	128.1	4.61	3.9	.81	1 1/16	2½	½	⅞	.70
10 × 2⅝	10.27	10.00	3.180	.820	115.2	3.34	4.6	.67	15/16	2½	7/16	¾	.69
	7.33	10.00	2.886	.526	90.7	3.52	3.4	.68	15/16	2½	7/16	¾	.62
	4.47	10.00	2.600	.240	66.9	3.87	2.3	.72	15/16	2½	7/16	¾	.64
9 × 2½	7.33	9.00	2.812	.612	70.5	3.10	3.0	.64	⅞	2½	7/16	¾	.61
	4.39	9.00	2.485	.285	50.7	3.40	1.9	.67	⅞	2½	7/16	¾	.59
8 × 2¼	6.23	8.00	2.619	.579	47.6	2.77	2.2	.60	13/16	2¼	⅜	¾	.59
	4.76	8.00	2.435	.395	39.8	2.89	1.8	.61	13/16	2¼	⅜	¾	.56
	3.36	8.00	2.260	.220	32.3	3.10	1.3	.63	13/16	2¼	⅜	¾	.58
7 × 2⅛	5.79	7.00	2.509	.629	33.1	2.39	1.8	.56	13/16	2	⅜	⅝	.58
	4.32	7.00	2.299	.419	27.1	2.51	1.4	.57	13/16	2	⅜	⅝	.53
	2.85	7.00	2.090	.210	21.1	2.72	.98	.59	13/16	2	⅜	⅝	.55
6 × 2	3.81	6.00	2.157	.437	17.3	2.13	1.1	.53	¾	2	⅜	⅝	.55
	2.39	6.00	1.920	.200	13.0	2.34	.70	.54	¾	2	⅜	⅝	.52
5 × 1¾	2.63	5.00	1.885	.325	8.8	1.83	.64	.49	11/16	2	5/16	½	.48
	1.95	5.00	1.750	.190	7.4	1.95	.48	.50	11/16	2	5/16	½	.49
4 × 1⅝	2.12	4.00	1.720	.320	4.5	1.47	.44	.46	⅝	2	5/16	½	.46
	1.82	4.00	1.647	.247	4.1	1.50	.38	.45	⅝	2	5/16	½	.46
	1.56	4.00	1.580	.180	3.8	1.56	.32	.45	⅝	2	5/16	½	.46
3 × 1½	1.75	3.00	1.596	.356	2.1	1.08	.31	.42	⅝	—	¼	½	.46
	1.19	3.00	1.410	.170	1.6	1.17	.20	.41	⅝	—	¼	½	.44

TABLE 7-5.—UNEQUAL ANGLES
(INCH UNITS)

Nominal Size	Thickness	Area of Section	Axis $x - x$			Axis $y - y$			Axis $z - z$
			I	k	x	I	k	y	k'
8 × 6	1	13.00	80.8	2.49	2.65	38.8	1.73	1.65	1.28
	¾	9.94	63.4	2.53	2.56	30.7	1.76	1.55	1.29
	½	6.75	44.3	2.56	2.47	21.7	1.79	1.47	1.30
8 × 4	1	11.00	69.6	2.52	3.05	11.6	1.03	1.05	0.85
	¾	8.44	54.9	2.55	2.95	9.4	1.05	0.95	0.85
	½	5.75	38.5	2.59	2.86	6.7	1.08	0.86	0.86
6 × 4	1	9.00	30.8	1.85	2.17	10.8	1.09	1.17	0.85
	⅞	7.98	27.7	1.86	2.12	9.8	1.11	1.12	0.86
	¾	6.94	24.5	1.88	2.08	8.7	1.12	1.08	0.86
	⅝	5.86	21.1	1.90	2.03	7.5	1.13	1.03	0.86
	½	4.75	17.4	1.91	1.99	6.3	1.15	0.99	0.87
	⅜	3.61	13.5	1.93	1.94	4.9	1.17	0.94	0.88
5 × 3½	⅞	6.67	15.7	1.53	1.79	6.2	0.96	1.04	0.75
	¾	5.81	13.9	1.55	1.75	5.6	0.98	1.00	0.75
	⅝	4.92	12.0	1.56	1.70	4.8	0.99	0.95	0.75
	½	4.00	10.0	1.58	1.66	4.0	1.01	0.91	0.75
	⅜	3.05	7.8	1.60	1.61	3.2	1.02	0.86	0.76
4 × 3	¾	4.69	6.9	1.22	1.42	3.3	0.84	0.92	0.64
	⅝	3.98	6.0	1.23	1.37	2.9	0.85	0.87	0.64
	½	3.25	5.0	1.25	1.33	2.4	0.86	0.83	0.64
	⅜	2.48	4.0	1.26	1.28	1.9	0.88	0.78	0.64
	5/16	2.09	3.4	1.27	1.26	1.7	0.89	0.76	0.65
	¼	1.69	2.8	1.28	1.24	1.4	0.89	0.74	0.65

TABLE 7-5.—(*Continued*)
(INCH UNITS)

Nominal Size	Thick-ness	Area of Section	Axis $x-x$			Axis $y-y$			Axis $z-z$
			I	k	x	I	k	y	k'
3½ × 3	¾	4.31	4.7	1.04	1.21	3.1	0.85	0.96	0.62
	⅝	3.67	4.1	1.06	1.17	2.8	0.87	0.92	0.62
	½	3.00	3.5	1.07	1.13	2.3	0.88	0.88	0.62
	⅜	2.30	2.7	1.09	1.08	1.8	0.90	0.83	0.62
	5/16	1.93	2.3	1.10	1.06	1.6	0.90	0.81	0.63
	¼	1.56	1.9	1.11	1.04	1.3	0.91	0.79	0.63
3 × 2½	½	2.50	2.1	0.91	1.00	1.3	0.72	0.75	0.52
	⅜	1.92	1.7	0.93	0.96	1.0	0.74	0.71	0.52
	5/16	1.62	1.4	0.94	0.93	0.90	0.74	0.68	0.53
	¼	1.31	1.2	0.95	0.91	0.74	0.75	0.66	0.53
2½ × 2	½	2.00	1.1	0.75	0.88	0.64	0.56	0.63	0.42
	⅜	1.55	0.91	0.77	0.83	0.51	0.58	0.58	0.42
	5/16	1.31	0.79	0.78	0.81	0.45	0.58	0.56	0.42
	¼	1.06	0.65	0.78	0.79	0.37	0.59	0.54	0.42
	3/16	0.81	0.51	0.79	0.76	0.29	0.60	0.51	0.43
2 × 1½	⅜	1.17	0.43	0.61	0.71	0.21	0.42	0.46	0.32
	5/16	1.00	0.38	0.62	0.69	0.18	0.42	0.44	0.32
	¼	0.81	0.32	0.62	0.66	0.15	0.43	0.41	0.32
	3/16	0.62	0.25	0.63	0.64	0.12	0.44	0.39	0.32

TABLE 7-6.—EQUAL ANGLES
(INCH UNITS)

Nominal Size	Thickness	Area of Section	Axis $x - x$ and Axis $y - y$			Axis $z - z$
			I	k	x	k min.
3 × 3	½	2.75	2.2	0.90	0.93	0.58
	⅜	2.11	1.8	0.91	0.89	0.58
	¼	1.44	1.2	0.93	0.84	0.59
2½ × 2½	½	2.25	1.2	0.74	0.81	0.47
	⅜	1.73	0.98	0.75	0.76	0.48
	5/16	1.47	0.85	0.76	0.74	0.49
	¼	1.19	0.70	0.77	0.72	0.49
2 × 2	⅜	1.36	0.48	0.59	0.64	0.39
	5/16	1.15	0.42	0.60	0.61	0.39
	¼	0.94	0.35	0.61	0.59	0.39
	3/16	0.71	0.28	0.62	0.57	0.40
1½ × 1½	⅜	0.98	0.19	0.44	0.51	0.29
	¼	0.69	0.14	0.45	0.47	0.29
	3/16	0.53	0.11	0.46	0.44	0.29
	⅛	0.36	0.08	0.46	0.42	0.30

sections, d represents the depth of the section, b the flange width, t the web thickness, n the mean thickness of the flange, m the distance from the toe of the fillet at the juncture of the flange and web, and G and g the usual rivet gages. Axes xx and yy are neutral axes, I the moment of inertia of the section, and k the radius of gyration. The section modulus Z of any section may be obtained

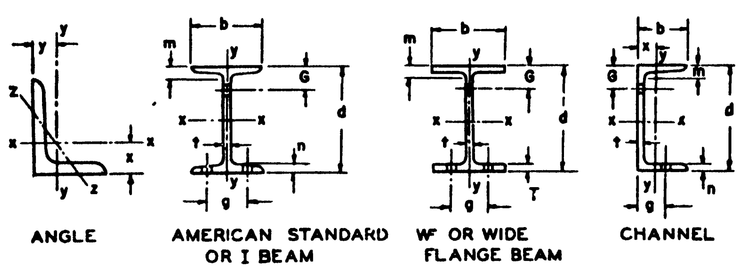
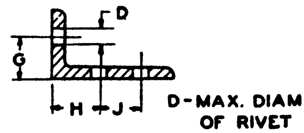


FIG. 7-1. Representative Structural Sections.

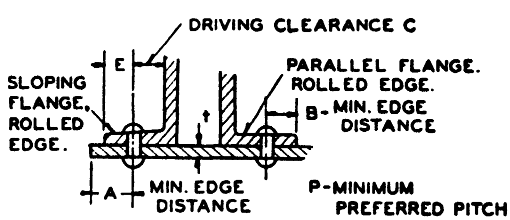
by dividing the moment of inertia I by the distance from the neutral axis under consideration to the extreme fiber; this distance is $d/2$ or $b/2$ for the WF and I-beam sections and $d/2$ or $b - x$ for the channel section. Structural angles are specified by the outer lengths and the thickness of the legs. The weight of any structural member, per foot of length, may be obtained by multiplying the gross sectional area by the factor 3.4, which represents the weight of a structural member one inch square and one foot long.

7-4. Connections. Connections for structural members are usually made by rivets, bolts, or welds. Riveting practice for these is similar to that for pressure vessel joints; with the difference that the holes for the rivets are usually punched to a diameter ⅛ in. greater than the diameter of the rivet. Rivet gages (or spacing) and other data are given in Figs. 7-2 and 7-3, and conventional methods of rivet representation on drawings are illustrated in Fig. 7-4. Square head bolts, with square or hexagonal nuts in reamed holes, in which the clearance between the bolt and hole diameters does not exceed 0.01 in., are considered as effective as hot driven rivets. (A hot driven rivet is presumed to fill the hole after it is headed.) Unfinished bolts in punched holes ⅛ in. larger than the body diameter of the bolt have only two thirds the load capacity of carefully fitted bolts and are usually used for temporary fastenings during the process of erection. Welded joints are finding increased application as structural connections because of their simplicity and low cost.



WIDTH OF LEG	8	7	6	5	4	3½	3	2½	2	1¾	1½
G	4½	4	3½	3	2½	2	1¾	1⅜	1⅛	1	7/8
H	3	2½	2¼	2							
J	3	3	2½	1¾							
D	1⅛	1	7/8	7/8	7/8	7/8	7/8	7/8	5/8	1/2	3/8

Fig. 7-2. Rivet Gages for Structural Angles.



RIVET DIAM.	3/8	1/2	5/8	3/4	7/8	1	1⅛	1¼	1⅜	1½
C	7/8	1	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2
A •••		1	1⅛	1¼	1½	1¾	2	2¼		
B		7/8	1	1⅛	1¼	1½	1¾	2		
E •		3/4	7/8	1	1⅛	1¼	1½	1¾		
P ••		1¾	2	2½	3	3½	4	4½		

• MAY BE DECREASED ⅛" FOR HOLES NEAR END OF BEAM

•• MAY BE REDUCED TO A MINIMUM OF 3D

••• MAXIMUM A = 12t; 6" MAXIMUM

Fig. 7-3. Rivet Gages and Edge Distances for Channel WF, and I Beam Sections.

7-5. Allowable Loads for Structural Rivets and Bolts. The stresses in structural members and connections are computed by applying the principles of stress analysis, developed in Chapters 2, 5, and 6, and modified by empirical data embodied in the AISC Code. Rivet selection is based upon the diameter of the rivet before driving, and the allowable strength in single shear is given by

$$F_s = S_s \frac{\pi D^2}{4} \tag{7-1}$$

and in compression or bearing by

$$F_b = S_b D t \tag{7-2}$$

where S_s and S_b are the allowable shear and bearing stresses, psi., from Table 7-1, D the rivet diameter before driving, and t the plate thickness or length of bearing of the rivet in the plate. Rivets in double shear are permitted higher unit bearing stresses, as shown in Table 7-1, because the tendency to twist the rivet in the holes is minimized. The strength of rivets in tension may be obtained from Eq. 7-1 by substituting the allowable unit tensile stress S_t for S_s. Equations 7-1 and 7-2 may also be used to determine the strength of bolts, but the root area of the thread must be substituted for the area of the body if the bolt is subjected to tension.

Riveted and bolted joints are often subjected to loads so applied that a serious degree of eccentricity of the connection may occur. The effect of such eccentricity, and the methods employed to compensate for them, will be considered in Sections 7-7 and 7-11.

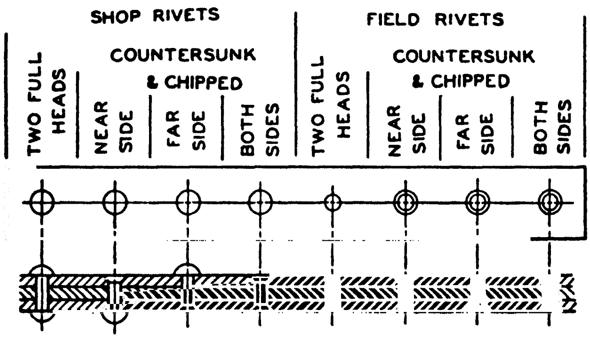


FIG. 7-4. Conventional Representation of Rivets.

7-6. Analysis of Welded Structural Joints. The growing importance of welded joints necessitates a careful analysis of the stresses induced by various methods of load application. The underlying theory regarding stress distribution in welds is rather complex, and for design purposes simple analyses based upon direct and flexural stress equations are usually used. To guard against over-stressed joints, conservative working stresses are obtained from structural and pressure vessel design codes, and any increase in the theoretical throat dimension caused by the bulge of the weld is disregarded in computing weld area. Representative methods of load application and types of welds are illustrated in Figs. 7-5 to 7-10.

Fig. 7-5 shows double-welded V- and single-welded U-butt joints subjected to direct tension. The area in tension is equal to the product of the throat dimension t and the length L of the weld, and the unit tensile stress is

$$S = \frac{F}{tL} \tag{7-3}$$

In Fig. 7-5 it is assumed that the forces F are approximately coaxial and uniformly distributed along the length L of the weld, thus eliminating any flexural tendency.

In Fig. 7-6, A and C show double-welded lap joints. The end welds of A are preferred to the side or parallel welds of C, since uniform stress distribution in long welds parallel to the line of action of the load cannot be obtained. Both types of welds are subjected to flexural as well as direct stresses because of the eccentricity of the loads; this condition is far more serious in the joint at A than in the one at C. The dominant stress in fillet welds is shear on the throat t of the weld, and Eq. 7-3 applies for fillet welds subjected to direct stress. Since fillet welds are usually specified by leg length m, it is customary to specify working stresses as allowable shear per lineal inch per ⅛ in. of weld leg. From the AWS and AISC Codes the allowable shearing stress in the throats of

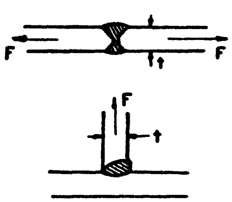


Fig. 7-5. Butt-welded Joints Subjected to Tension.

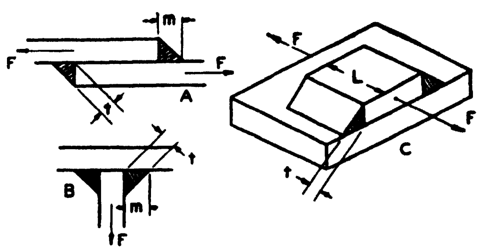


Fig. 7-6. Fillet-welded Joints Subjected to Tension.

structural welds are 11,300 psi. for average strength welds made with uncoated electrodes, and 13,600 psi. for high strength welds made with coated electrodes. A weld 1 in. long, with a ⅛-in. leg, has a throat area of $1 \times 0.125 \times 0.707$, or 0.0884 sq. in. The allowable shear in average strength welds is thus $0.0884 \times 11{,}300$, or 1000 lbs. per lineal inch per ⅛ in. of fillet leg. Similarly, high strength welds have a permissible shearing strength of $0.0884 \times 13{,}600$, or 1200 lbs. per lineal inch per ⅛ in. of fillet leg. Thus a ⅜-in. average strength fillet weld can resist a shearing stress of 3000 lbs.; a ½-in. high strength weld can resist a shearing stress of 4800 lbs.

To calculate the stress for the double-welded joints shown in Fig. 7-6, Eq. 7-3 becomes

$$S = \frac{0.707F}{mL} \tag{7-4}$$

where S is the allowable shearing stress, psi., in the throat of the weld, based upon the length m of the leg of the weld. Since the allowable unit stresses of 11,300 and 13,600 psi. are based upon the throat dimension t, which is $0.707m$,

the corresponding allowable shearing stresses based upon the leg dimension is 0.707 × 11,300, or 8000 psi., for average strength welds, and 0.707 × 13,600, or 9600 psi., for high strength welds.

The unit stress in a butt weld subjected to flexure, Fig. 7-7, may be found from the flexure equation,

$$S = \frac{6M}{Lt^2} \tag{7-5}$$

where M is the bending moment, in in.-lbs., and L is the length of the weld.

7-7. Welded Joints Subjected to Combined Stresses. Transverse fillet welded joints subjected to flexure, as illustrated in Fig. 7-8, are often encountered in machine elements. The stresses in the welds of Fig. 7-8A are computed by assuming first that the flexural moment Fa is counteracted by a couple composed of forces acting at the center of the fusion zones of the welds. The magnitude of these resisting forces is equal to the product of their throat area tL and unit stress S_1; the moment arm of the couple is b, and

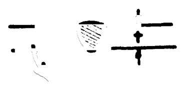


FIG. 7-7. Butt Weld Subjected to Flexure.

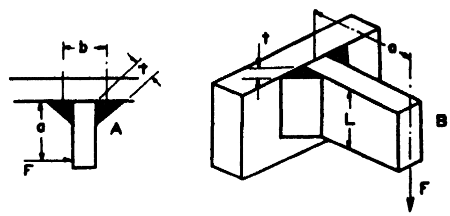


FIG. 7-8. Transverse Fillet Welded Joints Subjected to Flexure.

$$S_1 = \frac{Fa}{btL} = \frac{Fa}{0.707bmL}$$

or

$$S_1 = \frac{1.41\,Fa}{bmL}$$

In addition to the stress S_1 caused by the flexural load, there is a direct shear on either weld of

$$S_2 = \frac{F}{2tL} = \frac{0.707\,F}{mL}$$

The resultant shearing stress S_r across the throat of the weld is

$$S_r = \sqrt{S_2^2 + S_1^2}$$

and, by substitution

$$S_r = \frac{F}{bmL}\sqrt{\frac{b^2}{2} + 2a^2} \tag{7-6}$$

This expression gives the resultant shear in terms of the leg dimension rather than the throat dimension of the weld. This is often more convenient since a simplified stress value may be used.

Example 7-1. Find the required leg dimensions of an average strength weld for the 4-in. wide bracket of Fig. 7-9.

Solution. The arrangement of the welds and the method of application of the load F in Fig. 7-9 are essentially the same as in Fig. 7-8A. The moment arm a is 5 in.; the moment arm b of the couple may be taken as 6 in.; the length L is 4 in.; and the allowable resultant shearing stress is 8000 psi. By Eq. 7-6,

$$m = \frac{F}{bLS_r}\sqrt{(b^2/2) + 2a^2} = \frac{10{,}000}{6 \times 4 \times 8000}\sqrt{(6^2/2) + 2 \times 5^2} = 0.431 \text{ in.}$$

requiring ½-in. welds at the top and bottom. If desired, the stress S_r may be recalculated, based upon a moment arm b of $6 + ¼ + ¼$ in., or 6.5 in.

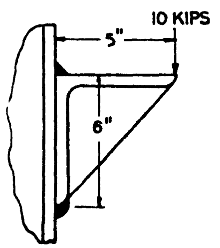


Fig. 7-9. Bracket Held by Fillet Welds.

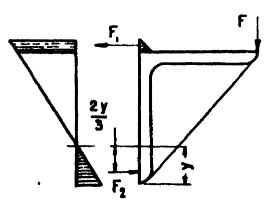


Fig. 7-10. Force Distribution in Bracket.

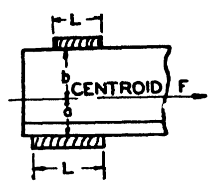


Fig. 7-11. Side-welded Structural Angle.

It has been assumed that the welds transmit the entire load from the bracket to the column. This assumption is on the safe side for beams and channels the ends of which are flame cut, because no consideration for bearing area should be given to such surfaces. For milled surfaces or for a seat condition, such as is shown in Fig. 7-9, where the machined face of the bracket rests against a column flange, some portion of the surfaces in contact may be assumed to furnish bearing resistance. For this condition the approximate distribution of the tensile force F_1 in the upper weld and the varying bearing resistance F_2 afforded by the lower portion of the vertical leg of the bracket are shown in Fig. 7-10. In this case the lower weld is of service only on account of its resistance to vertical shear. Such an analysis is essentially similar to that of Example 6-1, and while it is more rigorous than the preceding analysis that resulted from Eq. 7-6 and involves more complex computation, it gives essentially the same results.

In the transverse fillet welded joint shown in Fig. 7-8B, the welds resist a flexural moment Fa and a direct shear F. The stress induced by flexure is

resisted by the weld throat area tL, where the section modulus Z is twice $tL^2/6$. By substitution in the flexure equation:

$$S_1 = \frac{M}{Z} = \frac{6Fa}{2tL^2}$$

The stress induced by direct shear is

$$S_2 = \frac{F}{2tL}$$

The resultant shearing stress S_r, from Eq. 6-3, is

$$S_r = \frac{F}{2mL^2}\sqrt{2(L + 3a)^2} \qquad (7\text{-}7)$$

This expression, like Eqs. 7-4 and 7-6, is based upon the leg dimension m of the weld, and the value of the induced stress S should be compared to allowable working stresses of 8000 psi. and 9600 psi. for average and high strength welds.

If, in addition to the force F, an axial force F_x is applied perpendicular to the cross section of the projecting member or beam, the resultant shear stress will be $S_r + S_x$, where

$$S_x = \frac{F_x}{2tL} = \frac{0.707F}{mL} \qquad (7\text{-}8)$$

In a single angle attached to a gusset plate the load may be considered to act along the centroid of the angle, as illustrated in Fig. 7-11. To eliminate eccentricity in the joint the weld lengths may be specified so that

$$bL_1 = aL_2$$

where L_1 and L_2 are the weld lengths, and b and a are the distances from the line of the weld to the centroid or neutral axis of the member. Comparative tests show, however, that such refinement in calculation for single angle connections is not essential in ordinary construction, and equal lengths of weld are usually specified. If an axial load is applied to a member composed of two angles placed back to back, no serious eccentricity exists, and the welds on each side may be of the same length.

Tension and Compression Members

7-8. Design of Tension Members. Bars or rods of rectangular cross section are the simplest tension members obtainable, and are selected on the basis of sufficient area to resist the load. For welded attachment, the required area is equal to the gross area of the section. If a single line of rivets is employed to transfer the load, the required area is equal to the gross area minus the projected area of one rivet hole. Structural angles are also extensively used as tension members. If two angles are used back to back, as shown in Fig. 7-12,

there is no serious eccentricity in the connection, and the entire net area of the angle may be considered as furnishing effective tensile resistance. For a member composed of two angles, with a single row of rivets in each angle, the net area of the member is equal to the gross area of both angles minus the projected area of the rivet hole in each. For a member composed of two angles with two or more rows of rivets, any rivet areas in the plane of the section, plus an additional area to account for the possibility of diagonal failure along a line *A-A*, Fig. 7-12, must be deducted. The area A_d, deductible for diagonal failure, is obtained from the following, which is based upon the specifications of the AISC Code:

$$A_d = A_h\left(1 - \frac{H^2}{4J}\right) \qquad (7\text{-}9)$$

where A_h is the projected area of one rivet hole, H the rivet pitch as indicated in Fig. 7-12, and J the rivet gage distance, Figs. 7-2 and 7-12. The rivet pitch H

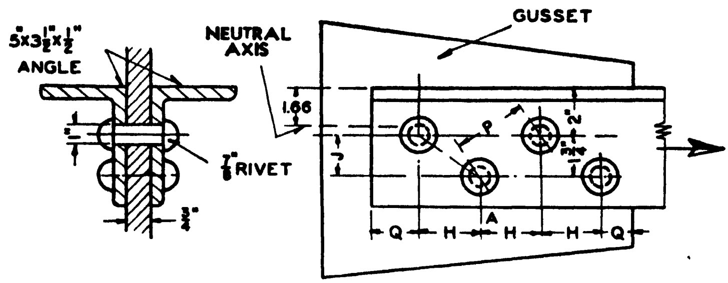


FIG. 7-12. Double-angle Tension or Compression Member.

depends upon J and the diagonal pitch P, from Fig. 7-3, and is equal to $P^2 - J^2$.

Example 7-2. Analyze the stresses and design the rivet arrangement for the tension member shown in Fig. 7-12, considering it subjected to a pull of 135,000 lbs.

Solution. The gross area of a 5 × 3½ × ½-in. angle, from Table 7-5, is 4 sq. in. From Fig. 7-2 the maximum diameter D of the rivet is ⅞ in., and the gage distance J is 1¾ in. (along the 5-in. leg). From Fig. 7-3 the minimum diagonal pitch P for a ⅞-in. diameter rivet is 3 in. The minimum pitch H for these conditions is $\sqrt{3^2 - 1.75^2}$, or 2.44, say 2½ in. Rivet holes are punched ⅛ in. larger than the diameter of the rivet; the projected area A_h of one rivet hole is then 1 × 0.5, or 0.50 sq. in. From Eq. 7-9 the area deductible for diagonal failure, for one angle, is

$$A_d = 0.50\left(1 - \frac{2.5^2}{4 \times 1.75}\right) = 0.055$$

The net area A_n of the two angles is

$$A_n = 2(4 - 0.50 - 0.055) = 6.9 \text{ sq. in.}$$

and the unit tensile stress S_t is

$$S_t = \frac{135{,}000}{6.9} = 19{,}600 \text{ psi.}$$

which is slightly below the allowable maximum of 20,000 psi. given in Table 7-1, AISC Code.

The shearing and bearing strengths of the rivets, from Eqs. 7-1 and 7-2, are

$$F_s = 2 \times 15{,}000 \times \pi \left(\frac{0.875^2}{4}\right) = 18{,}040 \text{ lbs.}$$

$$F_b = 40{,}000 \times 0.875 \times 0.750 = 26{,}020 \text{ lbs.}$$

It should be noted that the rivets are subjected to double shear, permitting the use of two shear areas in Eq. 7-1 and the higher of the two bearing values (from Table 7-1) in Eq. 7-2. The thickness of the gusset, ¾ in., is used for computing the bearing area of the rivet, since it is less than the sum of the thicknesses of the angle legs. Although the diameter of the hole is used when the net section of the member is computed, the nominal rivet diameter is used for computing shear and bearing stresses in the rivet.

Since the shear strength F_s is less than the bearing strength, it must be used to determine the number of rivets. Consequently

$$\frac{135{,}000}{18{,}040} = 7.49, \text{ or 8 rivets are required.}$$

The edge distance Q may be found by equating the product of twice the distance from the edge of the rivet hole and the thickness of the leg of the angle to the shearing area A_s of the rivet. For the angle:

$$2t\left(Q - \frac{D}{2}\right) = \frac{\pi D^2}{4}$$

or

$$Q = \frac{\pi D^2}{8t} + \frac{D}{2}$$

and

$$Q = \frac{\pi \times 0.875^2}{8 \times 0.50} + \frac{0.875}{2} = 1.05 \text{ in.}$$

For the plate

$$2t\left(Q - \frac{D}{2}\right) = \frac{2\pi D^2}{4}$$

or

$$Q = \frac{\pi D^2}{4t} + \frac{D}{2}$$

and

$$Q = \frac{\pi \times 0.875^2 \times 0.75}{4} + \frac{0.875}{2} = 1.24 \text{ in.}$$

Fig. 7-3 gives a value of 1¼ in. for the minimum edge distance. Since distances Q for the angle and the plate are both less than 1¼ in., the AISC requirements are satisfied by using Q equal to 1¼ in.

In structural members composed of a single angle, particularly those with a single row of rivets, the line of action of the applied force is usually coincident with the rivet gage line and midway between the inner and outer surfaces of the attached leg. This introduces a serious degree of eccentricity, not only about the neutral axis parallel to the rivet axis, but also about the centroid parallel to the attached leg. Such eccentricity may be compensated for by using the net area of the connected leg plus one half of the area of the unconnected leg as the effective area in tension, illustrated in Fig. 7-13.

Example 7-3. Find the allowable tensile load that may be carried by the angle of Fig. 7-13, based upon both the AISC and NES Codes, and determine the number of rivets required.

Solution. From Table 7-5, the gross area of a 4 × 3 × ½-in. angle is 3.25 sq. in. The projected area A_b is 1 × 0.50, or 0.5 sq. in.; the deductible area in the unconnected leg is 0.625. The net area is 3.25 − 0.5 − 0.625, or 2.125 sq. in. From Table 7-1 the allowable tensile stresses S_t for the AISC and NES Codes are 20,000 and 24,000 psi., respectively, and the allowable tensile loads F_t are:

AISC Code $F_t = 20{,}000 \times 2.125 = 42{,}500$ lbs.

NES Code $F_t = 24{,}000 \times 2.125 = 51{,}000$ lbs.

The shearing and bearing strengths of the rivets, from Eqs. 7-1 and 7-2 and Table 7-1 are:

AISC Code $F_s = 15{,}000 \times \pi\left(\frac{0.875^2}{4}\right) = 9{,}020$ lbs.

NES Code $F_s = 17{,}000 \times \pi\left(\frac{0.875^2}{4}\right) = 10{,}200$ lbs.

$F_b = 32{,}000 \times 0.875 \times 0.50 = 14{,}000$ lbs.

The number of rivets is found from:

AISC Code $\frac{42{,}500}{9020} = 4.7$

NES Code $\frac{51{,}000}{10{,}200} = 5.0$

In each case 5 rivets would be required.

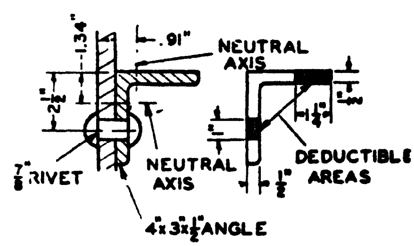
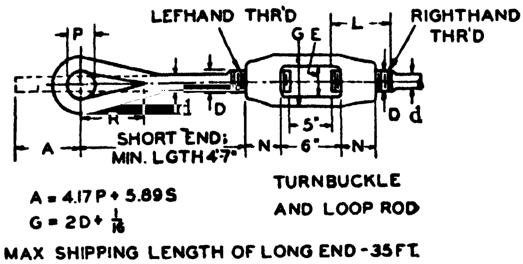


FIG. 7-13. Single-angle Tension or Compression Member.

7-9. Design and Application of Tension Rods. Tension rods are employed as secondary structural members, for example, sag rods in industrial buildings to support purlins in a direction parallel to the roof. They are also employed as tie rods to support long pipe spans or horizontal cylindrical vessels, or to provide auxiliary support for wooden beams. Tension rods may be obtained in either circular or square cross section; one form of tension rod of circular section is shown in Fig. 7-14. Loop rods have an integral loop or "eye" at each end. Rods with threaded outer ends may have an attached clevis at either end, Fig. 7-15. The connection between the separate rods is furnished by a turnbuckle, which has right and left threads so that some initial tension in the rod may be developed to eliminate sway or sag of the supported elements. Loop rods less than 1 in. in diameter at the threads are not furnished with upset ends. For such rods the size of the bar at the loop is the same as the nominal size of the thread. This mode of fabrication may introduce severe localized stresses at the root of the threads, and it is customary to select a rod of such size that the root diameter of the thread is at least 0.05 to 0.06 in. greater than the diameter actually required.

Example 7-4. A 6-in. standard pipe for an oil line is supported on columns *C* as shown in Fig. 7-16. The weight of the filled pipe is 40 lbs. per ft. of length. A loop rod is to be used to brace the pipe at the center. Select the proper rod from Fig. 7-14.

Solution. The pipe acts as a uniformly loaded continuous beam, with supports furnished by the columns C at the ends and by the supporting saddles S at the center, and is equivalent to a continuous beam of two equal spans each 15 ft. long. From Table 5-1 and Fig. 5-46 it is seen that the middle reaction is $10wL/8$. Substituting the uniform load of 40 lbs. per ft., and the span length (between the middle and end supports) or 15 ft., the vertical reaction at the saddle Q is $10 \times 40(15/8)$, or 750 lbs. This force must be exerted by the saddle and is transmitted from the saddle to the loop rods. The saddle force F and the tension T in the two parts of the rod are shown in the force diagram of Fig. 7-16; force T is found to have a magnitude of approximately 5625 lbs.



D	d	N	E	L
3/4, 7/8		1.5D − 1/16	D + 7/32	
1	3/4	1.5D − 1/8	1 9/32	4
1 1/8 TO 1 3/4	D − 3/8		1.25D	
2, 2 1/8	D − 1/2		1.88D	4 1/2
2 1/4, 2 3/8		1.5 D		5
2 1/2, 2 5/8				5 1/2
2 3/4, 2 7/8	D − 5/8			6
3 1/4	2 1/2, 2 5/8			6 1/2
3 1/2, 3 3/4	D − 3/4	6"		7
4	3 1/8, 3 1/4			7 1/2

Fig. 7-14. Turnbuckle and Loop Rods.

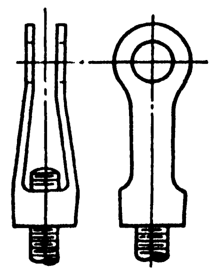


Fig. 7-15. Clevis for Tension Rod.

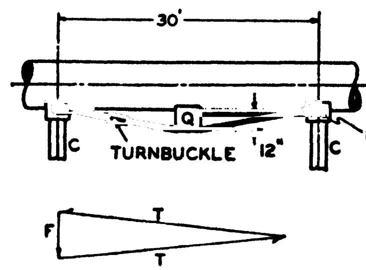


Fig. 7-16. Trussed Pipe Line.

From Table 7-1, the allowable tensile stress in the rod body or at the root of the thread is 20,000 psi., and a rod area of 5625/20,000, or 0.281 sq. in. is required. The rod diameter must be equal to $\sqrt{4 \times 0.281/\pi}$, or 0.597 in. As indicated in Fig. 7-14, the diameter d of the rod is equal to the nominal thread diameter for ⅞-in. rod, or 0.730 in., thus a ⅞-in. rod is satisfactory.

The diameter P of the pin is based upon the allowable shearing stress of 15,000 psi. from Table 7-1. This diameter is found from

$$P = \sqrt{\frac{4 \times 5625}{\pi \times 15{,}000}} = 0.691 \text{ in.}$$

which necessitates a ¾-in. diameter pin at the loop ends.

The rod and turnbuckle proportions given in Fig. 7-14 are such that further stress analysis is not necessary, since the parts are of ample strength for the rod size. It is essential that the two supports P and the saddle Q are in alignment, and that the pipe itself is perfectly level, after the turnbuckle is adjusted; otherwise the stresses in the loop rods

may be considerably greater than the computed values. The supports P should be welded to the pipe so that the pipe will take care of the horizontal component of the force in the rod.

A superficial analysis of this problem might result in a solution in which the 30-ft. pipe span would be treated as two single spans of 15 ft. each. Following this, the support Q would carry a pipe weight of 40×15, or 600 lbs., resulting in rod tension T of 4500 lbs. instead of 5625 lbs. Such an analysis is correct only if the two halves of the pipe span are disconnected at Q.

In some instances two saddles or supports are used, placed at the third points of the pipe span. For this case the pipe is treated as a uniformly loaded continuous beam with four supports, equivalent to a beam of three equal spans each 10 ft. long. From Table 5-1 and Fig. 5-46 it may be seen that the reactions at the supports are $11wL/10$. If two saddles are used in the foregoing problem, values of $11 \times 40 \times 10/10$, or 440 lbs., are obtained for the vertical forces on the loop rod. The tension in the loop rod is found from a force diagram similar in principle to that of Fig. 7-16 and is equal to 4400 lbs., which is considerably less than the 5625-lb. value for a simple support. As in the preceding discussion, it is vital that the pipe be level after erection, to eliminate variation in the magnitude of the rod stress.

7-10. Design of Compression Members. Structural columns and struts are designed or analyzed on the basis of the principles outlined in Chapter 5. The allowable load that may be carried by a column is equal to the product of the gross area and the allowable unit compressive stress. The latter is obtained from the allowable unit stress equations given in Table 7-1 (which are identical with Eq. 5-23 and Eq. 5-24), and is dependent upon the ratio of the column length L and the least radius of gyration k of the column section. Main members, such as the upper chord of a roof truss, are limited to a maximum L/k ratio of 120; secondary or bracing members, such as the interior struts of trusses, are limited to a maximum L/k ratio of 200.

If a single angle is used as a column, the least radius of gyration is usually the one taken with respect to the intersection of the centroidal axes parallel to the legs. Values of such radii of gyration, with respect to an axis zz, are given in Tables 7-5 and 7-6. In columns composed of two angles placed back to back, it is advisable to compute the radius of gyration about the centroidal axis parallel to the adjacent legs, although this value is usually greater than the radius of gyration about the centroidal axis parallel to the unconnected legs.

Example 7-5. Find the allowable load that may be carried by the member of Fig. 7-12 if it serves as a main member column whose effective length is 14 ft. Consider the gusset plate to be 1 in. thick instead of ¼ in. as indicated.

Solution. From Table 7-5 the radius of gyration of the pair of angles about the neutral axis xx parallel to the short legs of the angles is 1.58 in. The radius of gyration of the pair of angles about a central axis parallel to the long legs is found by applying Eqs. 5-6 and 5-7. If we consider the neutral axis of either angle as gg and the neutral axis of the pair as ee, the moment of inertia of one angle about gg, from Table 7-5, is 4.1 in.^{4} The distance from axis ee to gg is equal to one half the gusset plate thickness plus the distance y from Table 7-5, or $0.50 + 0.91$, or 1.41 in., as shown in Fig. 7-17.

From Eq. 5-6, $$I_{ee} = 4.0 + (4 \times 1.41^2) = 11.95 \text{ in.}^4$$

From Eq. 5-7, $$k_{ee} = \sqrt{\frac{11.95}{4.00}} = 1.73 \text{ in.}$$

The radius of gyration about axis xx (1.58 in.) is, therefore, the controlling value. The allowable load for the column is found from Eq. 5-23, and is equal to

$$P = 2 \times 4.00 \left[17{,}000 - 0.485 \left(\frac{14 \times 12}{1.58} \right)^2 \right] = 92{,}400 \text{ lbs.}$$

The L/k ratio is $14 \times 12/1.58$, or 106. This is satisfactory for main members since they can have an upper limit of 120 for L/k.

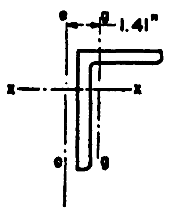


FIG. 7-17. Location of Controidal Axis of Double-Angle Column.

Example 7-6. Find the allowable compression load that may be carried by the member of Fig. 7-13 if it serves as a brace with an effective length of 10 ft.

Solution. By reference to Table 7-5 it is noted that the three radii of gyration of the section are: $k_{xx} = 1.25$; $k_{yy} = 0.86$; and $k_{zz} = 0.64$. The latter is the minimum or controlling value. The L/k ratio is $10 \times 12/0.64$, or 188, and since it lies between 120 and 200 the allowable load is obtained from Table 7-1.

$$P = \frac{18{,}000 \times 3.25}{1 + (10 \times 12/0.64)^2/18{,}000} = 19{,}800 \text{ lbs.}$$

It should be noted that in both Examples 7-5 and 7-6, the gross area of the member is used in computing allowable compressive loads.

7-11. Analysis and Design for Eccentric Loads. Columns are often subjected to eccentric loads in which the eccentricity is of considerable magnitude, as illustrated in Fig. 7-18, where a bracket is shown for supporting a length of pipe fastened to a *WF* section serving as a column. The eccentricity of the load induces a flexural moment at the point of attachment of the bracket and this greatly increases the possibility of column failure by buckling. To compensate for this eventuality, the AISC Code requires that members subjected to both axial and flexural stresses be so proportioned that

$$\frac{S_c}{S_a} + \frac{S_f}{S_x} \leqq 1 \tag{7-10}$$

where S_c/S_a is the ratio of the actual and permissible axial stresses, and S_f/S_x the ratio of the actual and permissible flexural stresses. This expression is the same as Eq. 5-17.

The rivets shown in Fig. 7-18 are subjected to both primary and secondary forces. The load F at a distance of 10 in. from the centroid C of the rivet group may be replaced by a force F at C, plus a couple whose moment is 10 F in.-lbs. The force at C results in secondary stresses in the rivets. The magnitudes of these stresses depend upon the respective moment arms about C.

Example 7-7. The 6×6-in. *WF* column shown in Fig. 7-18 carries an estimated concentric load of 18 tons, and the bracket shown is to be attached to support a length of pipe weighing 2 tons. Investigate the stress in the column and design the rivet arrangement for the bracket.

Solution. The eccentric load may be resolved into a direct force of 2 tons at the column axis, plus a moment of $2 \times 2000 \times 10$, or 40,000 in.-lbs., about the centroid C of the rivet group. From Table 7-3, the cross-sectional area of the column is 5.89 sq. in., and the

unit compressive stress S_a in the column due to the concentric load of 20 tons is $20 \times 2000/5.89$, or 6800 psi.

The moment of inertia I_{yy} of the column section is 13.5 in.^{4}, and the distance c from the neutral axis to the extreme fiber is 3 in. The unit flexural stress S_f from Eq. 5-12 is $40{,}000 \times 3 \div 13.5$, or 8900 psi. From Table 7-1 the allowable tension S_s in the extreme fiber of rolled beams is 20,000 psi. The radius of gyration of the column section, with respect to axis *yy*, which is the axis about which flexure will occur, is 1.51 in. (from Table 7-3). The maximum allowable unit stress in the column due to the axial load, from Eq. 5-17, is

$$\frac{P}{A} = S_a = 17{,}000 - 0.485\left(\frac{13 \times 12}{1.51}\right)^2 = 11{,}800 \text{ psi.}$$

Substituting in Eq. 7-10,

$$\frac{6800}{11{,}800} + \frac{8900}{20{,}000} = 1.021, \text{ which is higher than permitted.}$$

Substituting the value given by the NES Code for S_s,

$$\frac{6800}{11{,}800} + \frac{8900}{24{,}000} = 0.947$$

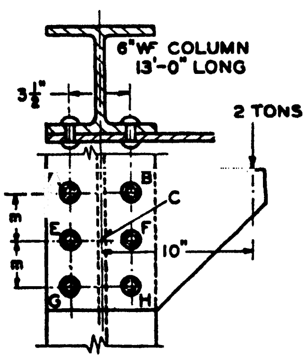


FIG. 7-18. Eccentrically-loaded Column and Bracket.

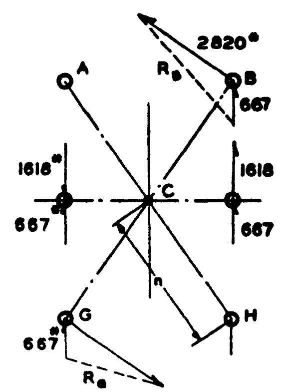


FIG. 7-19. Analysis of Forces on Rivets of Eccentrically Loaded Bracket.

which is within the specified limit. It is interesting to note that the actual compressive stress at the right edge of the column is $S_a + S_f$, or $6800 + 8900$, or 15,700 psi., which is considerably higher than the permissible stress of 11,800 psi. permitted by the column equation of Table 7-1.

The design of the rivet arrangement for the bracket involves a consideration of both primary and secondary forces on the rivets. With six rivets the primary force, or direct load on each rivet, is $2 \times 2000/6$, or 667 lbs. In considering the rotation of the bracket about the centroid of the rivet group, a unit force F_u at a unit distance from the centroid C may be assumed, and the force on any rivet at some distance n from C will be nF_u. To determine the distance n it will be necessary to make some assumption regarding the rivet arrangement. In Fig. 7-18 the distance between the vertical rivet centerlines, $3\frac{1}{2}$ in., is made equal to distance g, Table 7-3. The distance m between horizontal centerlines will be assumed as $2\frac{1}{2}$ in., since this figure represents the minimum preferred pitch (Fig. 7-3) for $\frac{3}{4}$-in. rivets, and a casual estimate indicates that rivet diameters in excess of this dimension will not be required. The distance n from the centroid C to the centers of rivets A, B, G, and H is

$$n = \sqrt{(1.75)^2 + (2.50)^2} = 3.05 \text{ in.}$$

The resisting force on each rivet *A, B, G*, and *H* is 3.05 × F_u; on *E* and *F*, 1.75 × F_u. The resisting moment of any rivet in the group is equal to the product of the resisting force and the moment arm *n*; the resisting moments of all the rivets must be equal to the external moment about *C*. The summation of the resisting moments, equated to the external moment, is

$$2 \times 1.75 \times 1.75 \times F_u + 4 \times 3.05 \times 3.05 \times F_u = 40{,}000 \text{ in.-lbs.}$$

Solving, F_u is equal to 924 lbs. The resisting force of rivets *A, B, G*, and *H* is 924 × 3.05, or 2820 lbs. each, and of rivets *E* and *F* is 924 × 1.75, or 1618 lbs. each. The space diagram of the primary and secondary resisting forces is shown in Fig. 7-19; the force systems at rivets *A* and *H* are like those at *G* and *B*. The resultant forces on rivets *E* and *F* are 951 and 2285 lbs., obtained by algebraic summation; the resultants R_B and R_G are 3250 and 2500 lbs., obtained by vectorial summation.

From Table 7-1, the allowable shearing and bearing stresses in these rivets, in accordance with the AISC Code, are 15,000 and 32,000 psi. The diameter *D* of the rivets for shear resistance is given by

$$D = \sqrt{4 \times 3250/15{,}000\,\pi} = 0.55 \text{ in.}$$

which necessitates the use of a ⅝-in. rivet. The necessary plate thickness *t* will be found from

$$t = \frac{3250}{0.625 \times 32{,}000} = 0.162 \text{ in.}$$

From the standpoint of bearing resistance alone, a 3/16-in. plate will be satisfactory.

7-12. Baseplates and Connections. Building columns are usually mounted on baseplates that are anchored to the foundation by bolts. Two methods of attaching wide flange column sections are shown in Fig. 7-20. The column may be attached to the baseplate by clip angles riveted or welded in place. Clip angles may also be placed against the outer faces of the flanges. If the end of the column is milled to provide a satisfactory seat on the baseplate, clip angles of nominal size only are required, since their purpose is merely to keep the column in position. If, however, the column end is finished by burning, no reliance can be placed upon the area of contact, and the clip angle rivets should be designed to transmit the full column load to the baseplate.

The construction shown in the lower portion of Fig. 7-20 is very satisfactory because it provides high moment resistance at the end of the column. The effect of the initial tension in the anchor bolts results in an entirely fixed column except for possible deformation of the angles. Theoretically, anchor bolts of almost any size may be employed. To guard against the weakness due to initial tension and the possibility that the base of the column might be subjected to a shock load or struck by a moving truck, anchor bolts smaller than ¾ in. are not used.

TABLE 7-7.—ALLOWABLE BEARING STRESSES FOR MASONRY, PSI.

Granite	800
Sandstone	400
Limestone	400
Concrete	600
Hard brick, in cement mortar	250

Column baseplates must have an area sufficiently great to keep the bearing stresses in the foundation within the maximum limits given in Table 7-7. The column load is assumed to be uniformly distributed within a rectangle of length $0.95d$ and breadth $0.80b$, shown in Fig. 7-21. The first step in designing a baseplate is to determine the required area for bearing pressure. The length and breadth of the baseplate are then selected so that dimensions m and n are approximately equal. The plate thickness t is determined by Eq. 7-11 or 7-12 (the maximum value governs the selection of the baseplate thickness).

$$t = \sqrt{0.00015pm^2} \qquad (7\text{-}11)$$

$$t = \sqrt{0.00015pn^2} \qquad (7\text{-}12)$$

where p is the actual unit pressure, psi., and m and n are edge distances (Fig. 7-21). Plates are available in thicknesses of ¼, ⅜, ½, ¾, 1, 1¼, 1½, 1¾, and 2 in.

Example 7-8. A column for an industrial building is to have a height of 12 ft. 6 in. and is to carry a load of 71,000 lbs. Select a suitable *WF* section, and design the baseplate and connection to a concrete foundation.

Solution. It will be necessary to make an initial assumption for the allowable compressive stress in order to select a tentative section from those listed in Table 7-3. If an allowable stress of 16,000 psi. is assumed, the required area is 71,000/16,000, or 4.43 sq. in. From Table 7-3, three 6 × 6-in. and one 5 × 5-in. *WF* sections have areas greater than this value. Consider the 6 × 6-in. section with an area of 4.57 sq. in., which has radii of gyration k with respect to the xx and yy axes of 2.48 and 1.46. The later is the controlling value, and the L/k ratio is 150/1.46, or 103, which is satisfactory for a main member. From Eq. 5-23

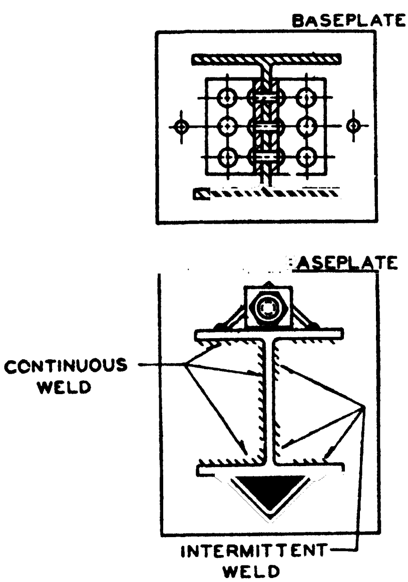


FIG. 7-20. Column Base Plate Attachments.

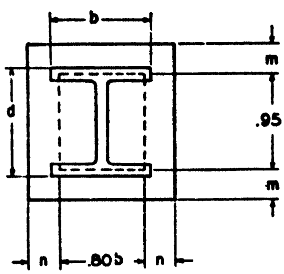


FIG. 7-21. Column Load Distribution Area.

$$S_c = 17{,}000 - 0.485(103)^2 = 11{,}860 \text{ psi.}$$

This value is appreciably lower than the original assumption of 16,000 psi. used to determine the required cross-sectional area, so the next larger size of section will be investigated.

From Table 7-3, the next 6 × 6-in. section has an area of 5.89 sq. in. and a least radius k of 1.51 about the yy axis. From Eq. 5-23

$$S_c = 17{,}000 - 0.485\left(\frac{150}{1.51}\right)^2 = 12{,}250 \text{ psi.}$$

The allowable load that the column will carry is

$$12{,}250 \times 5.89 = 72{,}100 \text{ lbs.}$$

which is greater than the required load, so the selection is satisfactory.

Assuming welded connections between the column and baseplate, the rectangular area subject to load (Fig. 7-21), has a length of 0.95 × 6, or 5.7 in., and a breadth of 0.80 × 6, or 4.8 in. From Table 7-7 the allowable bearing pressure for concrete is 600 psi., and the necessary area of the baseplate is therefore

$$\frac{71{,}000}{600} = 118 \text{ sq. in.} = (2m + 5.7)(2n + 4.8)$$

Assuming m equal to n,

$$4m^2 + 21m + 27.4 = 118,$$

or

$$m = 2.8 \text{ in.}$$

The length and breadth of the plate are 5.7 + (2 × 2.8), or 11.3 in., and 4.8 + (2 × 2.8), or 10.4 in. A plate 11-in. square will probably be satisfactory. The unit pressure for this size plate is $71{,}000/11^2$, or 586 psi. Dimensions m and n are equal to (11 − 5.7)/2, or 2.65 in., and (11 − 4.8)/2 or 3.1 in., respectively. Since dimension n is the larger, it will control the selection of a plate thickness from Eq. 7-12, which gives

$$t = \sqrt{0.00015 \times 586 \times 3.1^2} = 0.93 \text{ in.}$$

A 1-in. plate is required.

Beam Selection and Design

7-13. Integral rolled structural sections, such as I beams, wide flange or *WF* sections, and channels are used extensively for carrying floor and tank loads. Main carrying members are usually referred to as girders; smaller members of shorter span, as beams. The beams in the floors of buildings are usually known as joists; in roof construction they are termed purlins. A plate girder is a large built-up beam usually constructed of a vertical web plate, upper and lower flange plates, and connecting angles riveted together. It is used for heavy loads and long spans for which commercial rolled sections are not available.

Rolled beam and girder selection should be based upon the possibility of failure by flexure, horizontal and vertical shear, compression or bearing at the reactions, and local buckling or crimping of the beam web. In addition, the possibility of flange buckling, particularly important in *WF* sections of long span, should be considered.

7-14. Flexural stresses in a beam section are determined by applying Eq. 5-8. The effect of rivet holes in the tension flange may usually be disregarded if the holes are at or near the reaction; if holes are punched at the point of maximum moment, however, the resistance of the section should be deter-

mined by deducting the moment of inertia of the projected area of any holes in the tension flange from the moment of inertia of the gross section. Since rivet holes in the compression flange are presumed to be filled by the rivets, this procedure causes a shift in the neutral axis. The resulting computation is laborious, and experimental evidence does not indicate any appreciable effect in the strength of the section because of the shift in the neutral axis resulting from flange holes. The computation may be simplified by deducting equivalent hole areas from *both* flanges and thereby retaining the original position of the neutral axis.

Example 7-9. Find the section modulus of the 15 × 6-in. American Standard beam shown in Fig. 7-22 if two 1-in. holes are punched in the lower or tension flange.

Solution a. From Table 7-2 the area of the gross section is 17.68 sq. in., and its centroid is at the neutral axis *xx*. The projected area of the holes is 2 × 0.812 × 1, or 1.624 sq. in., and their centroid is a distance 7.094 in. from axis *xx*. Taking moments about the top of upper flange, from Eq. 5-4,

$$h = \frac{(17.68 \times 7.5) - (1.624 \times 14.594)}{17.68 - 1.624} = 6.77 \text{ in.}$$

which locates the neutral axis *nn* of the net section.

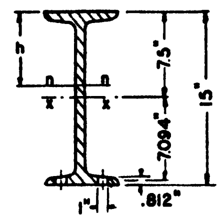


FIG. 7-22. *I* Beam Section with Holes in Lower Flanges.

The moment of inertia of the net section is equal to the difference between the moment of inertia of the gross section and of the holes, with respect to axis *nn*. From Eq. 5-6, for the gross section,

$$I_{nn} = I_{xx} + A(7.5 - h)^2 = 609 + 17.68(7.5 - 6.77)^2 = 618.4 \text{ in.}^4$$

For the holes,

$$I_{nn} = I_{gg} + A(7.094 + 7.5 - h)^2 = 2 \times 1\left(\frac{0.812^3}{12}\right) + (1.624 \times 8.164^2) = 108 \text{ in.}^4$$

The moment of inertia I_{nn} for the net section is 618.4 — 108, or 510.4 in.⁴, and the section modulus Z is equal to

$$Z_{nn} = \frac{I_{nn}}{15 - n} = \frac{510.4}{15 - 6.77} = 62.1 \text{ in.}^3$$

Solution b. The moment of inertia of a single hole in one flange, with respect to *xx*, is

$$I_{xx} = I_{gg} + A(7.094)^2 = (1 \times 0.812^3 \times 12) + (0.812 \times 7.094^2) = 40.8 \text{ in.}^4$$

The moment of inertia of the net section is equal to

$$I_{xx} = 609 - (4 \times 40.8) = 445.8 \text{ in.}^4$$

and the section modulus is 445.8/7.5, or 59.4 in.³, which differs very little from the value found in the preceding solution.

7-15. Shear and Buckling in Structural Beams. With relatively short spans the allowable beam load may be limited by the shearing or buckling strength of the web, instead of the maximum bending stress permitted in the flanges. From Table 7-1 the allowable unit shearing stress in the web of a beam is 13,000 psi., and the total maximum web shear V is given by

$$V = 13{,}000dt \qquad (7\text{-}13)$$

where d and t are the full depth and thickness of the web. Beams should be selected so that the compression stress in the web at the toe of the fillet, for webs without auxiliary stiffness, does not exceed the value of 24,000 psi. given in Table 7-1.

Using the relation shown in Fig. 7-23, the maximum permissible reaction R_e at the end of the beam and the maximum permissible load R_i between the reactions, along the beam span, are given by:

$$R_e = 24{,}000t(a + m) \tag{7-14}$$

$$R_i = 24{,}000t(a + 2m) \tag{7-15}$$

where t and m are the web thickness and toe distance, Fig. 7-1, and a is the length of bearing. If the above values are exceeded the beam webs should be reinforced, or the bearing length of the load or reaction should be increased.

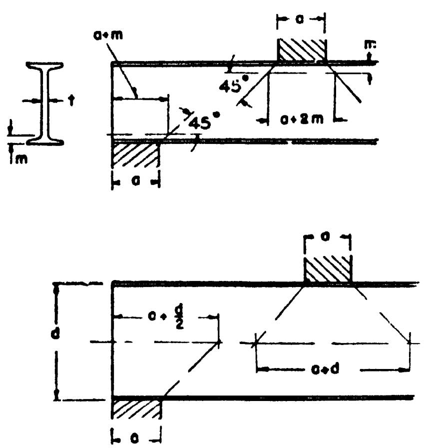


FIG. 7-23. Stressed Areas in Beam Webs.

For beam sections of considerable depth consideration should be given to the possibility of failure of the web by buckling or column action. In the lower portion of Fig. 7-23, for a bearing length of a, the lengths of the web at the outer flange and at the neutral axis are respectively a and $a + d/2$. The average length is $a + d/4$. The unit compressive stress is equal to the load divided by the average area of the web, or

$$S_c = \frac{P}{(a + d/4)t} \tag{7-17}$$

The allowable limits for S_c may be obtained from Table 7-1, or Eq. 5-23 or 5-24, by assuming the column length as some function of the depth of the section. For beam sections unsupported at the reactions or under the load, the column length is usually considered as equal to the depth d of the beam section. The radius of gyration of the web is found from Fig. 5-12 to be $t/\sqrt{12}$, or $0.29\,t$.

7-16. Stiffener Application. Stiffeners are vertical members attached to the web of a beam or girder as shown in Fig. 7-24. Stiffeners are not required if the ratio between the web depth d and the thickness t is equal to or less than 70, or beyond this limit if the unit shear does not exceed

$$S_s = \left[\frac{8000}{(d - 2m)/t}\right]^2 \tag{7-18}$$

(If S_s is less than 2200 psi., stiffeners are not required.)

Stiffener spacing must not exceed 84 in. or the following:

$$p = \frac{270{,}000}{S_s} \sqrt[3]{\frac{tS_s}{d - 2m}} \tag{7-19}$$

where p is the maximum stiffener spacing, and S_s is the unit vertical shearing stress from Table 7-1. If Eq. 7-18 or 7-19 is used for plate girders or built-up beams, the term $d - 2m$ should be replaced by the clear distance between the flange members.

The stresses in end stiffeners are computed by assuming that the entire end reaction is carried by the stiffener without any assistance from the web. The outstanding legs of the stiffeners are machined to bear against the flanges, or the two may be welded together. The area in bearing between the flange and the end of the stiffener must be such that the allowable bearing stress for milled contact surfaces, given in Table 7-1, is not exceeded. For deep girders

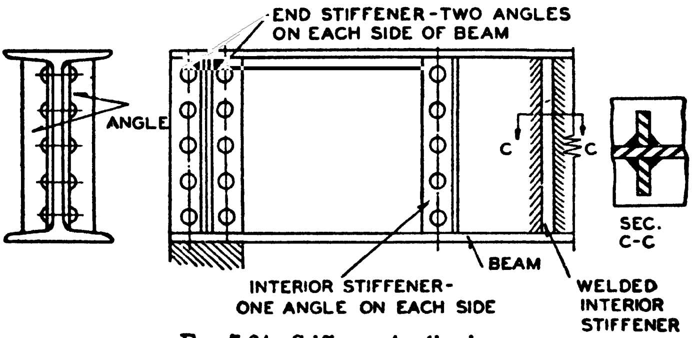


FIG. 7-24. Stiffener Application.

the stiffener may act as a column, and the selection should be based upon Eq. 5-17 or 5-18. The effective length of such a column is usually taken as one half the depth of the section, thereby allowing for the transfer of stress from the stiffener to the web. Since it is impossible for the stiffener to buckle in the plane of the beam or girder web, the value of the radius of gyration of the stiffener is computed with respect to a horizontal axis in the plane of the web. End stiffeners are used in pairs on the opposite sides of the web and may be riveted or welded in place. The connection should be of sufficient strength to transfer the entire end reaction to the web.

Interior stiffeners are usually selected with an outstanding leg width equal to the flange projection, or with a minimum outstanding width of 2 in. plus 1/30 of the girder depth. Riveted stiffeners are attached by a single row of rivets with a maximum pitch of 6 in. The stresses in interior stiffeners need not be computed if these proportions are adhered to, as they give satisfactory results.

Failure may occur in beams of long span by lateral buckling of the compression flange. This type of stress is controlled by the following:

$$S_c = \frac{22{,}500}{1 + \dfrac{(L/b)^2}{1800}} \qquad (7\text{-}20)$$

where L is the unsupported length of span of the beam, and b is the width of the flange. The maximum value of S_c is 20,000 psi., which is applicable to L/b ratios of 15 or less. The maximum permissible value of L/b is 40. A beam with a flange width of 9 in. therefore requires some form of lateral support every 30 ft. of span, even if the working stress is low. Span support may be effected by "framed-in" joists or by stiffeners.

7-17. Beam Deflection. In order to avoid serious cracks in concrete floors and plastered ceilings, the deflection of floor beams should be limited to not more than 1/360 of the span. For a uniformly loaded beam, with a nominal depth d, and a flange stress of 20,000 psi., the span L in feet that will result in a deflection of 1/360 of the span, based upon the data of Fig. 5-35, is

$$L = 1.61\,d \qquad (7\text{-}21)$$

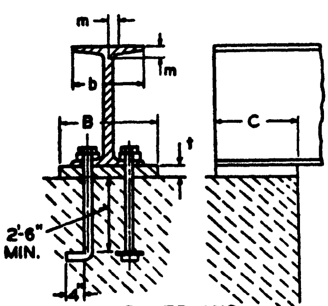


FIG. 7-25. Beam Baseplate and Anchorage.

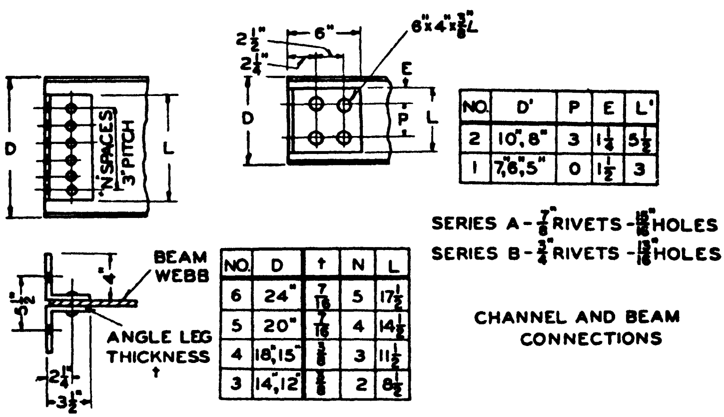
The deflection of supporting beams for riveted or welded pressure vessels must be limited to small amounts to eliminate openings and leakage at the joints and pipe connections. In some instances dissimilar beam sections may be employed to support tanks and other vessels, and the selection of a suitable section may have to be based upon identical deflection for each section chosen. Beam deflection computation is based upon the data of Figs. 5-35 and 5-36, using the modulus of elasticity E of structural steel as 29,000,000 psi.

7-18. Beam Connections. Beams may be mounted on baseplates resting on masonry or brick walls, Fig. 7-25, or fastened by double-angle connectors or clips, Fig. 7-25. A baseplate must have a bearing area sufficiently great so that allowable wall stresses for concrete or masonry are not exceeded. The length C of the baseplate, Fig. 7-24, is usually governed by the available wall thickness; with the width B thus determined, the thickness t of the plate is found from

$$t = \sqrt{0.00015p\,[(B/2) - m]^2} \qquad (7\text{-}22)$$

where p is the allowable bearing pressure, from Table 7-7, and m is the distance from the outer face of the beam flange to the toe of the web fillet.

Steel beams supported by masonry should always be properly anchored to the wall. Recommended alternate details are shown in Fig. 7-24. If the anchorage must be made to the vertical face of the wall, one or two sets of double-angle clips, fastened to the wall with 1-in. expansion bolts, may be substituted for the anchor bolts. If no seat for the end of the beam is provided and the expansion bolts carry the beam reaction, a very careful analysis must be made, similar to that of Example 6-1. Supporting brackets for girders and beams are widely used in welded connections and are essentially similar to the bracket shown in Fig. 7-9. The beam is usually anchored in place by two or four bolts passing through the lower flange of the beam and the horizontal leg of the bracket. Subsequent welding of the beam to the bracket is sometimes done. Brackets are often attached to columns by means of rivets. In such cases the stresses in the rivets should be checked for both tension and direct shear.



NO.	D'	P	E	L'
2	10",8"	3	1¼	5½
1	7",6",5"	0	1½	3

NO.	D	t	N	L
6	24"	7/16	5	17½
5	20"	7/16	4	14½
4	18",15"	⅜	3	11½
3	14",12"	⅜	2	8½

Fig. 7-26. Channel and Beam Connections.

Double-angle or clip connections are standardized in several series for American Standard, wide-flange, and channel sections of varying depth. Two series, *A* and *B*, are shown in Fig. 7-26, differing only in the diameter of the rivets employed. The load-carrying capacity of the clips is based upon the single-shear and the bearing capacity of the rivets in the outstanding legs of the clips or upon the bearing capacity of the rivets in the beam or girder web. Although the rivets in the web are subject to primary and secondary shearing stresses, and those in the outstanding legs to tension and direct shear, a computation for direct bearing and shear, with allowable stress values taken from Table 7-1, is satisfactory for ordin. v conditions.

Example 7-10. The beam section of Example 7-9 is to be used as a girder with a span of 20 ft., and carries a uniform load of 1900 lb. per ft. of length over the entire span. Investigate the stresses in the beam section, compute the deflection, and design two types of end connection.

Solution. The end reaction is equal to $1900 \times 20/2$, or 19,000 lbs., and the maximum moment, which occurs at the center of the span, is equal to $(19{,}000 \times 10 \times 12)$ -- $(19{,}000 \times$

5×12), or 1,140,000 in.-lbs. From Example 7-9, $Z = 59.4$ in.^{3}, and the maximum flexural stress S_t is 1,140,000/59.4, or 19,600 psi., which is slightly less than the allowable maximum of 20,000 psi. permitted by the AISC Code in Table 7-1. From Eq. 7-13 the maximum allowable web shear is

$$V = 13{,}000 \times 15 \times 0.59 = 115{,}000 \text{ lbs.}$$

which is considerably greater than the end reaction of 19,000 lbs., so shear failure is amply provided for.

Assuming the minimum bearing on the wall to be 3½ in., the maximum permissible end reaction is found from Eq. 7-14, where the distance m is obtained from Table 7-2.

$$R_e = 24{,}000 \times 0.59\ (3.5 + 1.625) = 73{,}000 \text{ lbs.}$$

which is considerably greater than the actual reaction of 19,000 lbs.

The possibility of failure by web buckling at the reaction is investigated by the use of Eq. 7-17, which gives a unit compressive stress of

$$S_c = \frac{19{,}000}{(3.5 + 15/4)0.59} = 4450 \text{ psi.}$$

The ratio of web depth to thickness is 15/0.59, or 25.4, and the allowable unit compressive stress is obtained from Eq. 5-23,

$$S_c = 17{,}000 - (0.485 \times 25.4^2) = 16{,}680 \text{ psi.}$$

A comparison of these values indicates that no danger of web buckling exists. No consideration need be given to providing end stiffeners, since the ratio of web depth to thickness is considerably less than 70.

The possibility of flange buckling is investigated by the use of Eq. 7-20. The span L is 20 ft., or 240 in.; the flange width b is 6 in. The ratio of span to flange width is 240/6, or 40, which is the maximum permissible value for an unsupported span. The allowable compressive stress in the upper flange for this ratio, from Eq. 7-20, is

$$S_c = 22{,}500 \div \left(1 + \frac{240^2}{1800 \times 6^2}\right) = 11{,}910 \text{ psi.}$$

which is considerably lower than the actual flexural stress of 19,600 psi. in the flange. Some form of support, therefore, such as angle stiffeners on each side of the beam, should be added. If a set of angles at the center of the beam is used, the unsupported span is reduced to 120 in., and the L/b ratio is 120/6, or 20. The allowable compressive stress in the flange, from Eq. 7-20, is

$$S_c = 22{,}500 \div \left(1 + \frac{120^2}{1800 \times 6^2}\right) = 18{,}410 \text{ psi.}$$

which is still too low. It will be necessary, therefore, to employ interior stiffeners at the "third" points, so that L may equal 240/3, or 80 in. The L/b ratio is then 80/6, or 13.67, which is less than 15, permitting the value of 20,000 psi. as the allowable compressive stress in the flange.

The beam deflection is computed from the equation for the deflection of a simply supported, uniformly loaded beam. From Fig. 5-35, Case 1, the deflection is

$$y = \frac{5(1900/12)240^4}{384 \times 29 \times 10^6 \times 515.3} = 0.46 \text{ in.}$$

The ratio of the deflection to the span is 0.46/240, or 1/510. Since this ratio is considerably smaller than 1/360, the beam could serve to support a plastered ceiling.

For the design of end connection, a baseplate similar to that of Fig. 7-25 may be used. If the plate rests on a concrete wall, the allowable bearing pressure, from Table 7-7, is 600 psi. The plate length C is 3½ in., and the value of the reaction is 19,000 lbs. The width B of the plate is

$$B = \frac{19{,}000}{600 \times 3.5} = 9.05, \text{ say 10 in.}$$

The actual pressure p on the wall is $19{,}000/(3.5 \times 10)$, or 543 psi. The thickness of the baseplate is obtained from Eq. 7-22; from Table 7-2, m is equal to 1.625 in.

$$t = \sqrt{0.00015 \times 543 \times (10/2 - 1.625)^2} = 0.912, \text{ say 1-in. thick plate.}$$

A plate 1 in. thick, with two 1-in. anchor bolts placed 3½ in. center to center, will be satisfactory.

For a second design of end connection, select angle clips from Fig. 7-26. Either Series *A*-4 or *B*-4 connections are of the correct proportions for a 15 × 6-in. beam. These connections have four rivets in each outstanding leg. For the 19,000-lb. reaction, the load per rivet is 19,000/8, or 2375 lbs. The shearing strength of a ¾-in. rivet is $15{,}000[\pi(0.75)^2/4]$, or 6630 lbs. for single shear. The bearing strength of four rivets in double shear in the web of the beam is $4 \times 40{,}000 \times 0.75 \times 0.59$, or 71,000 lbs. The ¾-in. rivets are amply strong for the 19,000-lb. end reaction and a Series *B* connection will suffice.

Flooring and Surfacing

7-19. Rolled steel plates with anti-skid surfaces are coming into wide use for flooring and deck and balcony surfacing in structures housing chemical equipment. The floor plate shown in Fig. 7-27 has raised, flat-surfaced lugs of maximum area, with square edges which provide the greatest possible resistance to slipping or skidding. Plates are available in widths w from 6 to 72 in., and in thicknesses of ³⁄₁₆, ¼, ⁵⁄₁₆, ⅜, ⁷⁄₁₆, ½, ⅝, and ¾ in. The ¼-in. plate weighs about 11 lb. per sq. ft., and the weights of plates of other thicknesses are in proportion. Rectangular plates are frequently used in mill floor construction supported by floor beams at two separate edges, or along all four edges, more or less securely fastened to the flanges of the supporting beams. For plates supported at the shorter edges the unit flexural stress S, in psi., for a uniformly distributed load of w psi. is given by

$$S = \frac{3B^2w}{4t} \qquad (7\text{-}23)$$

where B is the plate span and t the plate thickness. For plates supported along all four edges, the stress S for a uniformly distributed load is given by

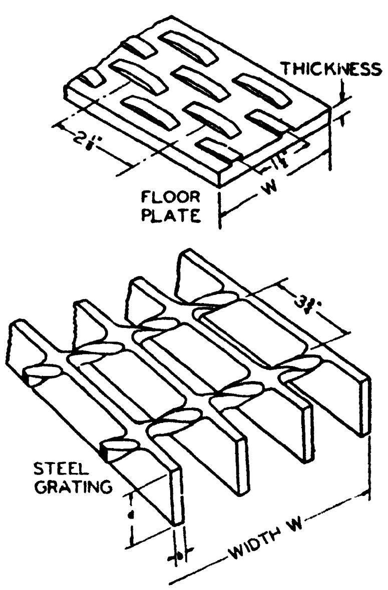


Fig. 7-27. Floor Plate and Steel Grating

$$S = K\frac{A^2B^2w}{2(A^2 + B^2)t^2} \qquad (7\text{-}24)$$

where A and B are the lengths of the edges and K is a constant dependent upon the manner of edge fixation. This relation is based upon theoretical and experimental data on flat plate analysis. If the edge fixation is such that full advantage can be taken of continuity, K may be taken as 0.563; for plates which rest on supports (simply supported), K should be taken as 0.75. For plates supported along all four edges, the stress S for a single concentrated load P in the center of the plates is given by

$$S = K \frac{3ABP}{2(A^2 + B^2)t^2} \qquad (7\text{-}25)$$

where K has a value of 1.25 for full fixation and 1.33 for simply supported plates.

7-20. Open flooring or steel grating, Fig. 7-27, is frequently used where high-strength, comparatively light-weight flooring is required. It is often employed for balconies, galleries, and flooring sections for supporting upper stages of process equipment, since it permits effective distribution of light and air and can be protected easily by painting or galvanizing. Steel grating is obtainable in lengths up to 20 ft. Two-foot widths W, with 21 bars, is a standard. Special close mesh grating, with 27 bars per ft. of width, is employed where close spacing of the bearing bars is desired. Odd width panels, in widths from 22$^{13}\!/_{16}$ to 1$^3\!/_8$ in., are also available. Steel grating, with bearing bars either $^1\!/_8$ or $^3\!/_{16}$ in. wide (dimension b, Fig. 7-26), is obtainable in depths d of $^3\!/_4$, 1, 1$^1\!/_4$, and 1$^1\!/_2$ in.; while depths of 1$^3\!/_4$, 2, and 2$^1\!/_4$ in. have bars $^3\!/_{16}$ in. wide.

The load capacity of standard 21-bar steel grating, 2 ft. wide, supported at the ends of the bars, is given by the following, where M is the bending moment in in.-lbs., and b and d are the bar width and depth.

$$M = 56{,}000\ bd^2 \qquad (7\text{-}26)$$

For close mesh grating, with 27 bars per 2 ft. of width,

$$M = 72{,}000\ bd^2 \qquad (7\text{-}27)$$

PROBLEMS—CHAPTER 7

1. A 3 × 3 × $^3\!/_8$-in. angle serves as a tension member in a truss, and is attached to a 6 × 6 × $^1\!/_2$-in. chord angle by fillet welds. If the permissible length of weld on either edge of the angle is 5 in., determine the permissible load on the tension member if ordinary strength welds are used.

2. Find the number of rivets (maximum diameter) based upon (a) the AISC Code, (b) considering gage line eccentricity, for the member of Problem 1 if the attachment to the chord member is accomplished by a $^1\!/_2$-in. gusset.

3. An 8 × 4-in. structural angle with the 8-in. leg seated against the face of a column serves as a bracket for a crane runway. If the bracket length is 7 in., what should be the leg dimensions of ordinary strength fillet welds for a load of 5000 lbs. applied 3 in. from the column face.

4. An 8-in. standard pipe filled with oil weighing 55 lbs. per cu. ft. is supported by two columns 36 ft. apart and is carried by two saddles at the third points of the span. Select a suitable loop rod for supporting the saddles.

5. If the centerline of the pipe of Problem 4 is 8 ft. from the ground, select a suitable WF section for the column.

6. Like Problem 5, using a pipe column.

7. Design a floor girder for a uniform load of 600 lbs. per foot of length for a span of 30 ft.

8. Design end connections for the girder of Problem 7.

9. Can expansion bolts be used for the end connections of Problem 7? Explain and prove.

10. Design a floor girder with a span of 12 ft. and a uniform load of 3000 lbs. per toot of length. Include end connections.

11. A pair of 7-in., 14.75-lb., channels, back to back and ½ in. apart, serve as a building column 15 ft. long. Find the permissible axial load.

12. What is the maximum length, and the corresponding load capacity, of a 3 × 2½ × ½-in. angle when used as a main member column? A secondary member column?

13. The column of Problem 11 carries a bracket, similar to Fig. 7-18, which is used to support a crane runway. The applied load is 2800 lbs. located 9 in. from the centroid of the column. Determine the permissible axial load on the column.

14. For the column of Problem 11, design a suitable baseplate to rest on a hard brick foundation.

15. For the column of Problem 5, design a suitable baseplate to rest on a concrete foundation.

16. For the bracket of Problem 13, design the rivet arrangement using six rivets of maximum size.

17. What maximum unit load may be carried by an anti-skid steel plate ½ in. thick, supported at all four edges, for a span of 10 ft. and a width of 3 ft.?

18. What maximum unit load may be carried by close-mesh steel grating 2 ft. wide with a span of 18 ft.?

19. Determine the stress in the rivets of the bracket of Problem 13 if the distance *m* (Fig. 7-18) is equal to 3 in. and if the rivet diameter is ⅝ in.

CHAPTER 8

TRUSSES AND TRUSS ADAPTATION

8-1. Introduction. Rolled section beams, or even large built-up beams or girders, are often inadequate or uneconomical for the heavy loads and long spans in such structures as roofs, bridges, and viaducts. Trusses are used in such cases. A truss is an arrangement of individual members, usually subjected to simple or direct stresses (either tension or compression), that acts as a beam. Truss joints may be pin-connected, riveted, or welded. The pin connection is formed by passing a cylindrical pin through all the component members at the joint in question. In welded trusses one member is usually butt or lap welded to another. In riveted joint trusses the members comprising the joint are riveted to gusset plates, as illustrated in Fig. 8-8. Theoretically the pin-connected truss is the only type that can be analyzed by the method of statics, but tests and construction experience have indicated that little or no error is introduced when riveted or welded joint trusses are handled as if the joints were pin-connected.

Roof truss nomenclature is illustrated in Fig. 8-1. The intermediate or secondary members joining the upper and lower chords, or main members of the truss, are known as web members. The purlins, or longitudinal members, support the roof between the trusses and are usually attached to the latter at the panel points. Sway bracing is used to provide lateral stability; in small trusses it may be omitted, since this function may be cared for by the purlins and roofing. Some trusses are supported on base plates resting on concrete or masonry walls; others are carried on columns (as illustrated). Some columns are secured against transverse sway by knee braces; such members are usually included in truss analysis. The pitch of a truss is the ratio of the vertical height (or rise) to the span. It is usually expressed as $\frac{1}{3}$ or $\frac{1}{4}$, which, for a span of 60 ft., means a rise of 20 ft. or 15 ft., respectively.

8-2. Truss Member Stress Analysis. Truss members develop internal resistance or stresses to counteract or equilibrate external forces. Such stresses may be analyzed by any of three methods: the method of joints, the method of sections, and the method of moments. The method of joints is based upon the principle that external forces acting at a joint must be equilibrated by direct stresses induced in the truss members. In the application of this method of analysis successive joints are considered to be free bodies, subject to the action of concurrent forces, and force polygons may therefore be constructed to determine the internal forces. In the method of sections, stresses in one or more members are obtained by taking a section across a panel of the truss and equilibrating the external forces to the left or right of the section to the hori-

zontal and vertical components of the members in the panel. In the method of moments, the stress in one member is ascertained by equilibrating the moment of the internal force to the moments of the external forces about a selected axis.

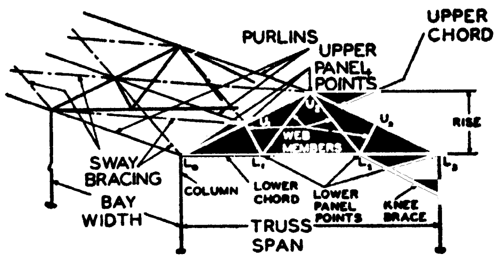


FIG. 8-1. Roof Truss Nomenclature.

The second and third methods are ordinarily used for finding the stresses in single members, or as auxiliary means to facilitate the solution by the method of joints.

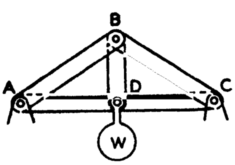


FIG. 8-2. Pin-connected Truss.

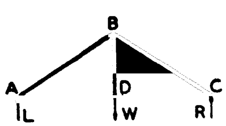


FIG. 8-3. Skeleton Layout of Pin-connected Truss.

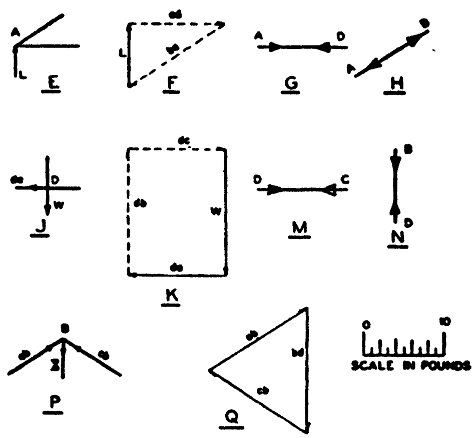


FIG. 8-4. Analysis of Pin-connected Truss by the Method of Joints.

8-3. Method of Joints. Fig. 8-2 shows a model of a pin-connected truss carrying a weight W and supported at ends A and C. The members AB and BC represent the upper, and AD and DC the lower, chords of the truss, connected by the web member BD. Fig. 8-3 shows a skeleton layout of this truss with the left and right reactions L and R replacing the supports at A and C. By vertical resolution and symmetry L and R are each equal to $W/2$. Fig. 8-4 shows the determination of the stresses in the members by the method of joints. The

detail at *E* shows a free-body or space diagram of joint *A*, which is subjected to one external force, the reaction *L*, and which serves as a connection for members *AB* and *AD*. The direction and magnitude of the forces along members *AB* and *AD* are unknown, but their position is defined as coincident with these members. Detail *F* shows a force polygon for joint *A*; the vector *L* is drawn parallel to force *L* in the funicular diagram, and the vectors *ad* and *ba*, representing the internal forces in members *AD* and *BA*, are drawn parallel to the positions of *AD* and *BA* in the space diagram. The direction of vectors *ad* and *ba* is determined by giving them the same general direction as the corresponding members and such as to close the force polygon. The magnitude of *ad* and *ba* may be scaled from the force polygon. Detail *G* shows a free-body space diagram for member *AD*; the direction of the internal force near end *A* is indicated by the solid arrowhead (placed at the end *A*) and is obtained from the force polygon. In order that *AD* may be in equilibrium, another internal force, equal in magnitude and opposite in direction to *ad*, as indicated by the outlined arrowhead near *D*, must be present. Two arrowheads pointing toward each other represent the internal resistance to a pair of external forces acting away from each other, and the resultant stress is tension in member *AD*. A similar free-body space diagram for member *AB* is shown in detail *H*. The solid arrowhead indicates the direction of the force *ba* at joint *A* and is placed at the end *A* of the member. The outline arrowhead represents the equilibrating internal resistance at end *B*. These arrowheads denote compression in member *AB*, since they represent the stress induced by external forces acting toward each other. (It should be borne in mind that these arrowheads on the members represent internal resisting forces and not externally applied forces.) Detail *J* shows a space diagram for joint *D*. The direction of the force in member *AD* is obtained from the space diagram for this element, detail *G*. Detail *K* shows a force polygon for the forces at joint *D*. The vectors *W* and *da* are laid down in order, followed by the vectors representing the forces in members *DB* and *DC*. It is important that a regular succession or order be followed in laying down the vectors, either as given above, or else as *da*, *W*, *dc*, and *db*. Details *M* and *N* show the space diagram for members *DC* and *BD*; both members are subjected to a tensile stress. The magnitude of these stresses may be scaled from the force polygon *K*. Since the truss is symmetrical, the stresses in members *AB* and *BC* are alike. Detail *P* shows a space diagram for joint *B*, with forces *ab*, *bd*, and *cb* acting at the joint. The force polygon in detail *Q* is not absolutely necessary, because the magnitude and direction of the vectors has been determined in the preceding stages of the solution, but it indicates that joint *B* is in equilibrium and serves as a check on the accuracy of the solution.

8-4. Method of Sections. Fig. 8-5 shows the application of the method of sections to the truss of Fig. 8-2. The detail at *E* shows a section *x-x* taken across panel *AB*, intersecting members *AB* and *AD*. Detail *F* shows a free-

body space diagram of that portion of the truss to the left of section *x-x*. This section of the truss must be in equilibrium, and the external vertical force *L* must be balanced by another vertical force. Member *AD* is horizontal and cannot provide a vertical resisting force; this force is furnished by member *AB*, as indicated by *L′*. Since member *AB* is not vertical, the imposition of *L′* induces a horizontal component *S*, which must in turn be equilibrated by a

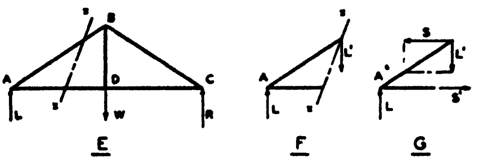


Fig. 8-5. Analysis of Pin-connected Truss by the Method of Sections.

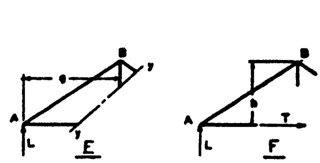


Fig. 8-6. Analysis of Pin-connected Truss by the Method of Moments.

horizontal force *S′*, acting in the opposite direction through member *AD* as shown in detail *G*. The resultant of *L′* and *S* acts along member *AB* and is equal to the internal force *ba*, Fig. 8-4F; the force *S′* is equal to *ad*. The stress in member *DB* is equal to the load *W*. It may be noted that *L* and *L′* and *S* and *S′* are couples whose moments are equal in magnitude and opposite in direction.

8-5. Method of Moments. Fig. 8-6 illustrates the application of the method of moments. Detail *E* shows a free-body space diagram of a section of the truss of Fig. 8-2. The moment of this section about joint *B* is equal to the product *gL*, which must be equilibrated by the moment of an internal force in one of the members. This resisting moment cannot be furnished by members *AB*, *BC*, or *BD*, as these all pass through joint *B* and their moment arm is consequently zero. The resisting moment must therefore be provided by an internal force *T* acting along *AD*, as in detail *F*. The moment arm of this force is *h*. The magnitude of *T* is given by *gL/h* and is equal to force *ad*, Fig. 8-4F.

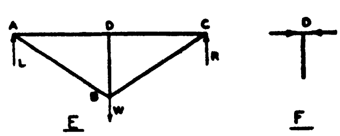


Fig. 8-7. Truss with Redundant Member.

8-6. Inactive Members. If the truss of Fig. 8-2 is inverted and the load applied at joint *B*, as illustrated in the skeleton layout in Fig. 8-7E, an analysis of the stresses in the members will show that members *AD* and *DC* are subjected to compressive stresses and *AB* and *BC* to tensile stresses, equal in magnitude to *ad* and *dc*, and *ab* and *bc*, Fig. 8-4. An examination of joint *D*, shown in Fig. 8-7F, will indicate that there is no stress in member *DB*, since no vertical forces exist at the joint. Member *DB* in this truss is termed an inactive member Inactive members are used when there is possibility of a change of position of the load; they may also serve (as indicated in Fig. 8-7E) to reduce the

unbraced length of compression members and thereby permit appreciable economy in material. It may be recalled from Section 5-21 that the allowable stress in a column is inversely proportional to a function of the square of the effective length, and the addition of an inactive member may be less expensive than the required increase in the column section.

8-7. Roof Truss Loads. Roof trusses are designed for dead loads, snow loads, wind loads, and moving loads. The dead load includes the weight of the roof and the weights of the truss members, gussets, purlins, and bracing. These weights are usually based upon the material comprising the half bays on either side of the truss, and are considered as concentrated at the panel points. For the truss of Fig. 8-1 these weights might be considered as divided into four equal parts, with full concentration at the ridge U_2 and the upper panel points U_1 and U_3, and with half concentration at the lower panel points L_0 and L_8. If the roof extends appreciably past the lower panel points L_0 and L_8 or if it is furnished with comparatively heavy eaves or gutters, such additional weights should be included in the dead load computation and full concentrations considered at L_0 and L_8, as well as at U_1, U_2, and U_3. For design purposes it is customary to estimate the dead load as so many pounds per foot of panel length. For analysis of existing structures the actual weights of the members and roof should be computed. In both cases the loads are represented by vertical forces at the panel points.

The weights of the structural steel members in pounds composing the truss are equal to the product of their area (from Table 7-5), their length in feet, and the factor 3.4. The approximate weight of representative roofing materials, in pounds per square foot, is given in Table 8-1.

TABLE 8-1.—APPROXIMATE WEIGHT OF REPRESENTATIVE ROOFING MATERIALS

	Pounds per Square Foot
Felt—2 layers	½
Corrugated galvanized iron	2
Felt and asphalt	2
Sheathing 1" thick, hemlock or spruce	2½
Sheathing 1" thick, yellow pine	3½
Slag roof, cement and sand	4
Slate ⅛" thick	4½
Slate 3/16" thick	6¾
Lead ⅛" thick	7½

The effect of snow on a roof varies with the geographical location of the structure, the humidity and altitude, and the slope of the roof. For average conditions the snow load may be taken as 25 lbs. per sq.ft. of horizontal projection of the roof surface for all slopes up to and including 20°. It is generally reduced 1 lb. for each degree of increase in slope above this figure, up to 45°, above which no snow load need be considered. In severe climates loads may be increased in accordance with actual conditions, for which data may be obtained from experimental results or from building codes of the locality.

Since the dead and snow loads are combined into vertical panel point forces, and both are based upon the roof area, a separate analysis of the stresses caused by the snow load in the truss members is not required. The type or kind of stress in each member is the same for the snow load as for the dead load, and the magnitude of the stress resulting from snow load is proportional to the stress magnitude induced by the dead load, in the same ratio as between the snow load and dead load panel point forces. The magnitude of the snow load stress in a particular member is equal to the product of the dead load stress in that member and the panel joint force ratio. For example, if the dead load stress in a member is 2 kips (2000 lbs.), with dead and snow load panel point forces of 1.5 and 2.1 kips respectively, the panel point force ratio is 2.1/1.5, or 1.4, and the resulting snow load stress in the member is 1.4 × 2.8 kips.

The effect of wind velocity on roofs and other exposed surfaces varies with their shape and slope and with the geographical location of the structures. The relation between the unit pressure P, in pounds per square foot (psf.), on a vertical plane surface, and the wind velocity V, in miles per hour, may be expressed by

$$P = 0.003\ V^2 \tag{8-1}$$

In the general region of the eastern and central states, average wind velocities exceeding 60 miles per hour may be anticipated, although maximum gust velocities exceeding 150 miles per hour are believed to have existed in tornadoes. For a wind velocity of 60 miles per hour, Eq. 8-1 gives

$$P = 0.003 \times 60^2 = 10.8 \text{ psf.}$$

The wind pressure assumed in industrial building design usually ranges from a minimum of 20 to a maximum of 30 psf. of vertical surface. The normal pressure on an inclined or curved surface cannot be determined by finding the component of pressure perpendicular to the roof, since this procedure assumes that a tangential component exists parallel to the roof and this has been shown by tests to be almost non-existent. A commonly used empirical expression for the pressure on an inclined roof is that of Duchemin, which is

$$P_n = P_h\left(\frac{2 \sin A}{1 + \sin^2 A}\right) \tag{8-2}$$

where P_n is the pressure, psf., normal to the surface of the roof; P_h the pressure, psf., on a vertical plane; and A the angle between the roof surface and a horizontal plane. The total pressure due to the wind is equal to the product of the unit pressure P_n and the roof area corresponding to one side of the roof and one bay length. The total pressure is represented by forces acting at the panel points, with half concentrations at the ridge and supporting panel points, and full concentrations at intermediate panel points. Should the roof extend appreciably past the lower supporting panel point, a full concentration may be considered there. If the latter condition exists, the total wind load acting on the truss of Fig. 8-1 would be equal to the product of P_n, the bay length, and the length of the upper chord plus the roof extension; the forces at panel points L_0 and U_1 would each be equal to two fifths of the total load, while the force at U_2 would be equal to one fifth the total load.

Theoretically, dead loads and snow loads induce only vertical reactions, and trusses subjected to loads of this character could be rigidly fixed at both supports. The wind load, acting normal to the surface of the roof, induces a horizontal as well as vertical reaction at the supports. If both ends of the truss are rigidly bolted to masonry or concrete walls, the horizontal thrust carried by each support cannot be determined by the methods of statics. For such construction it is usual to assume that the horizontal thrust is equally divided between the supports, but this assumption may cause serious errors in the stress analysis. This type of construction is also undesirable because it does not allow for the changes in member length that are caused by variations in temperature, humidity or stress. In bridge and viaduct trusses any elongation or contraction in the truss length is cared for by providing rollers under one or more supports. In roof trusses similar to that of Fig. 8-8 the structure is rigidly bolted to the masonry wall at one support (in this instance the left support), and the baseplate at the other support is furnished with slotted holes so that some expansion or contraction is possible. In such cases the movable support at one end will carry a vertical reaction only, and the horizontal thrust caused by the normal wind load and the vertical load must be carried by the fixed support. This construction makes the truss statically determinate and is preferred to that in which both ends are fixed rigidly. In trusses supported by columns or portals, Fig. 8-1, the horizontal reactions are assumed to be equally divided between the two supports because the columns will deflect sufficiently to insure an equal distribution of the thrust. Since the vertical reactions vary with the position of the wind loads, an investigation of the stresses caused by the wind acting from both the right and left is usually necessary.

Moving loads on trusses are often caused by crane and hoist loads. If crane runways are attached to the columns perpendicular to the trusses, or to the lower panel points of the truss, they are usually considered to induce additional forces at the lower panel points. In some instances hoist trolleys are operated on the flanges of the lower chord members; such members should be

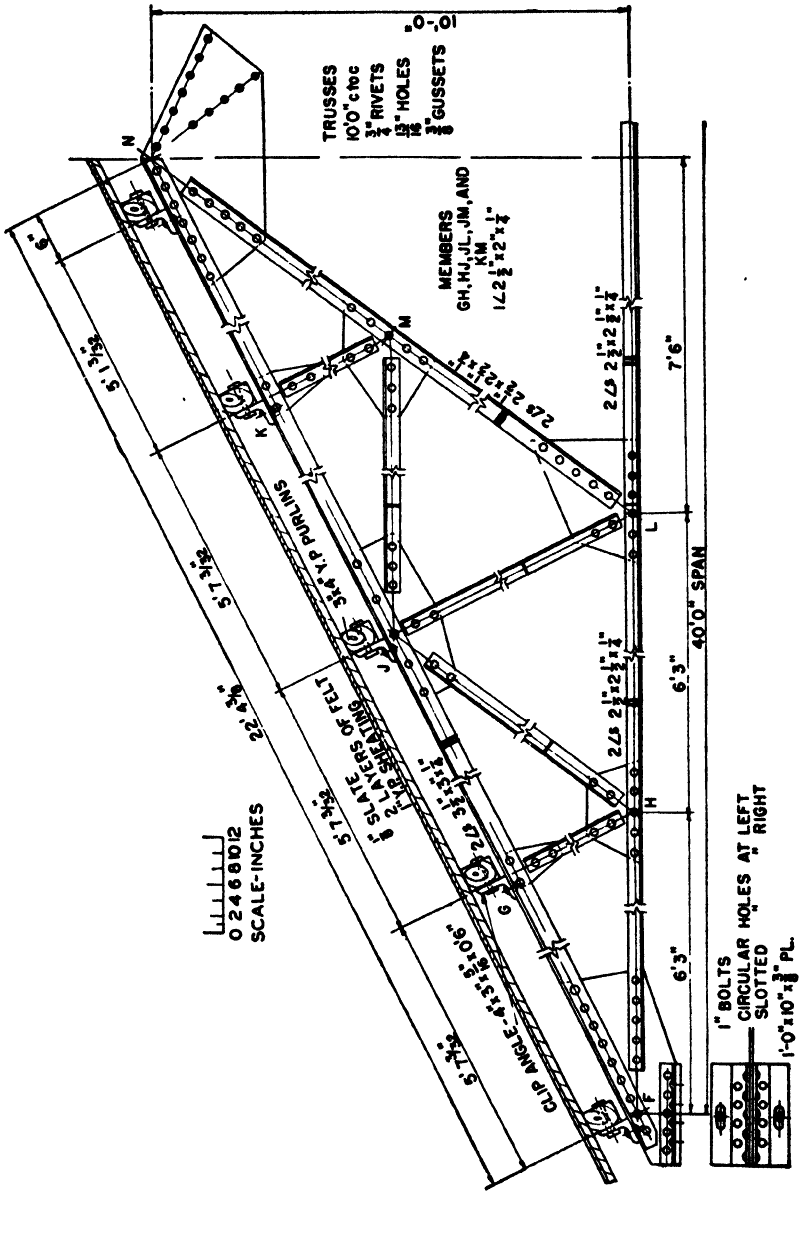


FIG. 8-8. Roof Truss for Industrial Building.

carefully investigated. A possible source of truss failure may present itself if a chain hoist or similar device for use in moving heavy equipment is attached at the midpoint of a lower panel member. Hoisting apparatus of this character should be attached to the lower panel points, and if this is not feasible, the flexural resistance of the lower chord should be investigated before heavy loads are applied.

In analysis and design a full concentration of snow is not regarded as likely to occur when the wind is blowing, and a so-called minimum snow load, whose values are equal to one half of the full or maximum snow load, is usually considered in combination with wind loads. For design, any one of three possible loadings is usually investigated. In the first the stresses in the members are computed based upon the dead load and the maximum snow load. In the second the stresses are computed due to the dead load, the minimum snow load, and the wind from one direction. The third is like the second, except that the wind from the other direction is considered. If the dead load, maximum snow load, and wind load stresses are combined and considered, the AISC Code permits the use of stresses one third greater than those given in Table 7-1.

Truss Adaptation

8-8. It is only infrequently that mechanical or chemical engineers are required to design roof trusses and similar structure; such work lies in the field of structural engineering. However, the necessity for analyzing a structure to permit its adaptation to some purpose other than the original does occur frequently, particularly when there is scarcity of materials and labor. Sometimes complete trusses are obtained from dismantled buildings and employed in otherwise new structures. In other cases such elements as sub-floors, galleries or balconies, crane and hoist girders, or supporting cradles or beams for tanks or machinery may be added to and carried by an existing truss. To illustrate, consider the truss shown in Fig. 8-8. Originally this structure was planned to care only for the roof loading caused by the truss and roof weights, and snow and wind loads. A question of providing needed space for oil storage tanks made it advisable to investigate the possibility of supporting the tanks between two adjacent trusses by attaching supporting beams at several of the lower panel points and carrying the tanks on cradles fastened to the beams. The problem involves the determination of the stresses in the truss to ascertain whether additional loads caused by the weight of the tanks can be carried by all or a majority of the members, designing required reinforcement for any members incapable of carrying additional stress, and also an estimate of the tank design, supporting beams, cradles, and necessary attachments. The solution will be developed on the basis of the foregoing generalizations, using notations and design data not specifically mentioned before.

8-9. The truss shown in Fig. 8-8 is one of four in an industrial building of brick walls, with a floor area of 40 × 30 ft. The span of the trusses is 40 ft., and the bay widths

are 10 ft. each. Two of the trusses are supported for their entire length along the front and rear walls of the building. The left ends of the two central trusses are anchored to the walls, while the right ends have slotted baseplates to permit expansion and contraction. The roof consists of ⅛-in. slate on 1-in. yellow pine sheathing, with two layers of felt between the sheathing and slate. The sheathing is nailed to 4 × 3-in. yellow pine purlins, which are bolted to clip angles riveted to the upper chord of the truss. The trusses are braced against transverse motion by sway bracing consisting of 2½ × 2 × ¼-in. angles, leading from joint *F* in one truss to joint *J* in the adjacent truss, from *J* in the first truss to *F* and *N* in the adjacent truss, and from *N* in the first truss to *J* in the adjacent truss.

8-10. Roof Loads. The analysis may be handled conveniently by first determining the dead and live roof loads and the stresses induced in the members, followed by a computation of the load-carrying capacities of the individual members to determine whether or not any additional loads can be carried. The dead load caused by the roof weight includes the truss member weights, as follows:

Member	Number	Size of Angles	Area of One Angle, sq. in.	Length	Total Weight, lbs.
FGJKN	Two	3½″ × 3″ × ¼″	1.56	23′	244
NSTUQ	"	3⅓″ × 3″ × ¼″	1.56	23′	244
FHL	"	2½″ × 2½″ × ¼″	1.19	12′2″	98
WXQ	"	2½″ × 2½″ × ¼″	1.19	12′2″	98
LMN	"	2½″ × 2½″ × ¼″	1.19	11′9½″	95
MVW	"	2½″ × 2½″ × ¼″	1.19	11′9½″	95
LW	"	2½″ × 2½″ × ¼″	1.19	14′9″	120
GH	One	2½″ × 2″ × ¼″	1.06	2′6″	9
UX	"	2½″ × 2″ × ¼″	1.06	2′6″	9
JH	"	2½″ × 2″ × ¼″	1.06	5′7½″	20
TX	"	2½″ × 2″ × ¼″	1.06	5′7½″	20
JL	"	2½″ × 2″ × ¼″	1.06	5′3½″	19
TW	"	2½″ × 2″ × ¼″	1.06	5′3½″	19
JM	"	2½″ × 2″ × ¼″	1.06	5′6″	20
TV	"	2½″ × 2″ × ¼″	1.06	5′6″	20
KM	"	2½″ × 2″ × ¼″	1.06	2′6″	9
SV	"	2½″ × 2″ × ¼″	1.06	2′6″	9
					1148

The weights of the ⅜-in. thick gusset plates are found by multiplying the approximate area by the thickness and the factor 0.284, as follows:

Joint	Size	Weight, lbs.
F and *A*	10″ × 2′2″	56
G and *U*	6″ × 8″	10
H and *X*	7″ × 1′2″	11
J and *T*	8″ × 1′9″	34
L and *W*	1′1″ × 1′4″	44
K and *S*	6″ × 8″	10
M and *V*	1′ × 1′3″	38
N	1′3″ × 2′9″	52
Baseplates	10″ × 1′	26
		281

For a panel length of 5 ft. 7 in., and a bay length of 10 ft. center to center, the length of the sway bracing angles are approximately 15 ft. The area of one angle

is 1.06 sq. in., from Table 7-5, and the weight of one angle is 1.06 × 3.4 × 15, or 54 lbs. Since there are eight such angles on each side of the truss, the total sway bracing is taken as 8 × 54, or 432 lbs.

The weight of the roof covering, from Table 8-1, is

1/8-in. slate	4½ lbs. per sq. ft.
2 layers of felt	½ lb. per sq. ft.
1-in. yellow pine sheathing	3½ lbs. per sq. ft.
Total	8½ lbs. per sq. ft.

The slant length f the roof based upon the truss length, is 22 ft. 4⅜ in. The roof eaves are considered to extend past points F and Q a distance equal to one half the panel length of 5 ft. 7 9/32 in. Each of the two central trusses carries a roof covering width of 5 ft. on either side of each truss. The roof covering area is 5 ft. 7 9/32 in. + 2(22 ft. 4⅜ in.) (10 ft.), or 503.2 sq. ft., and the roof covering weight is 503.2 × 8.5, or 4278 lbs.

The roof covering is supported by ten 4 × 3-in. yellow pine purlins each having a length of 10 ft. per truss. An additional purlin at each eave is added to compensate for the weight of the finishing strip at these edges. Estimating the weight of yellow pine at 44 lbs. per cu. ft., the purlin weight per truss is 10 × 12 × 44 × 4 × 3/144, or 440 lbs. The total weight per truss is

Truss framing	1148
Gussets	281
Wind bracing	432
Roof	4278
Purlins	440
Total	6579 lbs.

To this total approximately 10% is added to allow for rivet heads, clip angles, nails, and so forth, making the total dead weight 7200 lbs. This dead weight is considered equally divided among the nine upper panel points, or a unit panel load of 7200/9, or 800 lbs.

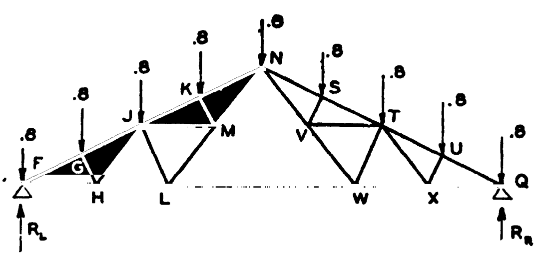


FIG. 8-9. Skeleton Layout for Industrial Building Roof Truss, with Dead Load Panel Point Forces.

8-11. Analytical Determination of Stresses. Fig. 8-9 shows a skeleton layout of the truss of Fig. 8-8, with the panel point loads shown in kips. The reactions R_L and R_R are vertical and equal to 3.6 kips each. Figs. 8-10 to 8-27 show the details of the analytical solution for the stresses in the truss members. Fig. 8-10 is a space diagram for the joint F; the truss considered as a free body. The direction and magnitude of the forces along members FG and FH are unknown, but their positions are defined since they are coincident with the members. Fig. 8-11 shows the force polygon for joint F, from which the direction and magnitude of the internal induced forces fh and gf in members FH and FG may be determined. Fig. 8-12 is a free-body diagram of member FH. The direction of the internal force at end F is obtained from Fig. 8-11, and indicated by a solid arrowhead. In order that FH may be in equilibrium, another internal force of opposite direction must be present

(the outlined arrowhead near *H*). The two arrowheads pointing toward each other show that the member is in tension. From Fig. 8-13 the member *FG* is found to be in compression.

Figs. 8-14 to Fig. 8-21 represent space and force diagrams for joints *G* and *H* and space diagrams for members *GJ, GH, HL,* and *HJ*. Fig. 8-22 shows a space diagram for joint *J*, in which the direction and magnitude of the three forces along members *KJ, MJ*

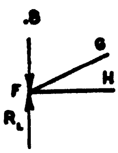


FIG. 8-10. Space Diagram, Joint F.

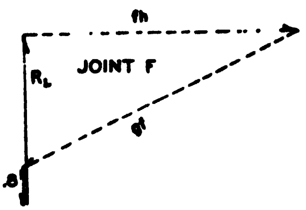


FIG. 8-11. Force Diagram.



FIG. 8-12. Space Diagram, Stress in Member FH.

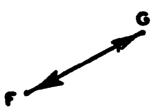


FIG. 8-13. Space Diagram, Stress in Member FG.

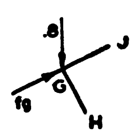


FIG. 8-14. Space Diagram, Joint G.

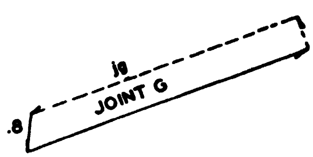


FIG. 8-15. Force Diagram, Joint G.

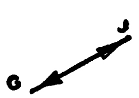


FIG. 8-16. Space Diagram, Stress in Member GJ.



FIG. 8-17. Space Diagram, Stress in Member GH.

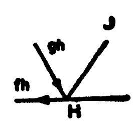


FIG. 8-18. Space Diagram, Joint H.

and *LJ* are unknown. Since there are more than two directions and two magnitudes unknown (four unknown definitives), solution by the method of joints is not possible. By using the method of moments the forces at *L* can be found, after which the joint *J* can be analyzed. Consider the entire left half of the truss to be a free body and solve for the internal force in member *WL* by taking moments about *N*, the ridge of the truss. This solution is illus-

trated in Fig. 8-23. The moments of all the external forces about joint N are equal to $R_L \times 20 - 0.8(20 + 15 + 10 + 5)$, or 32 ft. kips, which must be equilibrated by the internal resistance in member WL. Since WL is at a distance of 10 ft. from N, the force in this member is 3.2 kips acting toward the right. With the forces HL and LW known, it is possible to solve for the forces at joint J, as shown in Figs. 8-25 and 8-26, and to

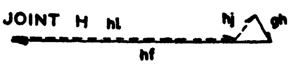


Fig. 8-19. Force Diagram, Joint H.



Fig. 8-20. Space Diagram, Stress in Member HL.

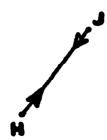


Fig. 8-21. Space Diagram, Stress in Member HJ.

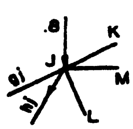


Fig. 8-22. Space Diagram, Joint J.

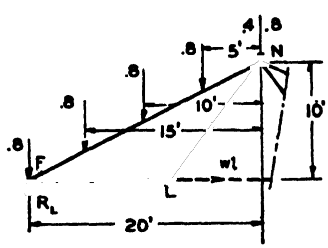


Fig. 8-23. Space Diagram Showing Force in Member WL by Method of Moments.



Fig. 8-24. Space Diagram, Stress in Member LW.

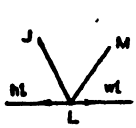


Fig. 8-25. Space Diagram, Joint L.

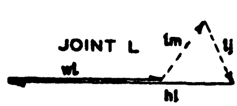


Fig. 8-26. Force Diagram, Joint L.



Fig. 8-27. Space Diagram, Stress in Member JL.

determine the stress in member JL. With the force in member JL known, the analysis of the rest of the joints and members may be completed in similar fashion as for joints F, G, and H. The stresses in the members of the right half of the truss correspond to similar members in the left half. A summary of the stresses in the truss members is listed in Fig. 8-28 under Dead Load.

Member	Number and Size of Angles, in.	Loading: Dead 1	Maximum Snow 2	Minimum Snow 3	Wind Left 4	Wind, Right 5	Combination for Maximum Stress (column numbers) 6	Maximum Load 7	Allowable Load 8
FG *QU*	Two 3½ × 3 × ¼	−6.26	−7.44	−3.72	−3.78 −2.10	−2.10 −3.78	1,3,4 1,3,5	−13.76	−46.55
FH *QX*	Two 2½ × 2½ × ¼	+5.60	+6.66	+3.33	+4.73 +1.88	+0.19 +3.00	1,3,4 1,2	+13.66 +12.26	+38.80
GJ *UT*	Two 3½ × 3 × ¼	−5.90	−7.00	−3.50	−3.78 −2.10	−2.10 −3.78	1,3,4 1,3,5	−13.18	−46.55
GH *UX*	One 2½ × 2 × ¼	−0.72	−0.86	−0.43	−0.84 0	0 −0.84	1,3,4 1,3,5	−1.99	−14.80
HL *XW*	Two 2½ × 2½ × ¼	+4.82	+5.72	+2.86	+3.79 +1.88	+0.19 +2.06	1,3,4 1,2	+11.47 +10.54	+38.80
HJ *XT*	One 2½ × 2 × ¼	+0.78	+0.94	+0.47	+0.94 0	0 +0.94	1,3,4 1,3,5	+2.19	+12.50
JL *WT*	One 2½ × 2 × ¼	−1.49	−1.78	−0.89	−1.71 0	0 −1.67	1,3,4 1,3,5	−4.09 −4.05	−8.00
LM *WV*	Two 2½ × 2½ × ¼	+1.64	+1.94	+0.97	+1.92 0	0 +1.87	1,3,4 1,3,5	+4.53 +4.48	+38.80
JK *TS*	Two 3½ × 3 × ¼	−5.51	−6.54	−3.27	−3.78 −2.10	−2.10 −3.78	1,3,4 1,3,5	−12.56	−46.55
JM *TV*	One 2½ × 2 × ¼	+0.80	+0.95	+0.48	+0.94 0	0 +0.94	1,3,4 1,3,5	+2.22	+12.50
KN *NS*	Two 3½ × 3 × ¼	−5.15	−6.12	−3.06	−3.78 −2.10	−2.10 −3.78	1,3,4 1,3,5	−11.99	−46.55
KM *SV*	One 2½ × 2 × ¼	−0.72	−0.86	−0.43	−0.84 0	0 −0.84	1,3,4 1,3,5	−2.09	−14.80
MN *NV*	Two 2½ × 2½ × ¼	+2.36	+2.82	+1.41	+2.86 0	0 +2.81	1,3,4 1,3,5	+6.63 +6.58	+38.80
LW	Two 2½ × 2½ × ¼	+3.20	+3.80	+1.90	+1.88	+0.19	1,2	+7.00	+38.80

FIG. 8-28. Tabulation of Stresses in Roof Truss Members.

8-12. Graphical Determination of Stresses—Bow's Notation. The graphical solution for determining the stresses in truss members is used extensively. A combined force polygon is usually substituted for the individual diagrams shown in Figs. 8-11, 8-15, and 8-19 to save time and eliminate drawing errors. In graphical solutions the method of designating the members shown in Fig. 8-9 is not so convenient as a much more commonly used method known as Bow's Notation, illustrated in Fig. 8-29. In this notation the areas between the force lines in the space diagram are lettered, and the areas between the truss members are numbered. To illustrate, the left reaction is AN; the panel point load at the extreme left is AB; the members meeting at joint U_1 are B-1, 1-2, and C-2; the joints are designated L_1 L_2 and U_1, U_2, and so forth, denoting lower and upper panel points. Corresponding letters and numbers are placed at the extremities of the vectors in the force polygon of Fig. 8-29. The panel point loads AB, BC, CD, and so forth are laid out in order in the force polygon,

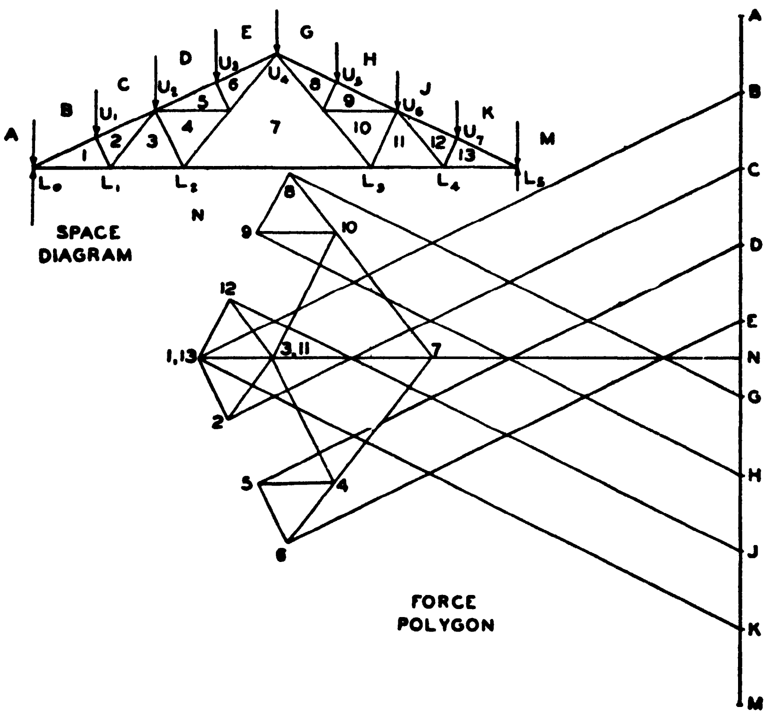


Fig. 8-29. Graphical Solution of Member Stresses, with Bow's Notation.

and the right and left reactions are represented by MN and NA. N-1 and B-1 in the force polygon are drawn parallel to the corresponding members in the space diagram and scaled to obtain the magnitude of the vectors. If joint U_1 is next considered, vectors from points 1 and C in the force polygon may be drawn parallel to members 1-2 and C-2 and the magnitude scaled. This procedure may be continued for joint L_1, but too many unknown definitives are again present at joint U_2. One method of solution is similar to that previously described. The magnitude of the force in member 7-N may be computed by the method of moments and the vector N-7 laid off on the vector line N-1 in the force polygon. The solution may then proceed by analyzing joint L_2, followed by U_3 and U_2.

8-13. Member Substitutions. An alternative method of solution for joint U_2, purely graphical in nature, is to delete temporarily members 4-5 and 5-6 and to substitute a theoretical member 4′-6′, as shown in Fig. 8-30. This procedure reduces the unknown

definitives at joint U_3 to four—the direction and magnitude of the forces in members 3-4′ and D-4′. It is then possible to obtain temporary or substitute solutions for the forces in members 3-4′, D-4′, 4-6′, E-6′, and 6-7, as shown in the corresponding force polygon of Fig. 8-31. The vector 6′-7, represents the actual force in member 4-7, for it has the same direction and magnitude regardless of the internal construction of the web members of the truss. The triangle L_0, L_2, U_4 is rigid and will hold its form regardless of any change in the internal member arrangement. Enough of the force polygon, Fig. 8-31, is drawn to locate point 7, which is transferred to the force polygon of Fig. 8-29, and the solution can then be completed. (In Fig. 8-31 the vector 3-4′ does not, of course, give the true value of the stress in member 3-4′, nor does the vector D-4′ represent the force in member D-5.)

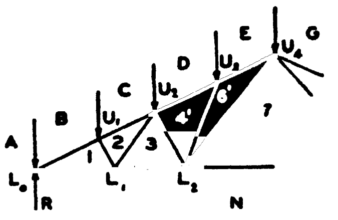


FIG. 8-30. Graphical Solution of Stresses.

8-14. Snow Load Stress Determinations. The usual value for snow loads is 25 psf. of horizontal projection for roof slopes up to 20°. The tangent of the angle between the roof and a horizontal plane is equal to twice the rise divided by the span, or 0.50, for which the angle A is approximately 26° 30′. From Section 8-7, the unit snow load is equal to $25 - (26.5 - 20)$, or 18.5, or

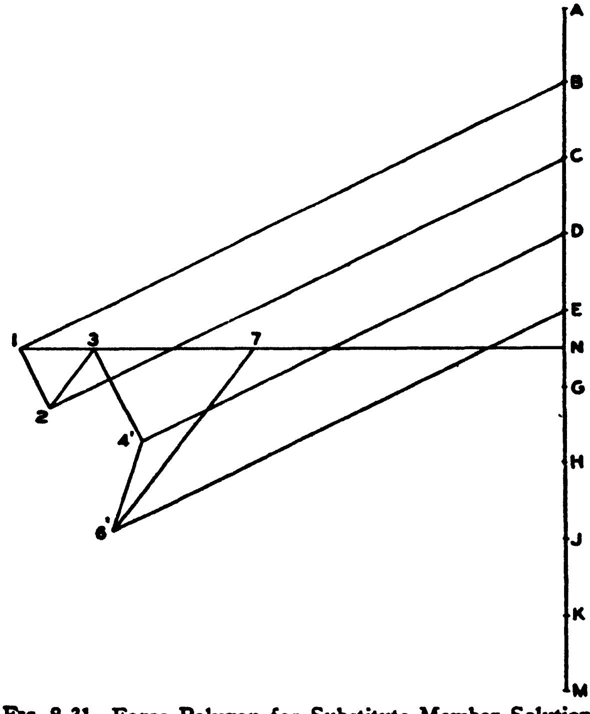


FIG. 8-31. Force Polygon for Substitute Member Solution.

approximately 19 lbs. per ft. of horizontal projection of roof area. Since the roof extends over the ends of the truss for a distance equal to one half of the panel width on each side, the area is $(40 + 5)\,10$, or 450 sq. ft., and the total snow load, based upon the horizontal projection of the roof area, is 450×19, or 8550 lbs. The snow load per panel point is then 8550/9, or 950 lbs.

The snow load panel point forces have the same direction and character as the dead load forces, so a separate analysis is not required to determine the magnitude and direction of the stresses in the members. The type of stress in each member is the same for the snow load as for the dead load, and the magnitude of the stress may be computed by multiplying the stress due to dead load by a factor representing the ratio between the

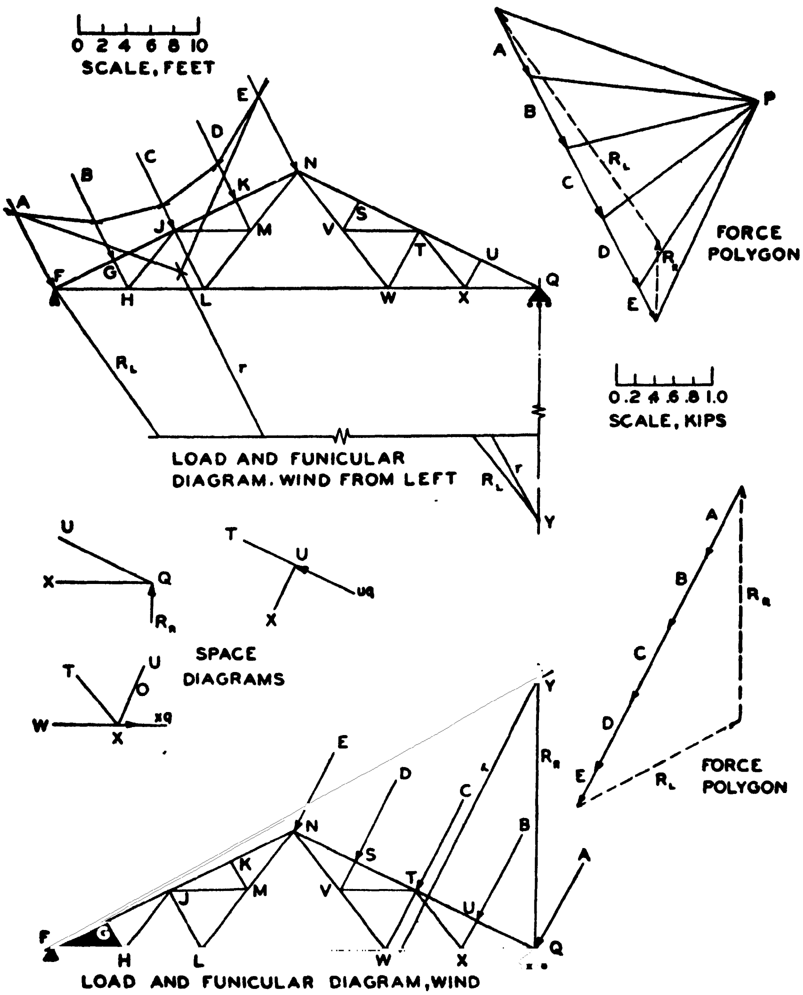


Fig. 8-32. Skeleton Layout for Truss with Wind Loads from Left and Right.

snow load and dead load panel point forces, in this case 950/800, or approximately 1.19. For example, the stress due to dead load in members *FG* and *QU* is 6.26 kips, compression; the stress due to snow load is 1.19 × 6.26, or 7.44 kips, compression. The snow load stresses thus found are listed in Fig. 8-28 in the column "Max. Snow." The minimum snow load stresses, based upon panel point forces equal to one half the maximum snow load, are listed in Fig. 8-28, in the column "Min. Snow."

8-15. Wind Load Stress Determinations. For computing the wind load assume a horizontal wind pressure of 20 psf. on a vertical surface. For a roof angle of 26° 30′ the pressure normal to the roof surface is found, from Eq. 8-2, to be

$$P_n = 20 \frac{2 \sin 26° 30'}{1 + \sin^2 26° 30'} = 14.9 \text{ psf.}$$

The roof area corresponding to any intermediate panel point is 5 ft. 7⅝₂ in. × 10 ft., or 55.9 sq. ft. The normal forces at panel points *F, G, J,* and *K* are each 55.9 × 14.9, or 834, approximately 840 lbs. Since the ridge *N* carries one half of a roof area on each side (and the wind effect is considered on only one side of the roof at a time), the force at *N* due to wind is 840/2, or 420 lbs.

The truss is rigidly bolted to the masonry wall at the left support, and the baseplate at the right support has slotted holes so that some expansion or contraction of the truss is possible. Permissible sliding at the right support will permit only a vertical reaction, and the horizontal thrust caused by the normal wind forces must be carried by the left reaction. Therefore it is necessary to compute the stresses in the members for the loads caused by

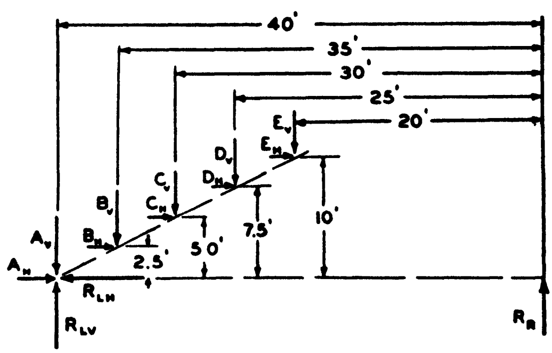


FIG. 8-33. Skeleton Layout for Truss with Wind Loads from Left.

the wind when acting both from the left and from the right, as shown in Fig. 8-32. The direction and magnitude of the reactions may be obtained graphically. In Fig. 8-32 the resultant *r* of the wind forces *A, B, C, D,* and *E* is obtained by drawing funicular and force polygons. The position of *r* is shown in the funicular polygon, and its magnitude (equal to $A + B + C + D + E$) and direction (opposite to *A, B, C, D,* and *E*) in the force polygon. The two reactions and the resultant must intersect, if extended, at a common point in the space diagram. The position of the resultant and the right (vertical) reaction are known, so the point of intersection *Y* is obtained, and the position of the left reaction determined by drawing it through points *Y* and *F*. The forces R_L and R_R in the force polygon are then drawn parallel to the reaction position lines in the space diagram. The magnitude of these reactions may be scaled from the force polygon. A similar procedure is followed in finding the reactions with the wind acting from the right.

It is sometimes more convenient to compute the magnitude of the reactions by using the method of moments. Fig. 8-33 shows the force system with the horizontal and vertical components A_H, A_V, B_H, B_V, and so forth, replacing the normal forces *A, B, C, D,* and *E*. The magnitude of the horizontal components A_h is 840 × sin 26° 30′, or 375 lbs.; the vertical component A_V is 840 × cos 26° 30′, or 750 lbs. E_V and E_H are 375 and 187.5 lbs., respectively. The horizontal component of R_{LH} or the left reaction must be equal and opposite to the summation of the horizontal force components, or

$$R_{LH} = A_H + B_H + C_H + D_H + E_H = 375 + 375 + 375 + 375 + 187.5 = 1687 \text{ lbs.}$$

The vertical reaction at the left support may be found by taking moments about R_B:

$$M = -(375 \times 20) - 750(25 + 30 + 35 + 40) + (187.5 \times 10) + 375(7.5 + 5.0 + 2.5) + 40R = 0$$

and the left vertical reaction is 97,500/40, or 2438 lbs. The reaction at the right support may be found by taking moments about the left support:

$$M = +(375 \times 20) + 750(15 + 10 + 5 + 0) + (187.5 \times 10) + 375(7.5 + 5.0 + 2.5) - 40R = 0$$

and is equal to 37,500/40, or 937 lbs.

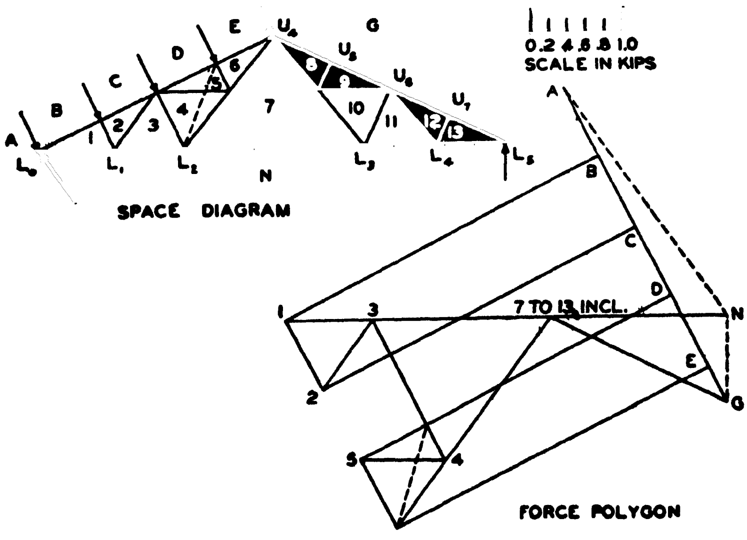


FIG. 8-34. Graphical Solution of Wind Load Stresses.

Checking by a vertical summation,

$$2438 + 937 - 750 - 750 - 750 - 750 - 375 = 0$$

The horizontal and vertical components of the left reaction, and the magnitude of the vertical reaction at the right, may be determined similarly for the condition shown in Fig. 8-32, when the forces due to the wind act on the right side of the roof.

The graphical analysis of the wind load stresses is shown in Fig. 8-34 for forces acting on the left side of the roof. The substitute member (shown by the dotted line), temporarily replacing 4-5 and 5-6, must be used to determine the stress in 7-*N*. The analytical determination of the stresses can be carried out in the same manner as in the case of the dead load stresses. After determining the stresses at joints *F*, *G*, and *H*, the analysis should be continued by determining the forces at joint *Q* and the stress in the member *QU*, as shown in Fig. 8-32. A consideration of joint *U* indicates that no force exists in member *UX*, and thus its stress is zero. Similarly, there cannot be any forces exerted by member *TX* at joint *X*, and the stress in this member is zero. Similarly the stress in members *TW*, *TV*, *SV*, *VW*, and *NV* is zero, as far as wind loads are concerned. Likewise, for wind forces acting from the right, no stresses exist in members *GH*, *HJ*, *JL*, *JM*, *KM*, *LM*, and *MN*.

The stresses due to dead, snow, and wind loads are tabulated in Fig. 8-28. The maximum resultant stresses in each member given in column 7 are computed by finding possible combinations for maximum stress that may exist. Such combinations are indicated in column 6.

8-16. Allowable Stresses in Truss Members. The next step is to calculate the actual allowable stresses that the truss members may carry, deducting the dead and live loads computed from the preceding analysis, and thus determine any additional safe loads that can be carried.

From Fig. 8-8 it is seen that the upper chord members *FG, GJ, JK, KN, NS, ST, TU,* and *UQ* are composed of two $3\frac{1}{2} \times 3 \times \frac{1}{4}$-in. angles, long legs back to back, separated by a $\frac{3}{8}$-in. gusset plate. Each of these members acts as a column 67.2 in. long. The radius of gyration of the column section with respect to the *x-x* axis (Fig. 8-35) is the same as for one angle, and from Table 7-5 its value is 1.11. The radius of gyration about the *g-g* or centroidal axis of the member is found from Eqs. 5-6 and 5-8 and is equal to

$$k_{gg} = \sqrt{(I_{yy}/A) + d^2} = \sqrt{k^2_{yy} + d^2} = \sqrt{0.91^2 + 0.978^2} = 1.34 \text{ in.}$$

Since this value is greater than that of k_{xx}, the latter is the controlling one.

The allowable unit stress in compression is given by the column formula in Table 7-1. The L/k ratio is 67.2/1.11, or 65.5, and the expression for allowable compressive stress for L/k ratios less than 120 governs the analysis. Substiuting,

$$S_c = 17{,}000 - 0.485 \times 65.5^2 = 14{,}920 \text{ psi.}$$

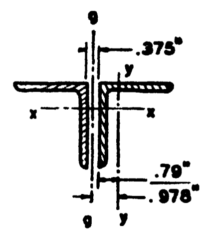


Fig. 8-35. Upper Chord Section.

The gross area of the member is 2×1.56, or 3.12 sq. in., and the allowable load capacity is $14{,}920 \times 3.12$, or 46,550 lbs. It should be noted that the entire gross area of this section may be considered effective in compression, because the length of the outstanding leg of the angle does not exceed 16 times the thickness. (Section 7-10.)

The lower chord members *FH, HL, LW, WX,* and *XQ,* and the web members *LM, MN, NV,* and *VW* are composed of two $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$-in. angles, and act as tension members. From Table 7-6, the area of one angle is 1.19 sq. in. Deducting for one hole for a $\frac{3}{4}$-in. rivet in each angle, the net area is $2[1.19 - (0.875 \times 0.25)]$, or 1.94 sq. in. Since the section of each member is composed of two angles, the slight degree of eccentricity caused by the non-coincidence of the centroid and the gage line may be disregarded and the full net area of the section considered in computing the stress. The allowable load capacity, using the allowable tensile stress from Table 7-1, is $20{,}000 \times 1.94$, or 38,800 lbs.

The web members *GH, UX, KM,* and *SV* are composed of one $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angle and act as columns 33.2 in. long. The minimum radius of gyration is taken with reference to the *z-z* axis from Table 7-6, and is 0.42 in. The allowable unit stress is

$$S_c = 17{,}000 - 0.485\left(\frac{33.2}{0.42}\right)^2 = 13{,}970 \text{ psi.}$$

The allowable load on each of these members is $13{,}970 \times 1.06$, or 14.8 kips.

The web members *JL* and *WT* are composed of one $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angle and act as columns 66.5 in. long. The minimum value of k is 0.42, and the L/k ratio is 66.5/0.42, or 158. The allowable unit stress is obtained from the column formula in Table 7-1 for L/k ratios greater than 120 and is

$$S_c = 18{,}000 \div \left[1 + \frac{1}{18{,}000}(158)^2\right] = 7540 \text{ psi.}$$

The allowable load is 7540×1.06, or 8.0 kips. (Since these web members are used as secondary or bracing members, the L/k value of 158 is satisfactory.)

The web members *HJ, XT, JM,* and *TV* are composed of one $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angle and act as tension members. Because of the eccentricity of load, the area subject to stress is considered equal to the net area of the connected leg plus one half the area of the

outstanding leg of 0.25(2.5 — 0.875) + 0.25(1.75/2), or 0.625 sq. in. The allowable load is 0.625 × 20,000, or 12.50 kips.

(Column numbers are a continuation from Fig. 8-28)

	Excess Strength = (8 — 7) 9	Tank Load Stress 10	Permissible Value of Tank Load *P* 11	Actual Tank Load Stress 12	Total Stress = (7 + 12) 13	Allowable Stress 14
FG *QU*	—32.79	—1.18*P*	—27.80	—26.80	—40.56	—46.55
FH *QX*	+25.14 +26.54	+*P*	+25.14	+22.70	+36.36 +34.96	+38.80
GJ *UT*	—33.37	—1.18*P*	—28.20	—26.80	—39.98	—46.55
GH *UX*	—12.81	0		0	—1.99	—14.80
HL *XW*	+27.33 +28.26	+*P*	+27.33	+22.70	+34.17 +33.24	+38.80
HJ *XT*	+10.31	0		0	+2.19	+12.50
JL *WT*	—3.93 —3.95	0		0	—4.09 —4.05	—8.00
LM *WV*	+34.27 +34.32	+0.31*P*	+110.50	+7.04	+11.57 +11.52	+38.80
JK *TS*	—33.99	—1.18*P*	—28.80	—26.80	—39.36	—46.55
JM *TV*	+10.28	0		0	+2.22	+12.50
KN *NS*	—34.56	—1.18*P*	—29.30	—26.80	—38.79	—46.55
KM *SV*	—12.71	0		0	—2.09	—14.80
MN *NV*	+32.17 +32.22	+0.31*P*	+103.60	+7.04	+13.67 +13.62	+38.80
LW	+31.80					
LZ *ZW*		+0.81*P* +0.81*P*	+39.2	+18.40 +18.40	+25.40 +25.40	+38.80
NZ		+0.5*P*		+11.35	+11.35	—13.14

FIG. 8-36. Tabulation of Loads and Stresses in Roof Truss Members.

8-17. Excess Stress Capacity of Truss Members. The excess capacity of each member of the truss is equal to the difference between the total allowable stress, listed in column 8, Fig. 8-28, and the maximum stress combination induced by the dead and live loads in the roof, as listed in column 7. This excess capacity or strength is given in column 9, Fig. 8-36. To illustrate, the total allowable stress in member *FG* is 46.55 kips; the excess strength of the member is 46.55 — 13.85, or 32.7 kips.

8-18. Preliminary Design Estimates. The maximum storage capacity can be attained if two tanks (their axes horizontal) are placed between the two central trusses, as shown in outline in Fig. 8-37; the tanks should be placed as high in the trusses as possible so as to conserve headroom in the building. At least two supports must be furnished for each tank; these supports may consist of cradles, carrying both tanks, and supported at their ends by beams extending from joints L and W of the two trusses. Such construction may necessitate comparatively large cradles and beams, and an alternate design calling for three beams, extending from L, W, and the midpoint Z of the lower chord, will probably prove more satisfactory. In this design, two cradles of comparatively short span will be required for each tank. Each of the outer beams will support one half of the weight of one tank; the central beam will carry the weight of one half of each tank. To eliminate the excessive flexural stress induced by the beam reaction at the midpoint of member LW, the central beam should be fastened to a gusset plate at Z, which in turn should be supported by an auxiliary tension member NZ.

If P represents the weight of one tank full of oil, and the weight of one half of the entire substructure necessary for the support of the vessels, the applied loads A, B, and C at joints L, Z, and W will be 0.25 P, 0.50 P, and 0.25 P, respectively. The reactions R_L and R_R at F and Q will then be equal to 0.50 P each. The essential features of the stress analysis for these forces is shown in Figs. 8-38 to 8-46; in Figs. 8-40 and 8-41, it is seen that the web members GH and JH carry no stress; in Fig. 8-42, at joint L, the member LM supplies the entire vertical internal resistance to offset the force A. The stresses in the truss members, in terms of the unit total load P, are listed in Fig. 8-36, column 10; the upper and lower chords, and the members LM, MN, NV, and VW, are the only members affected by the additional loads proposed.

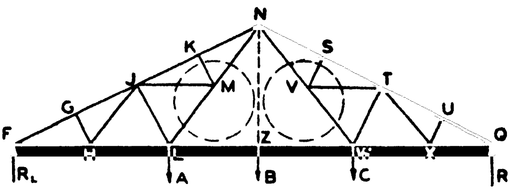


FIG. 8-37. Skeleton Layout for Truss with Tank Loads.

The minimum value of the unit tank load P may be obtained by equating the maximum tank load stress to the minimum excess strength or capacity of a particular member. From columns 8 and 9, Fig. 8-36, it is seen that member FG has an induced tank load stress of 1.18P kips, and an excess stress capacity of 32.77 kips; the resulting value of P is therefore 32.77/1.18, or 27.8 kips. Member FH, however, has an induced tank load stress of P kips, and an excess stress capacity of 25.14 kips; this value is therefore the controlling minimum figure for P. Column 11 gives the permissible value of the tank load P, based upon the excess strength of the members. Calculate riveted joints for shear and bearing strength of rivets; once the final design of tank is done, the load on each joint should be calculated and joints strengthened by welding where necessary.

8-19. Preliminary Tank Design. At this point it is necessary to make a design estimate for the tanks. A tentative scale layout will show that two tanks, each 7 ft. in diameter and 9 ft. long, with flat heads, are the largest feasible vessels that can be placed in the space available. Since the tanks are to be used for oil storage, their design should conform to the specifications of the API-ASME Code; the vessels may be made of structural or mild (Grade C) steel, and single-welded butt joints (without backing-up strips) for both longitudinal and circumferential seams should be used if possible. The pump pressure necessary to fill these tanks is approximately 20 psi., and since the vessels are for oil storage only, no corrosion allowance need be made. Grade C steel requires the use of a material factor F_m of 0.92, and the allowable design stress (Chapter 4, Table 4-1) is $55,000 \times 0.92 \times 0.25$, or 12,600 psi. From Eq. 4-3, the minimum thickness t of the vessel is:

$$t = \frac{20 \times 84}{2 \times 12,600 \times 0.70 - 20} = 0.095 \text{ in.}$$

(the efficiency of a single-welded butt joint being taken as 70%). From Eq. 4-4, however, the shell thickness is limited to a minimum of

$$t = \frac{84 + 100}{1000} = 0.184 \text{ in.}$$

To eliminate possible leakage caused by shell deflection, it is desirable to use a ¼-in. plate.

If a flanged flat head is used, butt welded to the shell, the head thickness may be found from Eq. 4-10,

$$t = 84\sqrt{0.25 \times 20/12{,}600} = 1.68 \text{ in.}$$

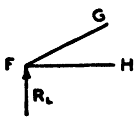


Fig. 8-38. Space Diagram, Joint F.

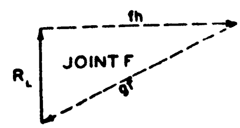


Fig. 8-39. Force Diagram, Joint F.

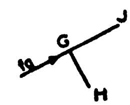


Fig. 8-40. Space Diagram, Joint G.

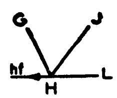


Fig. 8-41. Diagram, Space Joint H.

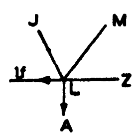


Fig. 8-42. Space Diagram, Joint L.

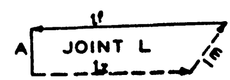


Fig. 8-43. Force Diagram, Joint L.

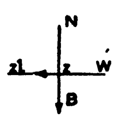


Fig. 8-44. Space Diagram, Joint Z.

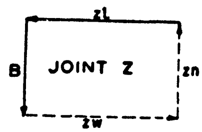


Fig. 8-45. Force Diagram, Joint Z.

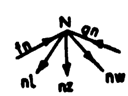


Fig. 8-46. Space Diagram Joint N.

This design will require a head thickness of 1¾ in., which is decidedly out of proportion to the shell thickness. Therefore, it may be advisable to redesign the vessel for dished heads. The inner crown radius of a dished head with an outer diameter of 84 in. is 78 in., and the knuckle radius of the thinner gage heads is 5⅛ in.; the ratio of knuckle radius to crown radius is 5.125/78, or 0.066. From Table 4-3, the value of the factor W is 1.74, and from Eq. 4-6, the necessary head thickness is

$$t = 20 \times 78 \times 1.74/2 \times 12{,}600 = 0.108 \text{ in.}$$

(The efficiency e of the head is taken as 1.0, since the head is seamless.) From a manufacturer's catalog, the smallest gage obtainable in this diameter is 5/16. (It should be noted

that the values of the knuckle and crown radii, as used above, are not the mean values, as called for in a rigid application of Eq. 4-6, but the head thickness is so much greater than the theoretical that such refinement in computation is unnecessary.)

The weight of the filled tank should be computed at this point to determine whether the estimated weight of 25,140 lbs. is greater than the substructure and tank weights. Since the use of dished heads will reduce the storage capacity to some extent, it may be advisable to increase the overall length of the vessel to 9 ft. 6 in. From Eq. 4-14, the depth of the dished head is approximated by

$$d = 78 - \sqrt{78^2 - 84^2/4} = 12.3 \text{ in.}$$

and the area A of the convex surface, from Eq. 4-16, is

$$A = 6.28 \times 78 \times 12.3 = 6030 \text{ sq. in.}$$

The weight of a cubic inch of steel is 0.284 lb., and the weight of the dished portion of a $\frac{5}{16}$-in. head is $6030 \times 0.313 \times 0.284$, or 536 lbs. The length of the shell is $114 - (2 \times 12.3)$, or 89.4 in., and its weight is equal to $\pi \times 84 \times 89.4 \times 0.25 \times 0.284$, or 1675 lbs.

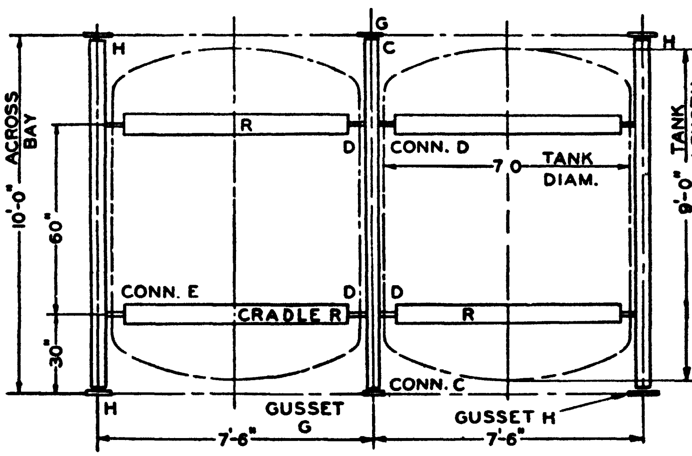


FIG. 8-47. Plan View of Cradle and Beam Arrangement.

The cubical contents of the tank are equal to the sum of the volumes contained in the heads and the volume of an 84-in. diameter cylinder 89.4 in. long. From Eq. 4-15, the volume of the head interior is

$$V = 1.05 \times 12.3^2[3(78) - 12.3] = 28{,}600 \text{ cu.in.}$$

The volume of the cylindrical portion of the tank is $\pi \times 42^2 \times 89.4$, or 495,000 cu. in. The weight of the oil contained in one tank is $(495{,}000 + 28{,}600 + 28{,}600)55/1728$, or 17,600 lbs. The total weight of one tank is $17{,}600 + 536 + 536 + 1675$, or 20,347 lbs. If the tank weight be assumed at 21,500 lbs., thereby allowing over 1000 lbs. for attachments, reinforcement, etc., the maximum permissible weight of the substructure, for one tank, will be $25{,}140 - 21{,}500$, or 3640 lbs.

8-20. Substructure Design. The layout for the tank cradles is shown in Fig. 8-47. Two cradles are employed for each tank; the layout shows them spaced 30 in. from each truss, 60 in. apart, the bay of the roof measuring 10 ft. The cradles will be made of flame-cut structural plate, with an upper edge curved to fit the tank; a flat strip flange is welded along this edge (Fig. 8-48) to support the tank. The cradles will be fastened to the beams spanning the bay of the roof by bolted angle clips. If possible, all connections made in the process of erection will consist of turned bolts in reamed holes; all shop connections can be welded. Since the weight of one filled tank is 21,500 lbs., the load on each cradle is 10,750 lbs. If the supporting beams are placed directly under the panel points

L, *Z*, and *W*, the span of the cradle is 7 ft. 6 in., or 90 in. If the tank weight is considered uniformly distributed over the supporting area of the cradle flange (which encompasses approximately 120° of the tank periphery and has a consequent horizontal length of about 72 in.), the load condition of the cradle will be that of a simply supported beam 90 in. long with a uniformly distributed load of 10,750 lbs. symmetrically applied over the central 72 in. of its length. The end reactions are 5375 lbs. each, and the maximum bending moment is at the center and is equal to (5375 × 45) — (5375 × 18), or 145,300 in.-lbs.

The weakest section is at the center of the cradle, and it may be advisable to design the cradle web to take the entire load and use the flange only as a support for the tank. If

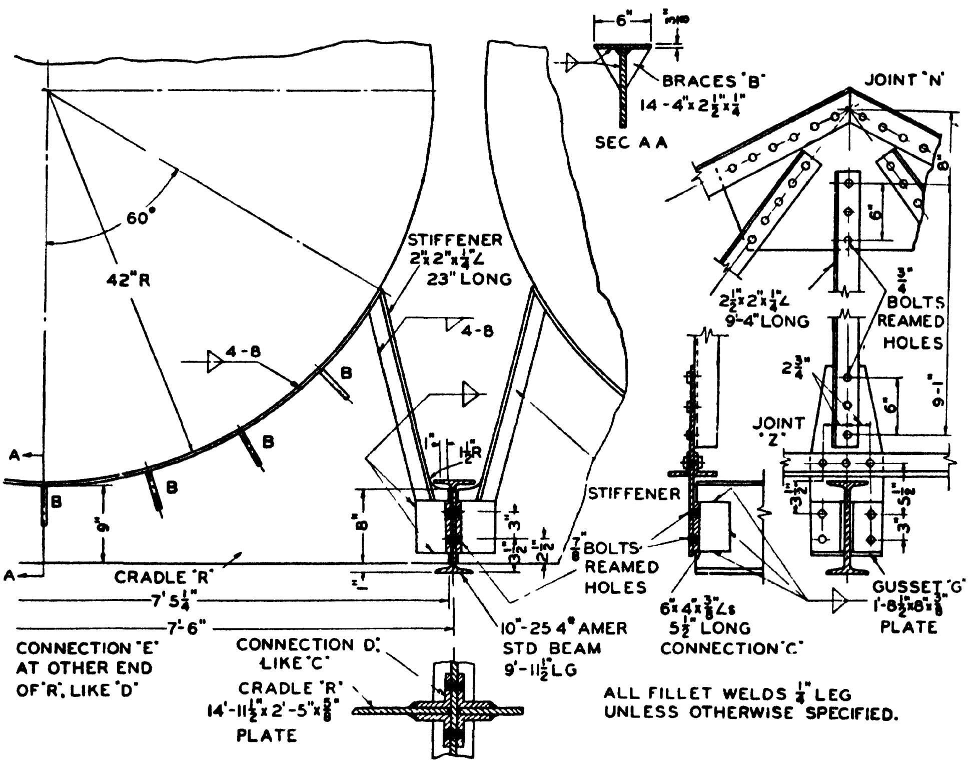


FIG. 8-48. Cradle Construction Details.

a tentative web thickness of ⅝ in. and an allowable flexural stress of 20,000 psi. is assumed, the required depth at the center of the cradle, from Eq. 5-8 and Fig. 5-12, will be

$$d = \sqrt{\frac{6M}{Sb}} = \sqrt{\frac{6 \times 145{,}300}{20{,}000 \times 0.625}} = 8.35 \text{ in.}$$

The central beam (supported at the panel points *Z* in adjacent trusses) carries the reactions of both sets of cradles, and is therefore designed as a simply supported beam 120 in. long, with concentrated loads of 10,750 lbs., plus the weight of the cradles, at points 30 in. from each reaction. The weight of one cradle may be estimated as follows: if a minimum depth of 9 in. is assumed at the center, then the average depth of the web, by scaling Fig. 8-48, may be taken as 14 in., and the volume of one web is 14 × 0.624 × 90, or

790 cu. in. The length of the flange is $(120°/360°)(\pi \times 84)$, or 88 in.; assuming a width of 6 in. and a thickness of ⅜ in. for the flange, its volume is $88 \times 0.375 \times 6$, or 198 cu. in. The weight of a single cradle is $(790 + 198)0.284$, or 280 lbs. The load at the point where the cradle is supported is equal to the weight of one tank and cradle, or $10{,}750 + 280$, or 11,030 lbs.; and this is also the magnitude of the end reaction at either end of the central beam. The bending moment is a maximum over the central 60-in. portion of the beam and is equal to $(11{,}030 \times 60) - (11{,}030 \times 30)$, or 330,900 in.-lbs. The required section modulus Z, from Eq. 5-9, is 330,900/20,000, or 16.56 in.^{3}

A reference to Table 7-2 indicates that an 8×4-in. American Standard I Beam with a section modulus of 68.1/4 or 17.03 in.^{3} will serve for this beam; an 8-in. depth, however, may not provide sufficient room for attaching the cradles. It may therefore be advisable to use the next size beam—a $10 \times 4\frac{3}{4}$-in. section—with a section modulus of 122.1/5 or 24.4 in.^{3}, and a weight per foot of 7.38×3.4, or 25.4 lbs. Another important factor in the selection of the latter section is the fact that this section appears on the AISC list of simplified shapes (NES Code).

The outer beams extending between joints L and joints W on adjacent trusses are to be supported by clip angles welded to the beams, and bolted to plates butt welded to the gusset plates at those joints. Although the bending moment on these beams is only one half the moment on the central beam, it is convenient to use the $10 \times 4\frac{3}{4}$-in. beam and thus eliminate the necessity of designing different cradle and connection details. Since the cradle beam design will probably require no major changes, the cradle design can now be continued.

From the ASME-UPV Code, the allowable stress in bearing between the vessel shell and the cradle flange should not exceed 40% of the allowable crushing strength of the shell material. The allowable crushing strength is usually 180% of the allowable tensile stress, or $1.8 \times 12{,}600$, or 22,700 psi.; 40% of 22,700 psi. gives 9080 psi. for the allowable bearing stress. If the horizontal projection of the flange length is equal to 72 in., and the width (as previously assumed) is 6 in., the total bearing area of one cradle flange is 72×6, or 432 sq. in. By comparison with the total load, it is found that the unit load per square inch of horizontal projected area is 10,750/432 or 25 psi., which is very small compared to the bearing strength of the material of the shell; this load is considerably smaller than the permissible bearing strength of the shell.

There is some possibility (fabrication, truss deflection, etc.) that the tank load may concentrate at the center of the cradle and cause a maximum moment at that point. As a result, the cradle flange may be required to resist some part of the bending stress and longitudinal or horizontal shear may develop at the juncture of the web and flange. This possibility may appear remote, but a check of the stresses will be of interest and will be in the interest of safety.

The location of the centroid g-g of the T-shaped cradle section is found by taking moments about axis x-x, Fig. 8-49; from Eq. 5-4,

$$h = \frac{(9 \times 0.625 \times 4.5) \times (6 \times 0.375 \times 9.188)}{(9 \times 0.625) + (6 \times 0.375)} = 5.87 \text{ in.}$$

The moment of inertia of the cradle section with respect to the centroidal axis g-g is found from Eq. 5-6,

$$I_{gg} = (0.625 \times 9^3/12) + (9 \times 0.625)(5.87 - 4.50)^2 + (6 \times 0.375^3/12)$$
$$+ (6 \times 0.375)(9.188 - 5.87)^2 = 73.38$$

With a reaction of 5375 lbs., the maximum moment (assuming the load concentrated at the center) is 5375×45, or 242,000 in.-lbs. The unit flexural stress, from Eq. 5-8, is $242{,}000 \times 5.87/73.38$, or 19,400 psi. This stress is slightly less than the allowable flexural stress of 20,000 psi., from Table 7-1. Thus, even if the entire tank weight is concentrated at the center of the cradle span, the design is adequate.

The longitudinal shear at the juncture of the flange and web is obtained from Eq. 5-14, where h is the distance from axis g-g to the centroid of the flange, or $9.188 - 5.87$, or 3.318 in.; b is the thickness of the web, or 0.625 in.; A is the area of the flange or 6×0.375,

or 2.25 sq. in.; V is the total vertical shear, or 5375 lbs.; and I is the moment of inertia of the gross section, or 73.38. The longitudinal shear is given by

$$S_h = \frac{5375}{73.38 \times 0.625} \times 3.318 \times 2.25 = 872 \text{ psi.}$$

The shear per lineal inch of length is 872/0.625 or 1395 lbs. If ¼-in. fillet welds are used at the juncture of the flange and web, the allowable load per inch of length for one ordinary-strength weld is 2000 lbs. (1000 lbs. per ⅛ in. of leg). Since there are welds on both sides, the allowable load per inch of length is 4000 lbs. If the welds are intermittent with 4-in. lengths, 8 in. on centers, the ¼-in. fillet welds will still be amply strong.

For end connections, two Series A angles for a 10-in. American Standard beam (Fig. 7-26) can be used. Allowing about ¼ in. clearance between the end of the cradle web and the web of the cradle beam, the available length of edge for fillet welding on one angle is equal to approximately 17 in., or 34 in. for both angles. The shearing strength of ¼-in. fillet welds is 2000 lbs. per lineal inch, and thus these welds are far stronger than required.

For the sake of a complete analysis with every possibility of failure considered, the cradle may be assumed to act as a fixed beam with a concentrated load at the center. The bending moment at the connection (Fig. 5-42) will be Wab^2/L^2. Quantities a and b are equal to $L/2$, and the moment becomes $WL^3/8L^2$, or $WL/8$. This gives a moment equal to $10{,}750 \times 90/8$, or 121,000 in.-lbs. On the assumption that this moment will be resisted by the moment of the fillet welds at the upper and lower edges of the connection angles, the allowable force that may be induced in one weld is 5.75×2000, or 11,500 lbs. Also, the resisting moment of the upper and lower welds of both angles is a couple whose moment is equal to the product of the resisting force and the distance between the centers of the welds, or $2[11{,}500(5.5 + 0.25)]$, or 132,000 in.-lbs. Since this value is greater than the moment of 121,000 in.-lbs., the connection is safe, especially so since the resistance of 132,000 in.-lbs. does not include the resistance of the vertical weld. It is also very unlikely that the end connection of the cradle will even approach a fixed-end condition.

Fig. 8-48 shows that four bolts are to be used to carry the end connections of two adjacent cradles. These will be ⅞-in. diameter bolts in reamed holes in the web of the cradle beam, and they will be subjected to double shear and to bearing in the cradle beam web. The strength of one bolt in double shear is $2(\pi \times 0.438^2 \times 15{,}000)$, or 18,040 lbs.; the bearing strength of one bolt is $40{,}000 \times 0.875 \times 0.31$, or 10,900 lbs. The bolts are weakest in bearing and the total bearing strength of four bolts is $4 \times 10{,}900$, or 43,600 lbs. This value is considerably higher than the sum of the reactions of both cradles, which was 11,030 lbs., and these bolts will be sufficiently strong.

The flange is supported by triangular braces welded in place, as shown in Fig. 8-48. The maximum depth of the cradle web is 29 in. at the end of the flange, and since the ratio of depth to thickness is 29/0.625, or 46.5 (which is less than 70), no stiffeners are required. It may be desirable, however, to introduce angle stiffeners at the ends as in Fig. 8-48. If a $2 \times 2 \times$ ¼-in. angle is welded to the cradle web, the effective length of the angle is 23 in.; the radius of gyration of the angle (Table 7-6) is 0.61, and the L/k ratio is 23/0.61, or 37.7. (It should be noted that the minimum value of k, about the angular axis z-z is 0.39; the presence of the web, however, permits buckling only in a plane perpendicular to the web.) From Table 7-1, the allowable load for the angle as a column is

$$SA = 17{,}000 - (0.485 \times 37.7^2 \times 0.94) = 15{,}300 \text{ lbs.}$$

This value is satisfactory, as the end reaction is only 5875 lbs. If 3/16-in. fillet welds are used to fasten the angle to the web, a total weld length of 5375/1500 or 4 in. will be required to transmit the reaction. Three 4-in. lengths of weld will be used, centered 8 in. apart as indicated.

8-21. Connection Design. The end connections of the central cradle beam ZZ' may be assumed tentatively to be like those employed for the cradle end connections. Because of the comparative flexibility of the supporting gussets, the beam will act as a simply supported beam; the end reactions are equal to the sum of the cradle load and the weight of one half of the beam or $11{,}030 + (25.4 \times 5)$, or 11,160 lbs. The four bolts are subjected to single shear and single bearing in the connection angles. The strength of a bolt in single shear is $\pi \times 15{,}000 \times 0.875^2/4$, or 9020 lbs.; in single bearing $32{,}000 \times 0.875 \times 0.375$, or 10,500 lbs.; the connection with four bolts is therefore amply safe.

The connection at Z is made by inserting a gusset plate between the two angles that comprise the member LW, and connecting joint Z with joint N by a single angle bolted to the gussets at each joint. The force acting on member NZ is 11,350 lbs., and the necessary net area to withstand the induced tensile stress is 11,350/20,000, or 0.568 sq. in. Using a $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angle with the long leg connected to the gusset, and $\frac{3}{4}$-in. bolts at each end of the angle, the effective net area in tension is equal to the net area of the connected leg plus one half of the net area of the unconnected leg. This gives $0.25(2.5 - 0.75) + (0.25 \times 1.75 \times 0.50)$, or 0.657 sq. in., which is ample. (Note that the diameter of $\frac{3}{4}$ in. is deducted—the usual value of $\frac{1}{8}$-in. clearance between a rivet and the hole is not used in this case, as a turned bolt is considered a reasonably good fit in the hole.) The shearing and bearing strengths of $\frac{3}{4}$-in. bolts are $\pi \times 15{,}000 \times 0.75^2/4$, or 6630 lbs., and $32{,}000 \times 0.75 \times 0.375$, or 9000 lbs., respectively. Two bolts at each end will be ample, but three bolts at each joint, as indicated in Fig. 8-48, will not involve any great additional expenditure, and will provide a further margin of safety.

The connection at points L and W is detailed in Fig. 8-50; the plate which supports the outer beam LL' or WW' is butt welded to the gusset at L and W. The allowable unit

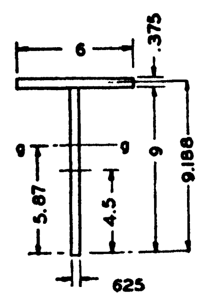


FIG. 8-49. Cradle Section.

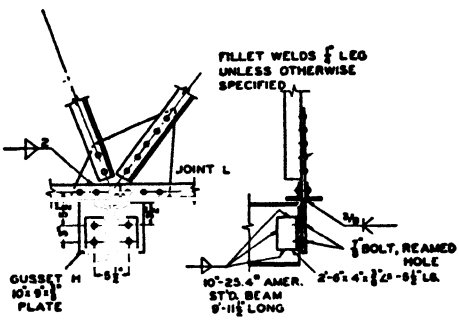


FIG. 8-50. Detail of Joint L with Welded Gusset Plate.

tensile stress in a butt weld is 11,300 psi. for average strength welds. The total load at this joint is equal to the reaction caused by one half of the tank and cradle weights and the weight of the cradle beam, or $11{,}030/2 + (25.0 \times 5)$, or 5640 lbs. The thickness of the plate is $\frac{3}{8}$ in., and the necessary length of weld, from Eq. 7-3, is $0.375 \times 5640/11{,}300$, or 1.87 in. The supporting plate is actually 10 in. long, welded along its entire length, to provide sufficient space for attaching the beam connection angles. The connection angles are made of the same size as those employed for the central beam, although the stresses in the welds and bolts are only one half as great.

8-22. Total Stress in Members. It is advisable to check the weight of the tanks and substructure. The weight of one filled tank is 21,500 lbs. and the weight of one cradle is 280 lbs. The cradle beams are approximately 10 ft. long and have a weight of 25.4 lbs. per ft. of length. The net weight of the substructure is $(4 \times 280) + (3 \times 10 \times 25.4)$, or 1882 lbs. If approximately 25% is added to compensate for connections, welds, and so forth, the weight of the substructure will be $21{,}500 + 2400/2$, or 22,700 lbs. The induced stresses in the members of the truss (based upon this value of P) are given in column 12, Fig. 8-36. The total stress in each member is given in column 13, and is equal to the sum of the maximum stress in column 7, Fig. 8-28, and the tank load stress of column 12, Fig. 8-36. Column 14 is the same as column 8, Fig. 8-28.

8-23. Investigation of Truss Joints. The load-carrying capacity of truss members is also limited by the capacity of riveted or welded joints at the extremities of the members. For members which extend from one gusset plate to another, the riveted joint should be designed to carry the full allowable load in the member; for continuous members which extend through a joint on one or both sides of a gusset plate, the riveted joint need only transmit the difference between the forces in the member on either side of the joint. The

AISC Code, however, does not permit less than two rivets to be used at any joint, no matter what the load. In the truss under consideration, the shearing strength of one rivet is $(2\pi \times 0.75^2/4)\,15{,}000$, or 13.25 kips, and the bearing strength, based upon a ⅜-in. gusset plate, is $0.75 \times 0.375 \times 40{,}000$, or 11.3 kips, for members composed of two angles. (The higher value of the unit bearing stress is employed because the rivet is in double shear.) Thus the bearing strength of the rivets will govern the load-carrying capacity of the joint.

The rivets in the majority of the joints all have excess capacity, as compared to the stress capacity of the members. For example, at joint *F*, the lower chord transmits a maximum force of 36,360 lbs. to the joint; since there are five rivets in *FH*, the load capacity of the joint is $11{,}300 \times 5$, or 56,500 lbs. Member *LZ* transmits a maximum force of 17,240 lbs. to joint *L*, whose rivet capacity is $11{,}300 \times 3$, or 33,900 lbs. Member *HL*, however, transmits 34,170 lbs. to joint *L*, for which the rivet capacity is 33,900 lbs. It may therefore be advisable to reinforce this joint by adding a 5⁄16-in. fillet weld at the upper edge of the angle on each side to take care of the excess load of 34,170 − 33,900, or 270 lbs. The capacity of a 5⁄16-in. weld is 1500 lbs. per inch of length, but 2 in. of weld length on each side may be added to provide ample security. The other joints will be found to be satisfactory upon investigation.

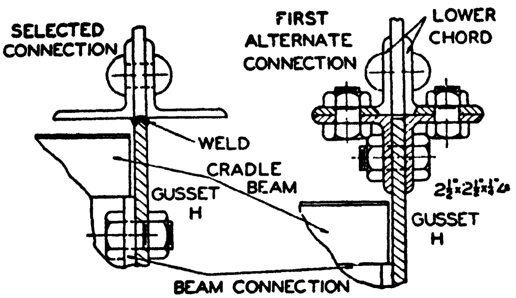


FIG. 8-51. Enlarged Views of Details for Joint L.

The necessary construction and erection details are shown in Figs. 8-48 and 8-50; ¼-in. fillet welds are specified in several locations where 5⁄16-in. welds were used in the computations; the 5⁄16-in. leg value was used because of the difficulty of obtaining a full fillet weld along a rounded angle edge.

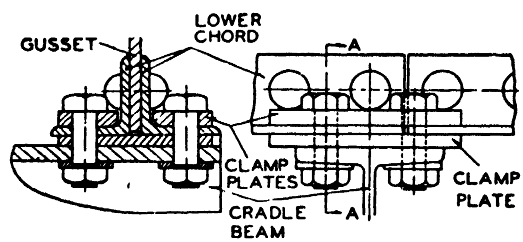


FIG. 8-52. Alternate Connection for Joint L.

Fig. 8-51, at the left, shows an enlarged detail of the welded plate at joints *L* and *W*. If this construction is not feasible, since overhead welding is often difficult, an alternate construction may be used (shown at the right) in which angles are bolted to the lower chord. This construction may require the cradle beams to be lowered slightly to provide clearance for the bolt heads. The connection angles may extend for any reasonable distance along the lower chord. Another construction is shown in Fig. 8-52, in which the cradle beam is bolted directly to the lower chord. The clamp plates are used to distribute the load and to serve as a splice for the lower chord.

PROBLEMS—CHAPTER 8

1. A roof truss for an industrial building has a span of 28 ft. 0 in. and a rise of 7 ft. 0 in. measured with respect to the gage lines of the angles composing the truss. The upper chord

has four panels and consists of two members *ABC* and *CDE*, each 15 ft. 8 in. long composed of two 4 × 3 × 5/16-in. angles, long legs back to back. The lower chord has three panels and consists of three members *AF*, *FG*, and *GE*. Members *AF* and *GE* are each 8 ft. 9 in. long, composed of two 3 × 2½ × ¼-in. angles, long legs back to back; member *FG* is 10 ft. 6 in. long and is a 3 × 2½ × ¼-in. angle with the long leg vertical. The web members consist of single angles 2½ × 2 × ¼ in. with the long leg vertical; members *BF* and *DG* are 3 ft. 11 in. long; members *CF* and *CG* are 8 ft. 9 in. long. The roof consists of 1 in. yellow pine sheathing with felt and asphalt covering and extends to the edges of the truss. The purlins are 4 × 3 × 5/16-in. angles extending entirely across the truss bays and are riveted to the upper chord. Single purlins are attached at joints *A*, *B*, *D*, and *E*, and two purlins at joint *C*. The purlin angles carry 4 × 3-in. oak railing strips. The truss ends rest on brick walls 1 ft. 0 in. wide; the baseplates at each end are 1 ft. 0 in. × 1 ft. ⅜ in. attached by two ⅞-in. anchor bolts in each plate. The plate at the left has circular holes, while the one at the right has slotted holes, 1 × 3 in., to permit expansion and contraction. At each end the plates extend 1½ in. beyond the span of the truss. The trusses are spaced 10 ft. 0 in. apart. The joints are made with ¾-in. diameter rivets in 13/16-in. holes in gusset plates ⅜-in. thick; all rivet pitches are 2½-in. except those at the joints, denoted by *, where 4½-in. pitches are used.

2. A horizontal cylindrical tank of maximum capacity is to be carried between two adjacent trusses by two cradles supported by two beams attached at joints *F* and *G*. Not more than 12 in. of the headroom of the truss can be used. There must be at least 6 in. clearance between the ends of the tank and the truss, and between the sides of the tank and the interior of the roof sheathing. The tank is made of ASME S-1 steel, has dished heads and a manhole in the side, and is to carry anhydrous ammonia at a 200-psi. pressure. The tank and all attachments are to be welded. The cradle shoe should bear along the stiffest portion of the tank. The supporting beams are to be bolted to the truss.

a. Make a layout of the truss, determine the stresses in the members, find the excess capacity of each, and determine the diameter of the tank.

b. Design the tank and make a fabrication drawing.

c. Design the substructure for the tank and make a fabrication drawing for construction and erection.

Joint	Member	Number of Rivets	Distance from First Rivet to Gage Line Intersection	Size of Gusset Plate Inches
A	*AB* *AF**	4 4	7 0	16 × 10
B	*ABC* *BF*	2 2	1¼ 3	9 × 6
C	*CB* *CD*	3 3**	3 3	17 × 12
D	*CDE* *DG*	2 2	1¼ 3	9 × 6
E	*EG** *ED*	4 4	0 7	16 × 10
F	*FB* *FC* *FA* *FG*	2 2 4 2**	4½ 5 2 2	17 × 9
G	*GE* *GC* *GD* *GF*	4 2** 2 2**	2 5 4½ 2	17 × 9

* 4½-in. rivet pitch.
**Field rivets

CHAPTER 9

PIPING

9-1. The transportation of fluids is one of the most important operations in chemical and mechanical engineering. Fluids require channels through which they may flow. In some cases, open channels may be used, but most fluids are transported through pipe or tubing. Pipe, tubing, and fittings for attachment purposes are available in ferrous, non-ferrous, and non-metallic materials.[54]

9-2. Cast Iron Pipe. Ferrous pipe is usually made of cast iron, wrought iron, or wrought steel. Cast iron pipe is used principally for underground lines conveying relatively non-corrosive liquids. It has an appreciably greater resistance to corrosion than ordinary iron pipe but is usually more expensive and is not so easily joined or connected as wrought iron and steel pipe. Cast iron pipe is available in 3 in. and larger inside diameters, and in lengths of 12 ft. The usual method of connecting sections is by the bell and spigot joint shown in Fig. 9-1. The detail dimensions of this joint are specified by the American Water Works Association. In assembly, the bottom of the space between the bell and spigot is calked with oakum, a compound of hemp fibers, and the remainder of the space is filled with molten lead; the latter is calked to fill the groove on the inside of the bell. A properly made joint will hold at pressures up to 100 psi. but is usually considered unsafe for other than very low pressures. It has the advantage that the pipe need not be perfectly aligned.

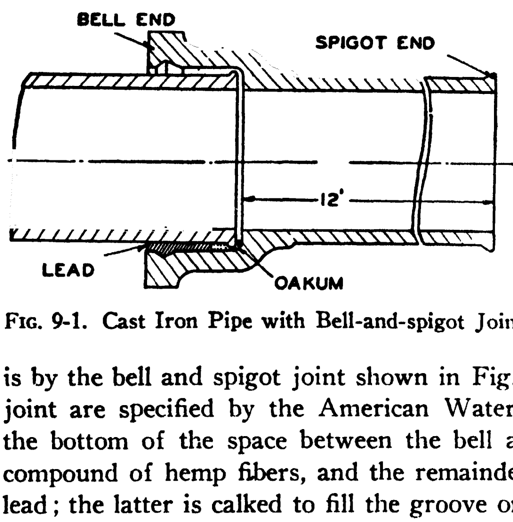


Fig. 9-1. Cast Iron Pipe with Bell-and-spigot Joint.

9-3. Steel and Wrought Iron Pipe and Tubing. Steel pipe is by far the most important and widely used piping material. Although special pipe of high-carbon and alloy steels is obtainable, most pipe of this character is made of mild steel. Steel pipe is manufactured in several standard wall thicknesses and is usually specified by its nominal inside diameter. Dimensional data on three weights of steel pipe are given in Table 9-1. The outer diameter of the pipe is constant for a given nominal size, and the increase in wall thickness is obtained at the expense of the actual inside diameter. This procedure has been adopted to insure interchangeability of threaded joints and fittings. The re-

sistance of steel pipe to corrosion may be increased by a protective coating of zinc; the pipe is then designated as "galvanized." The larger sizes of steel pipe are usually ungalvanized, or "black."

TABLE 9-1.—STANDARD IRON PIPE DATA

Nominal Inside Diameter In.	Actual Outside Diameter In.	Actual Inside Diameter, In. Schedule No. 40	80	160	Threads per Inch	Distance Pipe Enters In.
1/8	.405	.270	.205		27	3/16
1/4	.540	.364	.294		18	9/32
3/8	.675	.494	.421		18	19/64
1/2	.840	.623	.542	.244	14	3/8
3/4	1.05	.824	.736	.422	14	13/32
1	1.315	1.048	.951	.587	11 1/2	1/2
1 1/4	1.66	1.38	1.272	.885	11 1/2	35/64
1 1/2	1.9	1.61	1.494	1.088	11 1/2	9/16
2	2.375	2.067	1.933	1.491	11 1/2	37/64
2 1/2	2.875	2.468	2.315	1.755	8	7/8
3	3.5	3.067	2.892	2.284	8	15/16
3 1/2	4.0	3.548	3.358	2.716	8	1
4	4.5	4.026	3.818	3.136	8	1 1/16
4 1/2	5.0	4.508	4.28	3.564	8	1 7/64
5	5.563	5.045	4.813	4.063	8	1 5/32
6	6.625	6.065	5.751	4.875	8	1 1/4
7	7.625	7.023	6.625	5.875	8	1 3/8
8	8.625	7.982	7.625	6.875	8	1 7/16
9	9.625	8.937	8.625		8	1 9/16
10	10.75	10.019	9.75		8	1 11/16

The three weights listed in Table 9-1 were originally referred to as standard, extra-heavy, and double-extra-heavy pipe. The American Standards Association has recently adopted a nomenclature in which so-called "Schedule Numbers" are used, and correspond to the following approximation:

$$\text{Schedule Number} = 1000\, p/S \qquad (9\text{-}1)$$

where p represents the gage pressure and S the allowable working stress, psi. In addition to the Schedule Numbers listed in Table 9-1, Schedule 10, 20, 30, 60, 100, 120, and 140 pipe are also available in a limited range of sizes.

Above 12-in. nominal size, pipe dimensions and specifications are based upon the outer diameter and the wall thickness. In this range, pipe is usually referred to as OD pipe and is available in a wide range of diameters and wall thicknesses. Eighteen-inch OD pipe, for example, is available as Schedule 10 (¼-in. wall thickness), Schedule 20 (5/16-in. wall thickness), Schedule 30 (7/16-in. wall thickness) and so forth.

Steel pipe is usually made of strip steel, rolled and welded along the longitudinal seam. Steel pipe under 1½-in. nominal size is usually butt welded; larger sizes are lap welded. The standard length of pipe sections is 21 ft., threaded at both ends.

9-4. Wrought Iron Pipe. Wrought iron pipe is obtainable in the same sizes as steel pipe; it is more expensive, but is far more resistant to corrosion. A superficial visual examination will not suffice to distinguish between wrought iron and steel, and since the latter is often referred to as "wrought iron" by suppliers, the specification "genuine wrought iron" is necessary whenever this type is required.

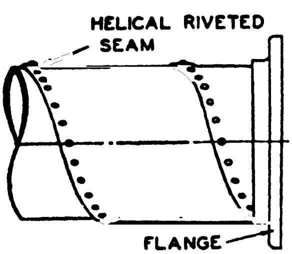


FIG. 9-2. Spiral-riveted Light-wall Flanged Pipe.

9-5. Light-wall Pipe. Fabricated pipe of large diameter and small wall thickness is referred to as light wall pipe, and usually has a helical riveted or welded seam, as shown in Fig. 9-2. This pipe is often, although incorrectly, referred to as spiral-riveted or spiral-welded pipe. The former is obtainable in inner diameters from 3 to 42 in., with wall thicknesses varying from No. 16 to No. 6 USSG (United States Standard Gage for Plates and Sheets). Spiral-welded pipe may be obtained in diameters from 4 to 30 in., in a range of wall thicknesses from No. 14 to No. 8 USSG. Pipe sections with attached bolting flanges are carried in supplier's stocks in 20-, 30-, and 40-ft. lengths.

9-6. Tubing. Tubing is differentiated from pipe by the fact that, for a given diameter, tubing wall thickness is usually less than pipe wall thickness. All sizes of tubing are specified by the actual outside diameter and the wall thickness. Seamless steel tubing may be obtained in outer diameters from ¼ to 1½ in. by 1/16-in. increments; from 1⅝ to 4 in. by ⅛-in. increments; and from 4¼ to 9 in. by ¼-in. increments. The wall thickness is usually expressed in terms of the Birmingham Wire Gage (BWG) in gage sizes from No. 24 to No. 0000; wall thicknesses from 5/32 to ⅜ in. by 1/32-in. increments, and from 7/16 to 1 in. by 1/16-in. increments are also available. Spiral-welded tubing can also be obtained in almost any size, to diameters as small as 0.025 in. Data on BWG and USSG sizes may be found in Table 9-2.

TABLE 9-2.—GAGE SIZES FOR TUBING AND PLATES

Gage No.	Birmingham Wire Gage BWG Size, In.	U. S. Std. Gage for Plates and Sheets USSG Size, In.
0000	0.454	0.4063
000	0.425	0.3750
00	0.380	0.3438
0	0.340	0.3125
1	0.300	0.2813
2	0.284	0.2656
3	0.259	0.2500
4	0.238	0.2344
5	0.220	0.2188
6	0.203	0.2031
7	0.180	0.1875
8	0.165	0.1719
9	0.148	0.1563
10	0.134	0.1406
11	0.120	0.1250
12	0.109	0.1094
13	0.095	0.0938
14	0.083	0.0781
15	0.072	0.0703
16	0.065	0.0625
17	0.058	0.0563
18	0.049	0.0500
19	0.042	0.0438
20	0.035	0.0375
21	0.032	0.0344
22	0.028	0.0313
23	0.025	0.0281
24	0.022	0.0250

9-7. Pipe Connections and Fittings. The function of a pipe connection is to provide a fluid-tight or pressure-tight joint. A pipe fitting is a pipe connection that will permit necessary disassembly and re-assembly for purposes of installation, maintenance, cleaning, or repair. Pipe connections are used for joining two or more sections of pipe, for effecting changes in the diameter or direction of the pipe line, or for closing the ends of the line. The three important methods of connecting pipe are screwed joints, welded joints, and flanged joints. Screwed joints have been standardized for pipe sizes up to 12 in., although screwed joints over 3-in. nominal size are rarely used for field connections because of the difficulty of cutting pipe threads by manually operated dies, and because of the difficulty of handling the larger sizes of pipe wrenches. Screwed joints are made with standard pipe threads as shown in Fig. 9-3, and differ from machine screw threads in that the threads are cut on a conical instead of a cylindrical pitch surface. With this type of thread, the joint becomes tighter as the pipe is screwed into the fitting.

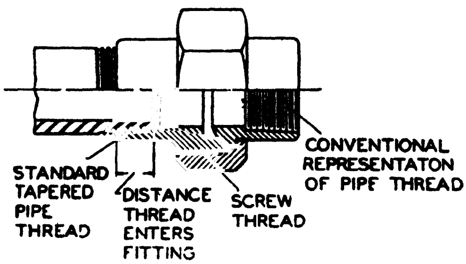


FIG. 9-3. Screwed Union.

Two sections of pipe in axial alignment may be joined by a coupling, which is a short sleeve internally threaded at each end. One coupling is usually furnished with each length of pipe. The use of couplings for pipe sizes over 2 in. is not considered good practice, but they are often used for shop joints in larger sizes. Since both ends of a coupling have right-hand threads, it cannot be used as the final connector in a closed line, and some form of union (Fig. 9-3) is employed for this purpose. The contact surfaces of a union are usually of mating spherical form, to permit a tight joint even though there may be a slight misalignment of the two pipes. In some cases, one of the halves of the unit is faced with a brass ring; in other instances, some form of packing is used.

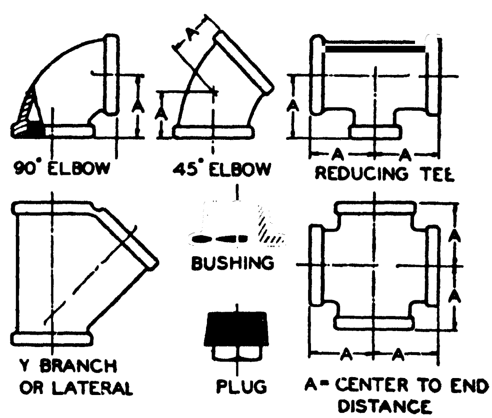


FIG. 9-4. Screwed Pipe Fittings.

For changes in the direction of a pipe line, 90° or 45° elbows are used, Fig. 9-4. Tees, crosses, and Y branches or laterals are used for branch connections. When openings in a tee or Y differ, it is usually referred to as a reducing

fitting. In these fittings the "run" is specified first, followed by the "outlet," i.e., a $2 \times 2 \times 1\frac{1}{2}$-in. tee, or a $2 \times 1\frac{1}{2} \times 1$-in. Y. A reducer may be used to change the size of pipe in a straight run but the most common method of effecting such changes is to use internally and externally threaded bushings, shown in Fig. 9-4. Plugs and caps are used to close the end of a line.

Nipples are short pieces of pipe threaded at both ends. The term is usually applied to lengths so short they cannot easily be threaded in an ordinary vise with hand tools, and are purchased in finished form. A close nipple is one so short the threads meet at the center; a short nipple is one with a straight section about ¼ in. long between threads; longer nipples are designated by overall length.

9-8. Rail Fittings. Globular or rail fittings are shown in Fig. 9-5. Pipe structures made from these fittings are often less expensive and more convenient to install than those made of fabricated or rolled structural members, such as angles and channels. The fittings are deeply and loosely threaded, so that assembly may be effected without the use of screwed unions. Rail

FIG. 9-5. Rail Fittings.

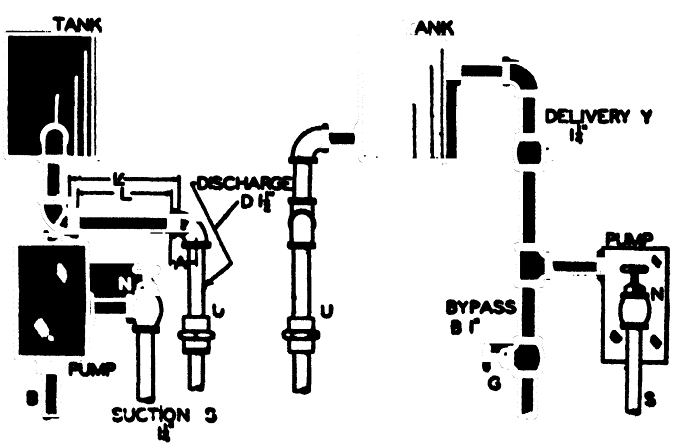


FIG. 9-6. Double-line Orthographic Representation of Piping System.

fittings are not to be used for fluid flow, but are extensively employed for stair railings, guard rails, and as legs or supports for benches and shelving.

9-9. Pipe Representation. Pipe systems are usually represented by double-line orthographic conventions, as shown in Fig. 9-6, or by single-line isometric conventions, as shown in Fig. 9-7. In the system shown (both figures), fluid is drawn by a pump through a 1¼-in. suction line equipped with an angle valve *N*, and forced into a tank through a 1¼-in. delivery line fitted with a check valve *C* to prevent return flow. The fluid flows from the tank through a 1½-in. discharge line. The globe valve *G* in the by-pass line is used as an auxiliary flow control. Complete conventional representation may be obtained from any standard text in engineering drawing. Threaded pipe fittings are available in 125- and 250-lb. (psi.) cast iron (C.I.), and 150-lb. malleable iron (M.I.) types. Screwed fittings of cast or forged steel, for water, oil and gas service, may be obtained in pressure capacities up to 6000 psi. Fitting dimensions required for installation are given in Table 9-3.

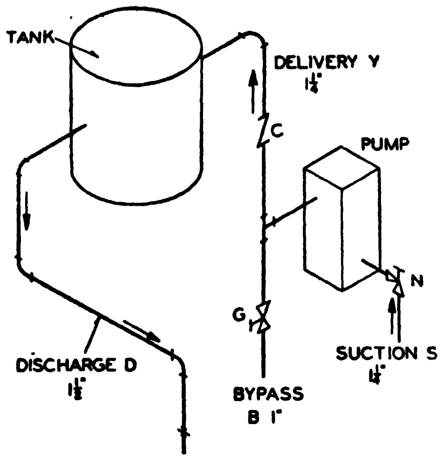


Fig. 9-7. Single-line Isometric Representation of Piping System.

9-10. Welded and Flanged Fittings. Welded fittings are coming into extensive use in piping systems. Ninety-degree elbows are available in either long radius or short radius types, as shown in Fig. 9-8, where *D* represents the *nominal* inside diameter of the pipe. Fittings are stocked in sizes from ¾ to 12 in., in Schedule 40, 80, and 160 weights. Welding type fittings may be beveled for welding, as shown in the elbows in Fig. 9-8, or a self-centering type of fitting can be used, as in the tee at the right. The centering ring or ridge

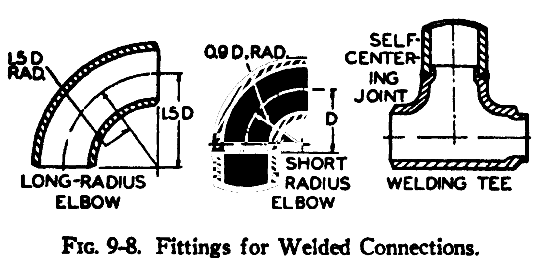


Fig. 9-8. Fittings for Welded Connections.

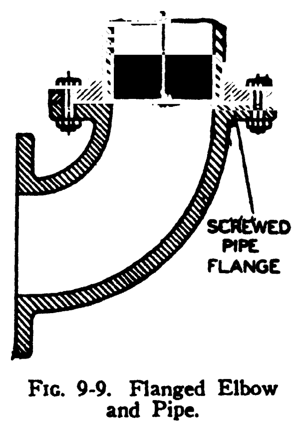


Fig. 9-9. Flanged Elbow and Pipe.

shown on the latter fitting facilitates alignment of the pipe line and the fittings, and often permits welding without the use of clamps or supports. Welding reducers, 45° elbows, long and short radius 180° return bends (mean bend radii of $3D$ and $2D$) are also available.

Table 9-3.—Center to End Dimensions of Pipe Elbows, Tees and Crosses, Inches. (See Fig. 9-4.)

Size	Screwed Fittings		Flanged Fittings				Welded
	125-lb. C.I. 150-lb. M.I. Screwed		150-lb.	150-lb. 300-lb.	300-lb.	400-lb.	
	90° Elbow, Tee and Cross	45° Elbow	90° Short Radius Elbow, Tee and Cross	90° Long Radius Elbow	90° Short Radius Elbow, Tee and Cross	90° Short Radius Elbow, Tee and Cross	Tees, Schedule 40, 80, 160
½	1.12	0.88					
¾	1.31	0.98					1⅛
1	1.50	1.12	3½	5	4	4	1½
1½	1.94	1.43	4	6	4½	4½	2¼
2	2.25	1.68	4½	6½	5	5½	2½
3	3.08	2.17	5½	7¾	6	6¾	3⅜
4	3.79	2.61	6½	9	7	7¾	4⅛
5	4.50	3.05	7½	10¼	8	8¾	4⅞
6	5.13	3.46	8	11½	8½	9½	5⅝
8	6.56	4.28	9	14	10	11½	7
10	8.08	5.16	11	16½	11½	13	8½
12	9.50	5.97	12	19	13	14¾	10
14			14	21½	15	16	11
16			15	24	16½	17½	12
18			16½	26½	18	19	

Flanged fittings are used for connecting flanged pipe and are described in Chapter 10. A 90° flanged elbow is shown in Fig. 9-9; 45° elbows, tees, crosses and laterals are also available in pressure ratings from 150 to 2500 psi., and in sizes from ½ to 24 in. Fitting dimensions required for installation are given in Table 9-3.

9-11. Valves. Valves are used to control fluid flow in pipe systems. It is so difficult to build satisfactory valves for many purposes that they have often been defined facetiously as "localized leaks." Valves may be used for

three purposes: to open or close a line completely, to control the rate of flow, or to serve as automatic or semi-automatic safety devices. Plug cocks and gate valves, Figs. 9-10 and 9-11, should be used only to open or close a line, and not for throttling or control service. Cocks are universally used on small lines for compressed air, rarely for steam or water. If the taper of the plug is too small, the plug tends to wedge in the body, making turning difficult; if the taper is too great, the line pressure tends to lift the plug out of its seat. To eliminate the former fault, which is most frequently encountered, special designs of cocks are available, in which lubricant is forced by a screw through the stem of the cock and grooves in the plug, thereby lifting and sealing the plug by hydraulic pressure. For ordinary service such construction is satisfactory, but in many cases it is difficult to obtain greases or other lubricants that will resist dissolution. Since the opening in the plug of the cock is approximately rectangular, the area of the flow opening changes very rapidly with a slight amount of rotation when the cock is barely opened, but has practically no change when the cock is almost at full opening.

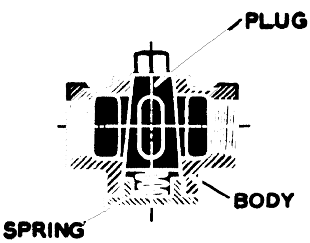


FIG. 9-10. Plug Cock.

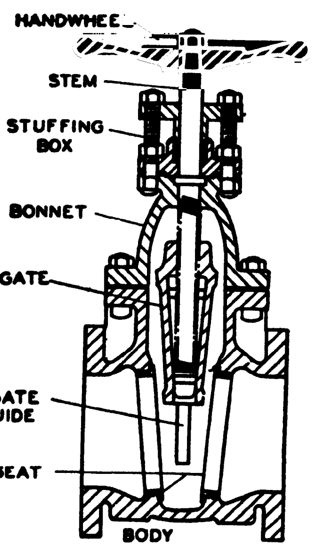


FIG. 9-11. Non-rising Stem Gate Valve with Outside Yoke and Bonnet.

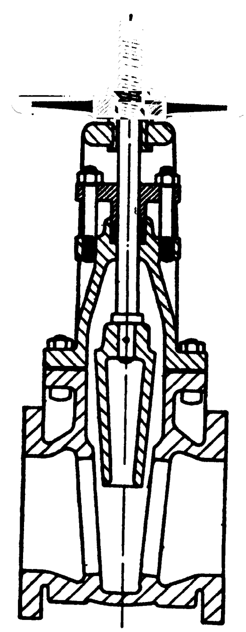


FIG. 9-12. Rising Stem Gate Valve.

Gate valves, Fig. 9-11, are used in the larger sizes of pipe lines, and are obtainable either in the stationary or non-rising stem type, or in the rising-stem

construction shown in Fig. 9-12. The latter has the advantage that a glance will indicate whether the valve is opened or closed. The non-rising stem construction, however, requires less overall clearance when the valve is open, and can be operated without any axial motion of the stem in the stuffing box. The pressure of the fluid on the gate may cause difficulty in opening and closing large valves by hand, and large valves are often equipped with small by-pass valves to permit pressure equalization on both sides of the gate before the main valve is opened. *QO*, or quick-opening, gate valves are opened and closed by a lever in a single operation, which permits more rapid operation than threaded-stem construction, but there is the disadvantage that a "water hammer" may result. For this reason, *QO* valves and cocks should not be used on comparatively short lines if the valve is to be opened and closed quickly.

Globe, angle, and needle valves are used for throttling or control service. The globe

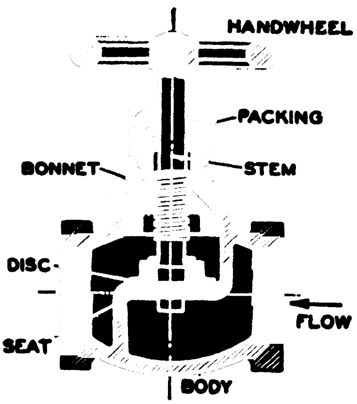
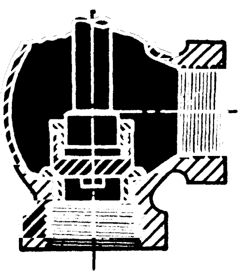
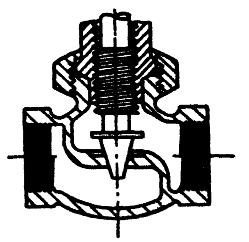


FIG. 9-13. Globe Valve with Screw-type Bonnet.


FIG. 9-14. Angle Valve with Inserted Seat.


FIG. 9-15. Needle Valve.

valve of Fig. 9-13 has a globular body with a horizontal internal partition having a circular passage into which a ring (the seat) is inserted. Inexpensive globe valves have no separable seat ring; better valves have this feature for ease in removal. Angle valves, Fig. 9-14, are essentially globe valves with connection openings at 90° instead of 180°. Valve disks for globe and angle valves may be of metal or of plastic composition. Needle valves, Fig. 9-15, are generally used for high pressure throttling service in the smaller sizes of pipe, and very accurate control can be obtained. Y-type valves are modified globe valves similar in appearance to a Y branch or lateral; they permit a more direct line of flow than globe valves, and their seats are more accessible for replacement.

Diaphragm or "pinch-type" valves, shown in Fig. 9-16, are used to handle corrosive substances, and are obtainable with a wide variety of linings. Closure is effected by the pressure of the valve stem *V* against the soft rubber diaphragm. The valve functions without lubrication and with low resistance to flow. The only maintenance required is occasional replacement of the diaphragm. The

usual diaphragm material is a natural or synthetic rubber composition, although other flexible plastics are available.

Check and safety valves are used to insure uni-directional flow or to guard against excessive pressures and pressure fluctuations. Two types of check valves are shown in Fig. 9-17; the type at the left is a ball check, that at the right a swing check valve. In each illustration the direction of flow is indicated by the arrow; a reversal of flow causes automatic operation of the check. Ordinarily, check valves are not absolutely tight, but will prevent the return of any large quantity of fluid. Safety valves are devices in which a valve stem is held against a seat by a spring, which may be adjusted to permit the valve to open at a predetermined pressure. Safety valves are usually mandatory for vessels containing steam or any substance being vaporized.

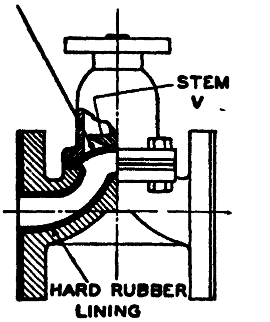
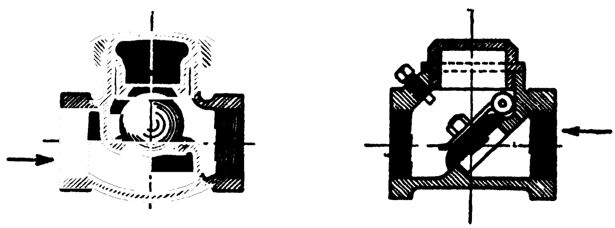
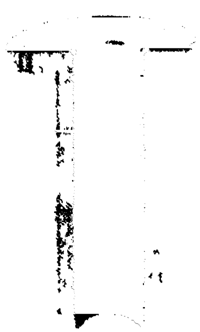


FIG. 9-16. Diaphragm Valve.


FIG. 9-17. Ball and Swing Check Valves.

9-12. Valve Application. The nature of the fluid handled is a determining factor in selection of the valve material, and often upon the type of valve used. Fluids containing foreign bodies which tend to clog or impede the flow through a restricted orifice may necessitate the use of gate valves for throttling service. In such cases, however, gate valves can operate satisfactorily only with the stem in a horizontal position. With the stem down, the bonnet may fill with sediment; with the stem up, the groove at the bottom tends to fill, preventing full closure. In some applications, for fluids which have a high corrosive effect on metals, ceramic or glass cocks are practically mandatory, although other types of valves give more effective control. Lined valves, Fig. 9-16, are often used for acid lines and are generally used in conjunction with lined pipe, an example of which is shown in Fig. 9-18. (Steel pipe with concrete or plastic lining is also obtainable.) Valves for pipe sizes 2 in. and under are usually made of brass. Larger sizes have iron bodies and bonnets, with seats, valve disks, and stems of brass or bronze. For service with fluids that affect


FIG. 9-18. Rubber-lined Pipe. (Courtesy of the American Hard Rubber Co.)

brass, such as ammoniacal or cyanide solutions, all-iron valves are obtainable, but are not satisfactory for general service because of the rusting of the contact surfaces. Monel metal fitted valves are used for superheated steam at high pressures; for very high pressures and temperatures, high-grade bronze, cast steel, or, in exceptional cases, drop-forged steel bodies and other parts are used.

9-13. Piping System Layout. There are certain general principles that should be adhered to in the design of piping systems and the selection and applications of fittings. Wherever possible pipe lines should be run in straight lines with right-angled turns. It is undesirable to run pipe lines at odd angles even though it shortens the line and reduces the number of fittings. When a pipe makes a right-angled turn, it is good practice to use a tee with one opening closed by a pipe plug or cap, or a cross with two closed openings, rather than an elbow. This will permit subsequent connections without breaking a number of joints, and will facilitate cleaning. Long lines of piping connected by threaded fittings often have unions at frequent intervals, so as to limit the quantity of pipe that must be disassembled for cleaning or repair. Flanged fittings have the advantage that any fitting serves the same purpose as a union. For connections that are not feasible with right-angled turns, the use of two 45° elbows will provide the necessary variation for most cases.

For screwed fittings, pipe should be carefully cut to the proper length and the interior edge burred or reamed to prevent constriction and sharp edges. In specifying the pipe length, consideration should be given to the distance that the pipe thread enters the fitting, see Fig. 9-6. The specified length should be L', which is equal to the distance L between the faces of the fittings plus twice the length of engagement of the thread (given in Table 9-1). Special fabricated fittings, such as expansion bends or coiled piping, particularly those with integral flanges, must be carefully specified so that they will fit properly. Two examples of such specification are shown in Fig. 9-28.

In all piping systems, valves should be installed in such a manner that they may be repacked, if necessary, without closing the line. In Fig. 9-13, a globe valve is shown in proper position with respect to the flow, and repacking under flow is possible; if the direction of flow were reversed the valve could not be packed. The gate valve of Fig. 9-11 offers no difficulty in this regard since it may be repacked when closed regardless of the direction of flow. Valves should not be used to brace a pipe, or carry the weight of the line, since distortion of the valve may result in inefficient operation and require an excessive amount of maintenance.

Whenever lines are closed down for repair or cleaning, great care should be taken to insure that the final closure valves are tight. In flanged piping. particularly in high pressure steam lines, a blank flange is usually inserted at the joint adjacent to repair or cleaning take-down, to provide positive assurance against valve failure. Sometimes a special blank flange, as shown in Fig. 9-19, may be used. For service when the line is open, the flange is inserted so that

the central hole is in alignment with the pipe axis; for closing off the line, the blank half is used. The use of this device has the advantage that a space is always available between the mating flanges of the fittings, the blind flange is always at hand, and the workman can tell at a glance whether or not the line is closed.

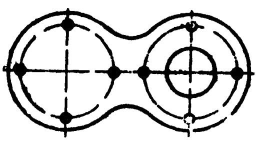


FIG. 9-19. Blank Flange for Temporary Line Closure.

9-14. Tube Fastening and Attachment. Seamless tubing is widely used in heat exchangers, boilers, and as piping. Tubing may be attached to tube sheets or headers in many ways, a few of which are shown in Fig. 9-20. The detail at *A* shows a tube which is expanded and beaded. Construction *B* shows a tube end which is expanded into grooves in the tube sheet. *C* shows a tube expanded beyond the tube sheet. Tubes may be flared, or flared and welded, as shown at *D*. Detail *E* shows an expanded and beaded tube lodged in a ferrule to give added tightness and flexibility. The construction at *F* shows a tube end provided with an interior ferrule to protect it against erosion or corrosion; a gasket is used to prevent leakage, and is held

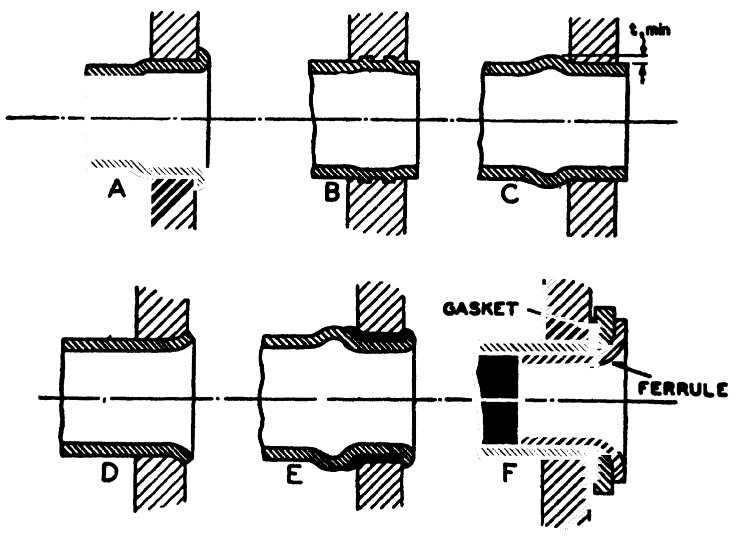


FIG. 9-20. Tube End Fastening Methods.

in place by a washer, against which the flange of the ferrule is forced. Properly spaced beaded or welded tubes have considerable holding capacity, and may eliminate the necessity for stays in the tube zone.

Tubing employed in flanged pipe lines may be threaded and screwed into a flange, and may also be brazed in place by an electric-furnace process as shown at the right in Fig. 9-21. The latter method will permit a considerable

reduction in the tube wall weight, as the thickness of the wall in a screwed fitting must be based upon the actual wall thickness at the root of the threads. The brazing is effected by forcing the flange on the tube, placing a copper-alloy wire ring in a counterbored seat, and subjecting the assembled unit to heat; the alloy ring melts and flows into the joint under the influence of capillary action. This method is usually employed for units produced in large quantities.

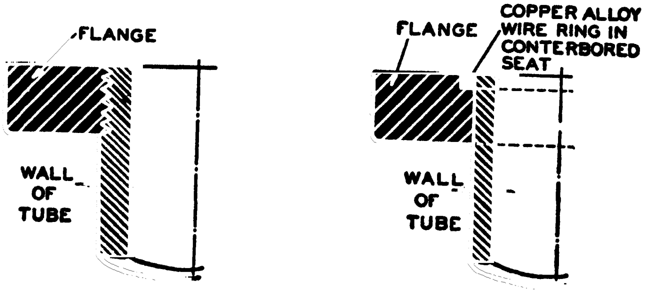


FIG. 9-21. Flanged Attachments for Tubing.

9-15. Non-ferrous Tube Fittings. Fig. 9-22 shows a fitting used for soft copper pipe or tubing; in other cases, a ball-shaped end is formed on the tubing, and a connection made by using a nut similar to that shown in Fig. 9-22. Solder fittings, illustrated in Fig. 9-23, utilize the phenomenon of capillary action for insuring penetration of molten solder to the entire contact surface. This illustration shows two copper tubes connected by a tubular coupling that

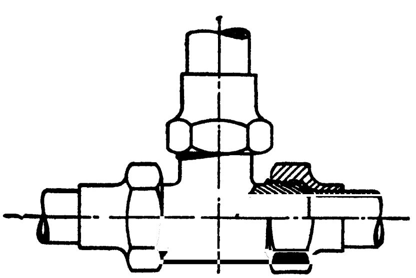


FIG. 9-22. Tee Connection for Flared Tubing.

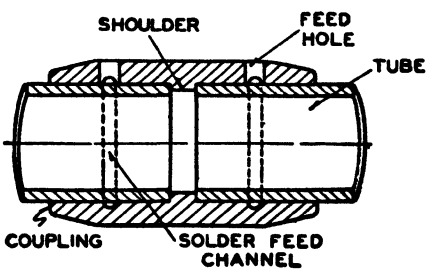


FIG. 9-23. Solder Fitting Coupling.

has a cylindrical bore with an inside shoulder against which the tube ends fit. Each half of the coupling has an annular groove or solder feed channel, with a solder feed hole entering the groove. The coupling is heated with a blow torch or gas torch, and solder wire is fed through the solder feed hole and melts as it comes into contact with the coupling. The liquefied solder is carried around the entire contact surface by capillary action. This type of fitting is extensively used for non-ferrous pipe and tube connection, and is available in

a wide variety of sizes, and in elbow, tee, cross, and coupling form. Valves and other accessories with solder fitting ends can also be obtained.

9-16. Ducts and Elbows. Sheet-metal ducts for exhaust air, dust collection, and other purposes are usually made of black or galvanized iron, with soldered or riveted seams. Ducts can be of any cross section, but are usually made square, rectangular, or circular. An elbow for connecting two cylindrical ducts is shown in Fig. 9-24; an economical method of development, and the usual nomenclature, is also shown. A detailed presentation of other fittings and their development may be found in any standard text in engineering drawing.

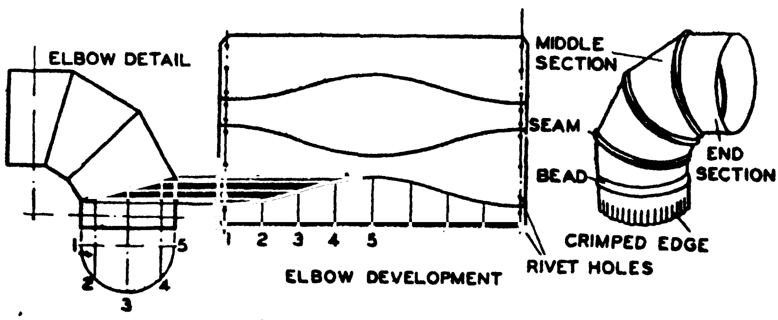


FIG. 9-24. Sheet Metal Elbow and Development.

9-17. Pipe and Tube Design Data. The maximum allowable internal unit working pressure p for ferrous tubes and for ferrous pipe used as tubes, in accordance with the specifications of the ASME-UPV Code, is given by

$$p = \frac{2.3tS}{D} - \frac{S}{30} \tag{9-2}$$

where S is the allowable stress, psi., from Table 9-4, t is the minimum wall thickness, and D the actual outer diameter of the pipe, in inches.

For non-ferrous tubes and pipes, p is given by

$$p = \frac{2tS}{D} \tag{9-3}$$

where S is obtained from Table 9-5.

The foregoing expressions are applicable only to outer diameters between ½ and 6 in., and for wall thicknesses not less than 0.049 in. Additional wall thickness must be provided when corrosion or wear due to cleaning operations is anticipated; when tube ends are threaded, an additional wall thickness of 0.8 divided by the number of threads per inch must be provided. If tubes are rolled into headers as indicated in Fig. 9-20B, the wall thickness may have to be increased to compensate for the reduction due to rolling. Tube design for external pressure is treated in Chapter 18.

TABLE 9-4.—ALLOWABLE STRESSES, PSI., FOR FERROUS MATERIALS FOR PIPES AND TUBES. ASME-UPV CODE

Spec. No.	Grade	ASTM Spec.	Weld	For Temperatures Not Exceeding Degrees F.						
				650	700	750	800	850	900	950
S-17	Steel	A83-38T	Lap	7300	7000	6650				
S-17	Steel	A83-38T	Seamless	9400	9000	8150	7150	5850	4400	2600
S-17	Wrought iron	A83	Lap	5600	5300	4800				
S-18	Steel	A53-36	Lap	7300	7000	6650				
S-18	Steel	A53-36	Butt	5400	5300	5050				
S-18	Steel	A53-36	Seamless	9600	9100	8250	7250	5850	4400	2600
S-19	Wrought iron	A72-39	Lap	5600	5300	4800				
S-19	Wrought iron	A72-39	Butt	4800	4600	4150				
S-32	A, Silicon 0.10%	A178-37	Resistance	8000	7650	7300	6700	5800	4750	3250
S-32	A	A178-37	Resistance	8000	7650	6950	6100	4950	3750	2200
S-32	B	A178-37	Resistance	6800	6500	5850				
S-32	C, Silicon 0.10%	A178-37	Resistance	10,200	9700	8850	7750	6300	4750	3250
S-32	C	A178-37	Resistance	10,200	9700	8450	7050	5400	3750	2200
S-34	P3a	A158-38T	Seamless	12,000	12,000	12,000	11,800	11,200	10,000	8000
S-40	A	A192-38T	Seamless	9400	9000	8600	7900	6800	5600	3800
S-45	P1	A206-39T	Seamless	11,000	11,000	11,000	10,700	10,500	10,000	8000
S-49		A210-38T	Seamless	12,000	11,400	10,400	9100	7400	5600	3800
S-48	T1	A209-38T	Seamless	11,000	11,000	11,000	10,750	10,500	10,000	8000
S-48	T1a	A209-38T	Seamless	12,000	12,000	12,000	11,500	11,000	10,000	8000

NOTE: Allowable stresses for API-ASME may be taken as 25% greater than the above.

TABLE 9-5.—ALLOWABLE STRESSES, PSI., FOR NON-FERROUS MATERIALS FOR PIPES AND TUBES. ASME-UPV CODE

Material	Spec. No.	For Metal Temperatures Not Exceeding Degrees F.								
		Subzero to 150	250	300	350	400	450	500	550	600
Muntz metal	S-24 S-47 S-59	10,000	9000	5500	2000	1500				
Red brass, high brass	S-24	7000	6500	5750	5000	3000	1000	800		
Admiralty	S-24 S-47	7000	6500	6250	6000	5500	4500			
Naval brass	*	11,000	10,000	10,000	6500	3000				
Steam bronze	S-41	7000	7000	6500	6000	5500	5000	4000	3000	
Steam bronze	S-46	6000	5500	5000	4500	3500				
Monel metal†	S-54	14,000	14,000	14,000	14,000	14,000	14,000	14,000	14,000	14,000
Cupro-nickel 70-30†	S-47	11,000	11,000	11,000	11,000	11,000	11,000	10,000	10,000	9000
Cupro-nickel 80-20†	S-47	10,000	10,000	10,000	10,000	10,000	10,000	9000	9000	8000
Copper, annealed, all types	S-20 S-22 S-23 S-47 S-66	6000	5000	4750	4500	4000				
Aluminum manganese alloy, annealed	S-39	2800	2400	2100	1800	1600				
Aluminum manganese alloy, quarter-hard or as rolled	S-39	3500	3000	2700	2400	2200				

* U. S. Navy Dept. Spec. 46B-6-j.
† Maximum permissible temperature Monel metal and cupro-nickel 750° F.

9-18. Design of Steam Piping. The ASME-PB Code specification for the minimum thickness t for steam piping is given by

$$t = \frac{pD}{2S} + C \tag{9-4}$$

where p is the maximum internal service pressure, psi., at the operating metal temperature, D the actual outer diameter of the pipe in inches, S the allowable stress from Table 9-4, and C an allowance for threading, mechanical strength, and corrosion. For ⅜-in. and smaller threaded pipe, and for plain end pipe or tubing 1-in. nominal size and smaller, C is equal to 0.05; for ½-in. and larger threaded pipe, C is equal to 0.80 divided by the number of threads per inch; and for plain end pipe over 1-in. nominal size, C is equal to 0.065. These values apply only to steel or wrought iron pipe whose nominal size is 4 in. or less. For pipe in excess of 4 in., the thickness is given by

$$t = \frac{D}{2}\left(1 - \sqrt{\frac{S-p}{S+p}}\right) + C \tag{9-5}$$

Steel or wrought iron pipe lighter than Schedule 40 shall not be threaded. For steam pressures greater than 250 psi., and for water pressures and temperatures greater than 100 psi. and 220° F., seamless pipe of a quality equivalent to S-17 or S-18 (ASME Material Specification, Table 9-4), and of a weight equivalent to Schedule 80 is a minimum requirement.

9-19. Design of Oil Piping. Oil piping designed in accordance with the provisions of the API-ASME Code may be specified by using Eqs. 9-4 and 9-5, but the values of the allowable unit stress S, as obtained from Table 9-4, should be multiplied by the factor 1.25, since all allowable stresses for petroleum liquids and gases are based upon an apparent factor of safety of 4, instead of the factor 5 recommended in the ASME-PB and ASME-UPV Codes. Cast iron pipe with flanged ends can be used for gas and oil service for underground applications where the metal temperature of the pipe line is less than 300° F.; no operating pressure limit is prescribed. Above ground, cast iron pipe may be used for oil or refinery gases for pressures not in excess of 150 psi., where metal temperatures do not exceed 300° F.

9-20. Design of Cast Iron Pipe. The required minimum thickness of cast iron pipe may be obtained from Eqs. 9-4 and 9-5; the stress S is 4000 psi for pipe cast vertically in dry sand molds and 6000 psi. for pipe cast centrifugally or horizontally. The value of the factor C is 0.18 for vertical or pit cast pipe, and 0.14 for centrifugally cast pipe. To take care of the effects of water hammer, an allowance for this phenomenon is added to the actual internal operating pressure p in Eqs. 9-4 and 9-5. For nominal pipe sizes from 4 to 10 in., inclusive, the water hammer allowance is 120 psi., for 12- to 14-in. sizes, 110 psi., for 16- to 18-in. sizes, 100 psi. The allowance decreases as the pipe size increases, but is taken as 70 psi. for pipe sizes from 42 to 60 in. Other values for water hammer allowance can be obtained from the American Standard Code for

Pressure Piping.[15] Dimensions of standard cast iron flanged pipe are given in Table 9-6.

TABLE 9-6.—DIMENSIONS OF FLANGED CAST IRON PIPE

INCHES

Nominal Size	Actual Outside Diam.	Wall Thickness	Flange Thickness	Flange Diameter	Bolt Circle Diam.	Size Bolts	No. of Bolts
4	4.80	0.40	0.72	9	7.125	5/8	4
6	6.90	0.43	0.72	11	9.125	5/8	4
8	9.05	0.45	0.75	13	11.125	5/8	8
10	11.10	0.49	0.86	16	13.75	5/8	8
12	13.20	0.54	0.875	18	15.75	5/8	8
16	17.40	0.62	1.00	22.5	20.00	3/4	12
20	21.60	0.68	1.00	27	24.5	3/4	16
24	25.80	0.76	1.125	31	28.5	3/4	16

9-21. Miscellaneous Design and Application Data. Circular flues for hot gases are designed in accordance with the provisions of the ASME-PB Code. For seamless or welded flues greater than 5 in. and equal to or less than 18 in. in diameter, the wall thickness t is found from the following, provided the ratio t/D is not greater than 0.023;

$$t = D\sqrt[3]{p}/216 \tag{9-6}$$

where p is the allowable pressure, psi., and D the outer diameter. For a t/D ratio greater than 0.023,

$$t = \frac{D(p + 275)}{17{,}300} \tag{9-7}$$

The foregoing expressions may be used for riveted flues if the sections do not exceed 3 ft. in length, and if the riveted joint efficiency e is greater than

$$e = \frac{PD}{20{,}000t} \tag{9-8}$$

The allowable pressure for a corrugated pipe is computed in the same manner as for a straight pipe, based upon the diameter of the straight non-corrugated sections. If the thickness in the corrugations is decreased because of fabrication, the decreased thickness should be used in design computations.

Steel or wrought iron pipe pierced with tube holes may be designed on the basis of the shell thickness equations in Chapter 3, using the efficiency of the

ligaments between tube holes as the design efficiency. In such cases the tube holes should not pass through the weld in the pipe.

Example 9-1. A lap welded S-17 steel pipe of 5-in. nominal diameter is subjected to a water pressure of 100 psi. Find the required Schedule Number.

Solution. From Table 9-4, the allowable stress S is 7300 psi. for temperatures below 650° F. The actual outer diameter of 5-in. nominal size pipe is 5.563 in. Eq. 9-2 gives:

$$t = \frac{D(p + S/30)}{2.3S}$$

Substituting,

$$t = \frac{5.563(100 + 7300/30)}{2.3 \times 7300} = 0.114 \text{ in.}$$

The actual inner diameter of the pipe cannot exceed 5.563 — (2 × 0.114), or 5.335 in. A Schedule 40 pipe has an inner diameter of 5.045 in., and is therefore satisfactory.

Example 9-2. A still has tubes made of S-24 red brass 1½-in. outer diameter, subjected to a pressure of 300 psi. at an operating temperature of 375° F. Find the required wall thickness, BWG size.

Solution. From Table 9-5, by interpolation, the allowable stress S is 4000 psi. From Eq. 9-3

$$t = \frac{300 \times 1.5}{2 \times 4000} = 0.0565 \text{ in.}$$

From Table 9-2, the nearest available size of BWG wall thickness is No. 17, or 0.058 in. It is common practice to specify tube wall thickness in even gage numbers, thus a No. 16 BWG would be used.

Example 9-3. An 8-in. flanged pipe steam main is used to carry superheated steam at a pressure of 600 psi. gage and a temperature of 700° F. What pipe specification should be used?

Solution. For steam pressures greater than 250 psi., seamless pipe of a quality equivalent to S-17 or S-18, and of a weight equivalent to Schedule 80 is a minimum requirement. From Table 9-4, S-17 seamless steel has an allowable stress S of 9000 psi. From Table 9-1, the actual outer diameter D of 8-in. nominal pipe is 8 625 in. The value of the constant C for plain end (or flanged) pipe is 0.065. From Eq. 9-5

$$t = \frac{8.625}{2}\left(1 - \sqrt{\frac{9000 - 600}{9000 + 600}}\right) + 0.065 = 0.346 \text{ in.}$$

The wall thickness of an 8-in. Schedule 80 pipe, from Table 9-1, is (8.625 — 7.625)/2 or 0.50 in., and this selection is satisfactory.

Example 9-4. What is the permissible operating pressure for a 4-in. centrifugally-cast flanged cast iron pipe carrying water at atmospheric temperature?

Solution. From Table 9-6, the outer diameter D and the wall thickness t of 4-in. cast iron pipe are 4.80 in. and 0.40 in. The constant C has a value of 0.14 in. and the allowable unit stress S is 6000 psi. for centrifugally-cast pipe. From Eq 9-4

$$p = \frac{2 \times 6000}{4.8}(0.40 - 0.14) = 650 \text{ psi.}$$

From this value, the water hammer allowance of 120 psi. must be deducted, giving an allowable operating pressure of 530 psi.

Pit-cast pipe of a similar size and character would have a gross pressure of

$$p = \frac{2 \times 4000}{4.8}(0.40 - 0.18) = 367 \text{ psi.}$$

with a resultant operating pressure of 367 — 120, or 247 psi.

9-22. Pipe Expansion. In pipe lines subjected to temperature changes, the stresses and forces occasioned by expansion may be of considerable magnitude. If free movement of the line is restricted, the pipe wall and joints are subjected to high stresses, and large thrusts may be exerted against anchors and equipment to which the pipe line is fastened. These effects are significant in high temperature installations, but are also present in water lines that are subjected to seasonal temperature variations. To provide for pipe line movement caused by thermal expansion, two methods are usually employed: the use of expansion joints, and changes in the direction or shape of the line.

9-23. Expansion Joints. Several types of expansion joints are shown in Figs. 9-25, 9-26, and 9-27. Slip-type expansion joints, Figs. 9-25 and 9-26, consist of a sleeve S free to move axially in a barrel or casing B. The sleeve is connected to one of the two pipes meeting at the joint by bolted flanged connections, or by pipe threads. In floating type joints, the casing may be attached directly to the other pipe. The fixed type of joint is securely anchored to an external base or foundation, and consequently requires a movable sleeve S at each end. Slip-type expansion joints are usually packed to prevent leakage and thus require more or less frequent maintenance. They are expensive in first cost, but permit considerable expansion of the pipe line. The line should be properly supported and guided so that no cramping or binding of the sleeve occurs as the line expands or contracts.

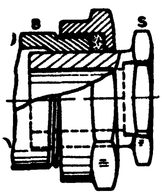


FIG. 9-25. Floating Slip Type Expansion Joint for Threaded Pipe.

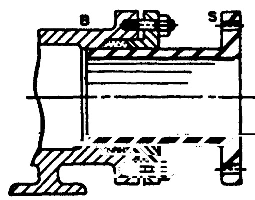


FIG. 9-26. Fixed Slip Type Expansion Joint for Flared Pipe.

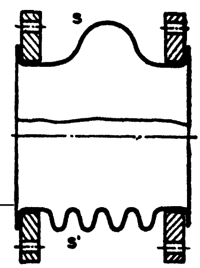


FIG. 9-27. Packless Expansion Joints.

The packless expansion joints shown in Fig. 9-27 permit pipe line expansion by means of the bellows action of corrugated members or sleeves S or S' held between flanged joints. The sleeve is usually made of copper for service in which the pressure is less than 50 psi. and the temperature below 300° F.;

steel sleeves are used for higher pressures. For some types of service, a Monel metal insert is used to protect the corrugated sleeve, in conjunction with spacing clamps on the exterior to limit the axial and radial movements of the corrugations. Packless expansion joints are less expensive than slip-type joints in first cost, and require practically no maintenance as no packing is used; they may be installed in a limited space, and have considerably more lateral flexibility than slip joints. Their use should be limited to low pressures and temperatures and to ⅜ to ½ in. of pipe expansion.

Rubber expansion joints are similar in principles to packless joints, but are applicable only to low pressures, temperatures below 180° F., and fluids not injurious to soft rubber. Within these limitations, however, they may compensate for slight pipe misalignment, and are of value in reducing or absorbing pipe line vibration.

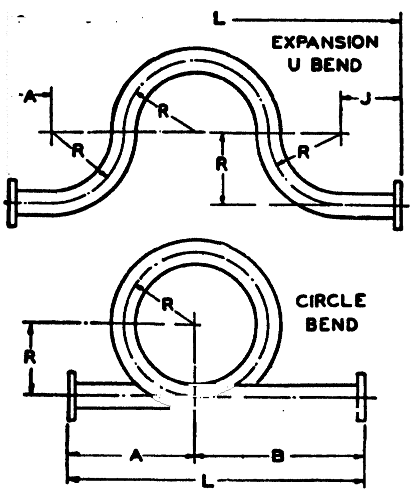


Fig. 9-28. Essential Specification Dimensions for Expansion Bands.

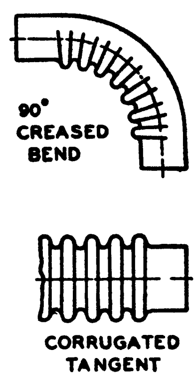


Fig. 9-29. Creased and Corrugated Pipe.

9-24. Pipe Line Flexibility. In high pressure, high temperature pipe lines, it is general practice to provide for thermal expansion by taking advantage of the inherent flexibility and the elastic properties of the pipe line itself. This may often be accomplished by planning the piping layout to take advantage of directional changes which will in themselves provide the necessary flexibility. In cases where it is not feasible to employ such means, expansion bends may be inserted. Two examples of such designs are shown in Fig. 9-28. For increase in flexibility without major changes in pipe lengths or system design, corrugated pipe and creased or corrugated bends, shown in Fig. 9-29, may be used. Regardless of the method used, a careful analysis of the forces and stresses is usually required and has become one of the major problems in the power and processing industries.

The relationship between the loads and the displacements in a piping system is based upon Castigliano's Theorem of Least Work, which states that the derivative of the total elastic energy with respect to any concentrated load gives the total deflection at that load. The complete theoretical analysis of pipe line stresses based upon this theorem is complicated, and is an advanced problem in design. This text will serve only to introduce the principles involved by consideration of pipe lines in a single plane. The analysis of multi-plane piping is so involved that reference should be made to the excellent presentations in the literature, or to industrial specialists in the field of piping design.[31,51,55]

9-25. Expansive Forces in Pipe Lines. The analysis of the forces and moments induced by thermal expansion for a simple single-plane pipe line is illustrated in Figs. 9-30 to 9-40. The first figure shows a pipe line consisting of two sections B and G, fixed at ends 1 and 3, and connected by a 90° elbow or bend C. If the restraining effect at end 1 is removed and if the portion G is considered rigid and unaffected by temperature change, the position of the pipe before and after expansion is shown in Fig. 9-31 by the solid and dotted lines, where the free end H has moved to a position H'. Two forces are required to bring the free end to its original position; these are vectors F_x and F_y in Fig. 9-32, and represent the actual restraining forces at joint 1. The attachment by which the end 1 is held in place also keeps section B in a horizontal or nearly horizontal position, and a restraining moment M must be present as indicated in Fig. 9-33, in addition to the horizontal and vertical forces.

The forces and moment induced by thermal expansion are not confined to any single joint; the actual displacement of the entire line is more nearly like that shown in Fig. 9-34, in which forces and moment are induced at joint 3 as well as at joint 1, and resisting stresses are developed in the entire length of the line. For convenience in computation, the forces F_x and F_y at the joints are usually referred to the elastic center O of the system, which is the centroid of the projected areas of the pipe comprising the entire line. The location of the elastic center O is found as illustrated in Fig. 9-35, where the coordinates x and y of the elastic center with respect to an origin O' are found by:

$$x = \frac{x_b A_b + x_c A_c + x_g A_g \ldots .}{A_b + A_c + A_g \ldots .}$$

$$y = \frac{y_b A_b + y_c A_c + y_g A_g \ldots .}{A_b + A_c + A_g \ldots .}$$

where A_b, A_c, A_g, etc., represent the projected areas, or the products of the lengths and outer diameters of the pipe sections, and x_b, x_c, y_b, y_c, etc., the distances from the origin O' to the centroids of the projected area of the sections. If the pipe is of uniform size, the lengths of the sections may be substituted for the projected areas. On the other hand, if the pipe varies in size or in wall thickness, a correction must be made to compensate for the variation in flexibility.

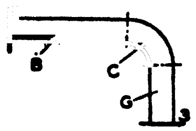


FIG. 9-30. Pipe Line.

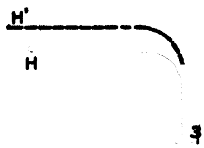


FIG. 9-31. Free Theoretical Movement of Pipe End.

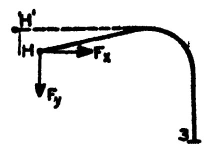


FIG. 9-32. Partial Restraint of Pipe End.

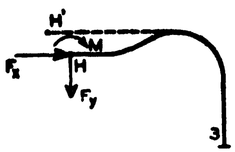


FIG. 9-33. Full restraint of Pipe End.

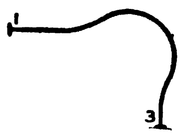


FIG. 9-34. Actual Expansion of Pipe Line.

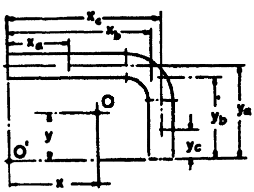


FIG. 9-35. Location of Elastic Center of Pipe System.

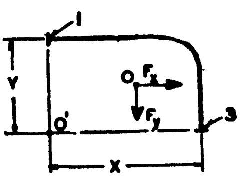


FIG. 9-36. Expansion Forces Acting at Elastic Center.

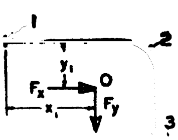


FIG. 9-37. Moment of Expansion Forces about Joint 1.

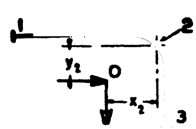


FIG. 9-38. Movement of Expansion Forces about Joint 2.

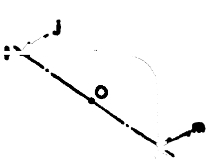


FIG. 9-39. Thrust Line of Pipe System.

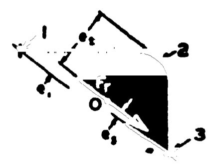


FIG. 9-40. Determination of Maximum Moments for Pipe System.

The magnitude of the forces F_x and F_y at the elastic center O depends upon the modulus of elasticity of the pipe material, the coefficient of expansion, the temperature difference, and the moments and products of inertia of the pipe system with respect to the elastic center. These magnitudes are given in Table 9-7 for the six cases shown in Fig. 9-41. In Table 9-7, V represents the virtual length of the system (composed of virtual lengths a, b, n, etc., from Eqs. 9-9 to 9-16), x and y are the distances from the origin O' to the elastic center O, P is the product of inertia of the system, N and Q are the moments

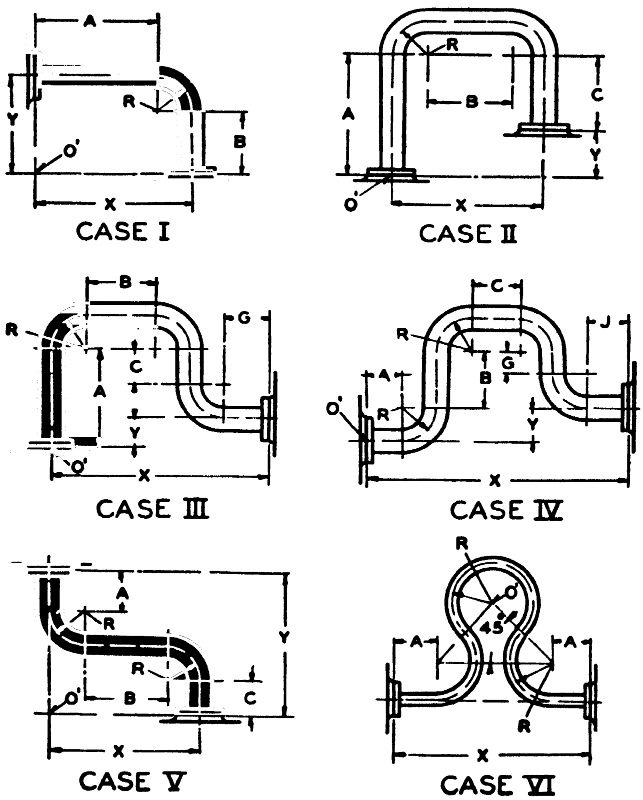


Fig. 9-41. Dimensional Characteristics of Piping Systems.

of inertia of the system with respect to the horizontal and vertical axes of reference, and X and Y are the horizontal and vertical distances between the anchors or points of fixation. E is the modulus of elasticity, T the operating temperature, f the coefficient of expansion, and I the plane moment of inertia of the pipe cross section. Values of the coefficient of expansion and the modulus of elasticity vary with the operating temperature; values of the combined factor EfT for several piping materials are obtained from Fig. 9-42. The solid-line curves are based upon modulii E at operating temperatures, and

TABLE 9-7.—EXPANSION STRESSES AND REACTIONS IN PIPING SYSTEMS (See Fig. 9-41)

SUMMARY OF SYMBOLS

A, B, C, G, J =	actual length of section of pipe
a, b, c, g, j =	virtual length of section of pipe (Eqs. 9-11, 9-12)
E =	modulus of elasticity of pipe material
F =	resultant force at elastic center (Eq. 9-23)
F_x =	horizontal force at elastic center
F_y =	vertical force at elastic center
f =	coefficient of expansion
h =	bend stiffness characteristic of plain pipe
I =	plane moment of inertia of pipe cross section
i =	stress intensification factor (Eqs. 9-18, 9-19)
k =	flexibility factor (Eq. 9-12)
N =	moment of inertia of pipe system with respect to vertical axis
n =	virtual length of pipe bend or loop
O =	elastic center
P =	product of inertia of pipe system
Q =	moment of inertia of pipe system with respect to horizontal axis
R =	mean radius of pipe bend
r =	mean radius of pipe wall
S =	allowable stress in pipe (Eq. 9-22)
S_a =	permissible stress at atmospheric pressure (Table 9-2)
S_f =	stress in pipe due to bending in plane of system (Eq. 9-17)
S_m =	maximum longitudinal stress in pipe (Eq. 9-21)
S_o =	permissible stress at operating temperature (Table 9-4)
S_p =	longitudinal stress in pipe due to internal pressure (Eq. 9-20)
T =	operating temperature
t =	thickness of pipe wall (Table 9-1)
V =	virtual length of pipe
X, Y =	distance between pipe supports or points of fixation
X_1, Y_1, etc. =	moment arms of forces F_x and F_y
x, y =	coordinates of elastic center
Z =	section modulus of pipe cross section

Case I

$$V = a + b + n$$

$$x = \frac{1}{V}\left[\frac{aA}{2} + bX + n(A + 0.637R)\right]$$

$$y = \frac{1}{V}\left[\frac{bB}{2} + aY + n(B + 0.637R)\right]$$

$$P = -Vxy + \frac{aAY}{2} + \frac{bBX}{2} + n[XY - 0.363R(X + Y) + 0.045R^2]$$

$$N = -Vx^2 + \frac{aA^2}{3} + bX^2 + n(X^2 - 0.727RX + 0.227R^2)$$

$$Q = -Vy^2 + \frac{bB^2}{3} + aY^2 + n(Y^2 - 0.727RY + 0.227R^2)$$

$$F_x = +EfTI\left(\frac{NX - PY}{NQ - P^2}\right)$$

$$F_y = -EfTI\left(\frac{QY - PX}{NQ - P^2}\right)$$

TABLE 9-7.—(*Continued*)

Case II

$$V = a + b + c + 2n$$

$$x = \frac{X}{V}\left(\frac{b}{2} + c + n\right)$$

$$y = \frac{1}{V}\left[\frac{aA}{2} + b(A + R) + c\left(Y + \frac{C}{2}\right) + n(2A + 1.273R)\right]$$

$$P = -Vxy + X\left[\frac{b(A + R)}{2} + c\left(Y + \frac{C}{2}\right)\right] + n(AB + 2AR + 0.637BR + 1.273R^2)$$

$$N = -Vx^2 + \frac{b(X^2 - RX + R^2)}{3} + cX^2 + n(B^2 + 3.273BR + 3R^2)$$

$$Q = -Vy^2 + \frac{aA^2}{3} + b(A + R)^2 + c\left(Y^2 + CY + \frac{C^2}{3}\right) + n(2A^2 + 2.546AR + R^2)$$

$$F_x = +EfTI\left(\frac{NX + PY}{NQ - P^2}\right)$$

$$F_y = -EfTI\left(\frac{QY + PX}{NQ - P^2}\right)$$

Case III

$$V = a + b + c + g + 3n$$

$$x = \frac{1}{V}\left[b\left(\frac{B}{2} + R\right) + c(B + 2R) + g\left(X - \frac{G}{2}\right) + n(2B + 4.363R)\right]$$

$$y = \frac{1}{V}\left[\frac{aA}{2} + b(A + R) + c\left(Y + \frac{C}{2} + R\right) + gY + n(Y + 2A + 1.637R)\right]$$

$$P = -Vxy + b\left(\frac{B}{2} + R\right)(A + R) + c(B + 2R)\left(A - \frac{C}{2}\right) + gY\left(X - \frac{G}{2}\right) + n[B(2A - C) + R(4.363A - 2.363C) - 0.318R^2]$$

$$N = -Vx^2 + b\left(\frac{B^2}{3} + BR + R^2\right) + c(B + 2R)^2 + g\left(\frac{G^2}{3} - GX + X^2\right) + 2n(B^2 + 4BR + 4.34R^2)$$

$$Q = -Vy^2 + \frac{aA^2}{3} + b(A + R)^2 + c\left(\frac{C^2}{3} - AC + A^2\right) + gY^2 + n[3A^2 - 2AC + C^2 + 1.273(A + C)R + 1.5R^2]$$

$$F_x = +EfTI\left(\frac{NX + PY}{NQ - P^2}\right)$$

$$F_y = +EfTI\left(\frac{QY + PX}{NQ - P^2}\right)$$

TABLE 9-7.—(*Continued*)

Case IV

$$V = a + b + c + g + j + 4n$$

$$x = \frac{1}{V}\left[\frac{aA}{2} + b(A + R) + c\left(A + \frac{C}{2} + 2R\right) + g(A + C + 3R) + j\left(X - \frac{J}{2}\right) + 2n(X + A - J)\right]$$

$$y = \frac{1}{V}\left[b\left(\frac{B}{2} + R\right) + c(B + 2R) + g\left(B - \frac{G}{2} + R\right) + jY + n(Y + 2B + 4R)\right]$$

$$P = -Vxy + b(A + R)\left(\frac{B}{2} + R\right) + c\left(A + \frac{C}{2} + 2R\right)(B + 2R) + g(A + C + 3R)\left(B - \frac{G}{2} + R\right) + jY\left(X - \frac{J}{2}\right) + n[AR + (B + R)(2A + C + 4R) + (X - J)(Y + R) - 0.637RY]$$

$$N = -Vx^2 + \frac{aA^2}{3} + b(A + R)^2 + c\left[\frac{C^2}{3} + C(A + 2R) + (A + 2R)^2\right] + g(A + C + 3R)^2 + j\left(\frac{J^2}{3} - JX + X^2\right) + 4n(A^2 + AC + \frac{C^2}{2} + 4AR + 3CR + 5.227R^2)$$

$$Q = -Vy^2 + b\left(\frac{B^2}{3} + BR + R^2\right) + c(B + 2R)^2 + g\left[\frac{G^2}{3} - G(B + R) + (B + R)^2\right] + jY^2 + n[1.273R(B + G) + 2(B + R)^2 + (Y + R)^2 + 3R^2]$$

$$F_x = +EfTI\left(\frac{NX + PY}{NQ - P^2}\right)$$

$$F_y = +EfTI\left(\frac{QY + PX}{NQ - P^2}\right)$$

Case V

$$V = a + b + c + 2n$$

$$x = \frac{X}{V}\left(\frac{b}{2} + c + n\right)$$

$$y = \frac{1}{V}\left[a\left(Y - \frac{A}{2}\right) + \frac{c}{2}C + (b + 2n)(C + R)\right]$$

$$P = -Vxy + b\left(\frac{B}{2} + R\right)(C + R) + \frac{cCX}{2} + n[BC + R(0.637B + 2C) + 1.363R^2]$$

$$N = -Vx^2 + b\left(\frac{B^2}{3} + BR + R^2\right) + cX^2 + n(B^2 + 3.273BR + 3R^2)$$

TABLE 9-7.—(*Continued*)

Case V—(Continued)

$$Q = -Vy^2 + a\left(\frac{A^2}{3} - AY + Y^2\right) + b(C+R)^2 + \frac{cC^2}{3} + 2n(C^2 + 2CR + 1.227R^2)$$

$$F_x = +EfTI\left(\frac{NX - PY}{NQ - P^2}\right)$$

$$F_y = -EfTI\left(\frac{QY - PX}{NQ - P^2}\right)$$

Case VI

$$x = 0$$

$$y = -R\frac{2.414a + 0.354n}{a + n/2}$$

$$F_x = +\frac{EfTIX(n + 2a)}{nR^2(1.318n + 8.465a)}$$

$$F_y = 0$$

the dotted-line curves are based on E at atmospheric or 70° F. temperature. When forces and stresses in the operating condition are desired, the value of EfT based upon E at operating temperature is used; for evaluation of the maximum range of stresses, the quantity EfT based upon E at atmospheric conditions will give results that are on the sate side.

The moment at any point in a system is equal to the algebraic sum of the products of the forces F_x and F_y and the distances from the point under consideration. In Fig. 9-37, the moment at point 1 is equal to $(F_x \times Y_1) - (F_y \times X_1)$; in Fig. 9-38, in the moment at point 2 is $(F_x \times Y_2) + (F_y \times X_2)$.

The forces F_x and F_y may be combined into a single resultant F, as follows:

$$F = \sqrt{F_x^2 + F_y^2} \tag{9-9}$$

This resultant force acts along a line at an angle θ with the horizontal axis, and,

$$\tan\theta = \frac{F_y}{F_x} \tag{9-10}$$

The position line of force F is shown in Fig. 9-39, is termed the thrust line of the system, and locates the points j and m of contra-flexure or zero moment. It may also be used to determine the point of maximum moment, as shown in Fig. 9-40. By inspection, it is obvious that the moment $F \times e_2$ about point 2 is greater than moments $F \times e_1$ or $F \times e_3$ about points 1 and 3. Therefore the moment about point 2 will be the basis of the design.

If corrugated or curved sections make up all or a part of the system, the elastic center may be determined by substituting virtual or equivalent lengths for the actual lengths, in a ratio corresponding to the increased flexibility of such sections.

The virtual (or flexibly equivalent) length of a plain straight section of pipe is equal to the actual length; the virtual length of a corrugated straight section is equal to five times the actual length. If A, B, C, G, and J, in Fig. 9-41, denote the actual length of a section, and a, b, c, g, and j, the corresponding virtual lengths, then, for plain straight pipe

$$a = A;\ b = B;\ \text{etc.} \tag{9-11}$$

and for corrugated straight pipe

$$a = 5A;\ b = 5B;\ \text{etc.} \tag{9-12}$$

For plain or creased 90° bends, the virtual length n is given by

$$n = \frac{\pi k R}{2} \tag{9-13}$$

where R is the mean radius of the bend, and k is a flexibility factor, given by

$$k = \frac{12h^2 + 10}{12h^2 + 1} \tag{9-14}$$

In this expression, h is the bend stiffness characteristic of plain curved pipe, given by

$$h = \frac{tR}{r^2} \tag{9-15}$$

where t is the thickness and r the mean radius of the pipe wall.

For corrugated 90° bends, the virtual length is

$$n = \frac{5\pi R}{2} \tag{9-16}$$

For plain or creased expansion loops of 270°, as shown n Case VI, Fig. 9-41, the virtual length is

$$n = 9.42\ kR \tag{9-17}$$

For similar corrugated loops

$$n = 47.124\ R \tag{9-18}$$

The stress S_f in the pipe due to bending in the plane of the system is given by

$$S_f = \frac{iM}{Z} \tag{9-19}$$

where M is the bending moment, in in.-lbs., and Z is the section modulus of the pipe cross section, in in.[3] The factor i is a stress intensification or concentration factor, and depends upon the degree of curvature of the pipe. For values of h (Eq. 9-15) equal to or less than 1.472,

$$i = \frac{\sqrt{(12h^2 + 10)^3}}{9(12h^2 + 1)} \tag{9-20}$$

For values of h greater than 1.472,

$$i = \frac{12h^2 - 2}{12h^2 + 1} \tag{9-21}$$

For plain straight pipe, i is equal to 1 0; for corrugated tangents and for creased bends, i is equal to 2 5.

The longitudinal stress in the pipe wall induced by the internal pressure is given by

$$S_p = \frac{pr}{2t} \tag{9-22}$$

in which p is the internal pressure, psi, and r and t are the mean radius and thickness of the pipe wall. The total maximum longitudinal stress S_m is given by

$$S_m = S_f + S_p \tag{9-23}$$

The allowable unit stress S, which would be equal to or greater than S_m, is given by

$$S = 0\,75(S_a + S_o) \tag{9-24}$$

where S_a and S_o are the permissible stresses, psi., at atmospheric and at operating temperatures, and may be obtained from Table 9-4.

Example 9-5. An oil refinery pipe line has the general dimensions shown in Fig. 9-43, and is subjected to an internal pressure of 200 psi at a metal temperature of 600° F. The line is made up of 8-in Schedule 80 pipe, S-17 seamless steel, with a corrosion allowance of 0 10 in The vertical portions are joined to the horizontal positions by long radius welding elbows Determine the resistance of the system to expansion, and evaluate the stresses in the pipe.

Solution The system shown in Fig. 9-43 corresponds to Case II, Fig 9-41 and Table 9-7 From these data the virtual lengths a, b, and c are equal to the actual lengths A, B, and C, which are respectively 40, 10, and 20 ft long From Fig 9-8, the mean bend radius R of a long radius welding elbow is 1 5 D which is equal to 1 5 × 8 or 12 in The thickness of the pipe wall, from Table 9-1, for Schedule 80 pipe, is (8 625 − 7 625)/2 or 0 50 in , the net thickness, after deducting 0 10 in for corrosion, is 0 40 in , the mean pipe radius is equal to (8 625/2) − (0 40/2) or 4 1125 in The flexibility of the elbow is somewhat greater after corrosion has reduced the effective wall thickness, but the increase in flexibility is insufficient to compensate for the reduction in strength, and the computation for flexibility will consequently be based upon the wall thickness after corrosion.

The bend stiffness characteristic of plain curved pipe, from Eq. 9-15, is

$$h = \frac{0.40 \times 12}{4.1125^2} = 0.284$$

The flexibility factor k, from Eq. 9-14, is

$$k = \frac{12 \times 0.284^2 + 10}{12 \times 0.284^2 + 1} = \frac{0.972 + 10}{0.972 + 1} = 5.56$$

The virtual length of a plain 90° bend, from Eq. 9-13, is

$$n = \frac{\pi \times 5.56 \times 1}{2} = 8.7 \text{ ft.}$$

From Table 9-7, Case II, the virtual length of the system is

$$V = 40 + 10 + 20 + 2 \times 8.7 = 87.4 \text{ ft.}$$

The distance X and Y, from the origin O' to the points of fixation, are 12 and 20 ft., respectively; the distances x and y, from the origin O' to the elastic center O, are given by

$$x = \frac{12}{87.4}\left(\frac{10}{2} + 20 + 8.7\right) = 4.63 \text{ ft.}$$

$$y = \frac{1}{87.4}\left[\frac{40 \times 40}{2} + 10(40 + 1) + 20\left(20 + \frac{20}{2}\right) + 8.7(2 \times 40 + 1.273)\right] = 28.8 \text{ ft.}$$

The product of inertia P is given by

$$P = -87.4 \times 4.63 \times 28.8 + 12\left[\frac{10(40+1)}{2} + 20\left(20 + \frac{20}{2}\right)\right]$$
$$+ 8.7[(40 \times 10) + (2 \times 40 \times 1) + (0.637 \times 10 \times 1) + 1.273] = 2550$$

The moment of inertia N of the system about the elastic center with respect to the x-x axis is

$$N = -87.4 \times 4.63^2 + \frac{10(12^2 - 12 + 1)}{3} + (20 \times 12^2)$$
$$+ 8.7(10^2 + 3.273 \times 10 \times 1 + 3) = 2633$$

The moment of inertia Q of the system about the elastic center with respect to the y-y axis is

$$Q = -87.4 \times 28.8^2 + \frac{40 \times 40^2}{3} + 10(40 + 1)^2 + 20\left[20^2 + (20 \times 2) + \frac{20}{3}\right]$$
$$+ 8.7[(2 \times 40^2) + (2.546 \times 40 \times 1) + 1] = 13{,}093$$

From Fig. 9-42 for S-17 carbon steel, the value of EfT at an operating temperature of 600° F., based upon a modulus of elasticity E at atmospheric temperature, is found from curve A to be approximately 17×10^6. The moment of inertia I of the pipe section is found from the equations in Fig. 5-12 to be

$$I = \frac{\pi d_o^4}{64} - \frac{\pi d_i^4}{64}$$

For an uncorroded pipe d_o is equal to 8.625/12, or 0.719 ft., and d_i is equal to 7.625/12, or 0.635 ft. Substituting,

$$I = \frac{\pi}{64}(0.719^4 - 0.635^4) = 0.00515 \text{ ft}^4$$

The forces F_x and F_y, based upon the uncorroded pipe section, at the elastic center O are given by

$$F_x = 17 \times 10^6 \times 0.00515 \times \frac{2633 \times 12 + (2550 \times 20)}{2633 \times 13{,}093 - 2550^2} = 259 \text{ lbs.}$$

$$F_y = 17 \times 10^6 \times 0.00515 \times \frac{13{,}093 \times 20 + (2550 \times 12)}{2633 \times 13{,}093 - 2550^2} = 918 \text{ lbs.}$$

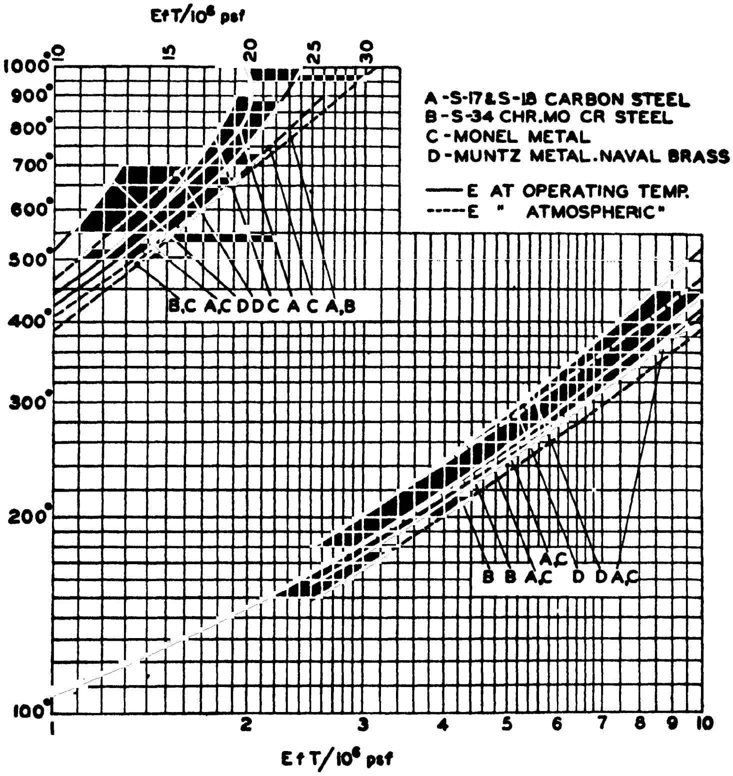


FIG. 9-42. Values of EfT Based upon Temperature, °F.

The forces F_x and F_y, based upon the corroded pipe section, are given by

$$F_x = 17 \times 10^6 \times 0.00423 \times \frac{2633 \times 12 + (2550 \times 20)}{2633 \times 13{,}093 - 2550^2} = 213 \text{ lbs.}$$

$$F_y = 17 \times 10^6 \times 0.00423 \times \frac{13{,}093 \times 20 + (2550 \times 12)}{2633 \times 13{,}093 - 2550^2} = 755 \text{ lbs.}$$

Fig. 9-44 shows a skeleton view of the pipe system of Fig. 9-43, with the elastic center O and the thrust line of the forces F_x and F_y. By inspection it is seen that point *2* is the point of contra-flexure or zero moment, and that the maximum moment will occur at the origin O' (since the perpendicular distance O' to w at the thrust line is greater than distances

3-u or 4-q). The moment of the forces at the elastic center about point O' for uncorroded pipe is given by

$$M_m = (F_x \times y) + (F_y \times x)$$

or

$$M_m = (259 \times 28.8) + (918 \times 4.63) = 11{,}720 \text{ ft.-lbs.}$$

The stress in the pipe due to bending is given by Eq. 9-19; the stress intensification factor i is 1.0 for plain straight pipe; the section modulus Z of the pipe section, from Section 5-10, is equal to the moment of inertia divided by the outer radius of the pipe, or

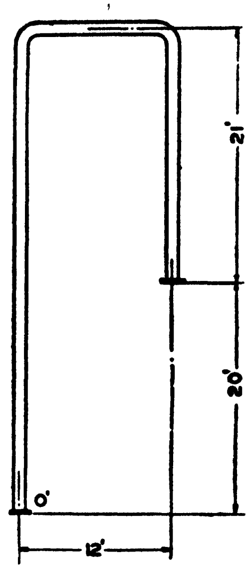


FIG. 9-43. Pipe Line.

$$Z = \frac{\pi}{64 \times 4.313} \times 8.625^4 - 7.625^4 = 24.51$$

The flexural stress is

$$S_f = \frac{1.0 \times 11{,}720 \times 12}{24.5} = 5750 \text{ psi.}$$

The longitudinal stress in the pipe wall, from Eq. 9-22, is

$$S_m = 5750 + 812.5 = 6562.5 \text{ psi.}$$

The allowable stress at atmospheric and at the operating temperature of 600° F., for seamless pipe of S-17 quality, from Table 9-4, is 9400 psi.

The permissible stress, from Table 9-4, for S-17 seamless steel pipe is 9400 psi. for any temperature below 650° F. The allowable stress S, from Eq. 9-24, is

$$S = 0.75(9400 + 9400) = 14{,}100 \text{ psi.}$$

which is more than twice the actual induced stress S_m calculated above.

It may be of interest to determine the induced stress when the pipe interior is at the point of maximum corrosion. The flexural stress in the pipe will be the same as in the preceding solution, because the decrease in the moment of inertia and section modulus of the pipe section is compensated for by the smaller values of F_x and F_y. The longitudinal stress, however, is

$$S_p = \frac{200 \times 4.1125}{2 \times 0.40} = 1030 \text{ psi.}$$

which gives a maximum induced stress of

$$S_m = 5750 + 1030 = 6780 \text{ psi.}$$

Although the maximum moment is exerted at the origin O', it will be advisable to check the flexural stress at point 3 because the stress intensification factor i for an elbow is appreciably greater than unity. If the point of tangency of the elbow and the vertical portion of the pipe be taken as the center of moments, the moment about that point is

$$M = 259(40 - 28.8) + 918(12 - 4.63) = 9660 \text{ ft.-lbs.}$$

The bend stiffness characteristic h, for the uncorroded pipe, from Eq. 9-15, is

$$h = \frac{0.50 \times 12}{4.065^2} = 0.362$$

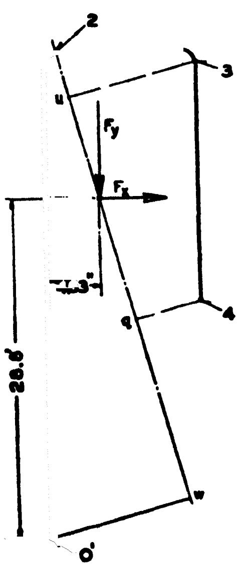


FIG. 9-44. Skeleton Diagram of Pipe Line.

The stress intensification factor i, from Eq. 9-20, is

$$i = \frac{\sqrt{(12 \times 0.352^2 + 10)^2}}{9(12 \times 0.352^2 + 1)} = 1.73$$

The flexural stress S_f, from Eq. 9-19, is

$$S_f = \frac{1.73 \times 9660 \times 12}{24.5} = 8200 \text{ psi.}$$

If the longitudinal stress of 812.5 psi. is added to the above, the resulting maximum stress will be 9012.5 psi., which is appreciably greater than the stress at the origin O', although it is still within the limits of safety. The stress in the elbow when the maximum corrosion exists is based upon the forces obtained by using the moment of inertia of the pipe section for this condition. The moment at the point of tangency of the elbow and the vertical portion of the line is

$$M = 213(40 - 28.8) + 755(12 - 4.63) = 7945$$

For a bend stiffness characteristic h of 0.284, the stress intensification factor i, from Eq. 9-20, is

$$i = \frac{\sqrt{(12 \times 0.284^2 + 10)^2}}{9(12 \times 0.284^2 + 1)} = 2.02$$

The section modulus Z of the corroded section is

$$Z = \frac{\pi}{64 \times 4.313}(8.625^4 - 7.825^4) = 20.2$$

The flexural stress, from Eq. 9-19, is

$$S_f = \frac{2.02 \times 7945 \times 12}{20.2} = 9530 \text{ psi.}$$

If the flexural stress is combined with the longitudinal stress based upon the corroded section, the maximum stress is

$$S_m = 9530 + 1030 = 10{,}560 \text{ psi.}$$

which is higher than any of the preceding values, although it is still less than the allowable stress 13,100 obtained from Eq. 9-24.

The analysis of other piping systems shown in Fig. 9-41 is similar to that given in Example 9-5. Several features of such analyses are worthy of additional comment. Because of their relative rigidity, systems with cast or forged elbows should be computed on the basis of a square corner system, with the mean bend radius R equal to zero. For systems such as those of Case II or Case IV, Fig. 9-41, in which the points of fixation are in alignment, the distance Y is taken as zero. The expansion U bend of Fig. 9-28 is analogous to Case IV, Fig. 9-42, for which distances B, C, G, and Y are equal to zero, and distance X is equal to L in Fig. 9-28.

9-26. Pipe Anchors and Supports. Most of the piping used for the transportation of fluids in the power and processing industries is exposed within the buildings, and is supported by the walls or the building framework, to which it is attached by anchors, supports, or hangers. The function of a rigid pipe anchor, Fig. 9-45, is to protect one end of a line against excessive

thrusts caused by expansion forces, particularly when pumps, turbines, blowers, and other process equipment may be seriously affected by the pipe deformation. Although the anchor shown in Fig. 9-45 will protect the adjacent unit from the effects of expansion in the line beyond the anchor, the thrust caused by

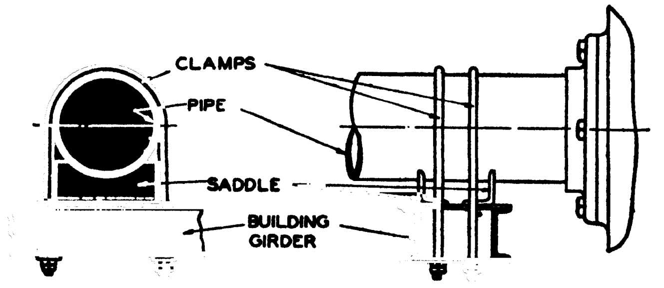


FIG. 9-45. Rigid Pipe Anchor.

expansion between the anchor and the connected unit may be serious when high temperature differentials exist.

Fig. 9-46 shows an adjustable, easily applied rigid hanger for supporting a length of pipe from an overhead beam. A similar spring-supported hanger is

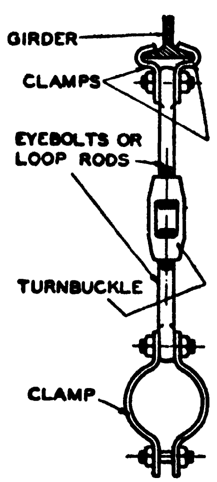


FIG. 9-46. Rigid Pipe Hanger.

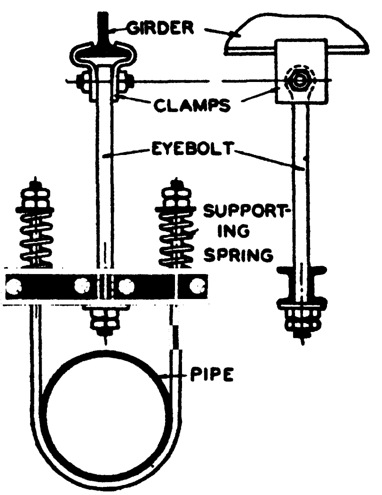


FIG. 9-47. Spring-supported Pipe Hanger.

shown in Fig. 9-47, and is superior to the rigid hanger for a system like that of Fig. 9-43 because of its greater flexibility. Any form of intermediate restraint in a line reduces its flexibility and tends to increase line stresses and thrusts. For lines subjected to high temperature differentials, roller supports

are often used to permit free axial movement of the pipe. One form of roller support is shown in Fig. 9-48, and consists of a bracket composed of welded structural plate and angles, bolted to a wall or column. The spool-shaped roller is supported by a shaft which is free to rotate in a pair of vertically adjustable eyebolts that serve as bearings. For maximum flexibility, spring-supported rollers are sometimes employed.

Pipe anchors and supports should be carefully selected and properly applied. One important factor to be considered is the deflection of the pipe due to its own weight and its contents, Horizontal runs of pipe carrying vapor where condensate is present should be supported to eliminate any appreciable sag, and to provide sufficient pitch for adequate drainage. Condensate flow in a direction opposite to vapor flow should be avoided, unless the line has a gradient of not less than 1 in. in 7 ft. For uni-directional vapor and condensate flow, a gradient of 1 in. in 15 ft. is usually ample unless the pipe deflection is apt to be excessive.

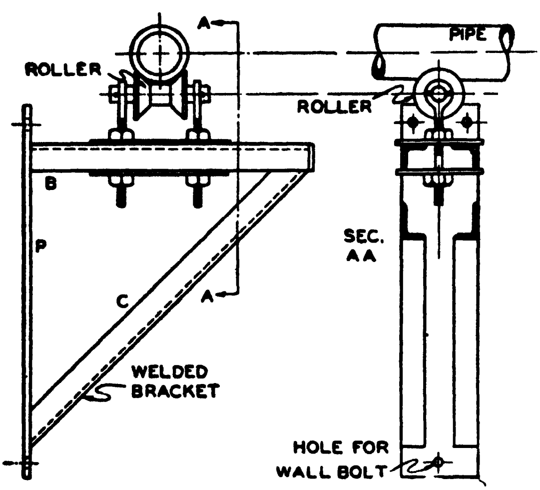


Fig. 9-48. Roller Support for Pipe.

9-27. Tube Attachments. Tubes are often held in place by welded or mechanically applied attachments or lugs; these may produce flexural and other stresses in the tube walls that must be added to the tensile stresses caused by the internal pressure. The required dimensions of such lugs are obtained by an adaptation of the equation for eccentrically loaded columns and struts given in Section 5-18, resulting in the ASME-PB Code specification

$$p = \frac{W \cos \theta}{L} \pm \frac{6We}{L^2} \qquad (9\text{-}25)$$

where W is the total load applied to the lug or attachment, p represents the maximum allowable load per inch of length of the attachment, and the other symbols as illustrated in Fig. 9-49. Allowable values of p for both tensile and compressive loads depend upon the ratio of the outer diameter and the square of the wall thickness of the tube, and may be obtained from Fig. 9-49.

Example 9-6. Find the required length L for the attachment shown in Fig. 9-51A. The supported tube has outer and inner diameters of 4.0 in. and 3.4 in., and a reaction of 1200 lbs. acts as shown by arrow W.

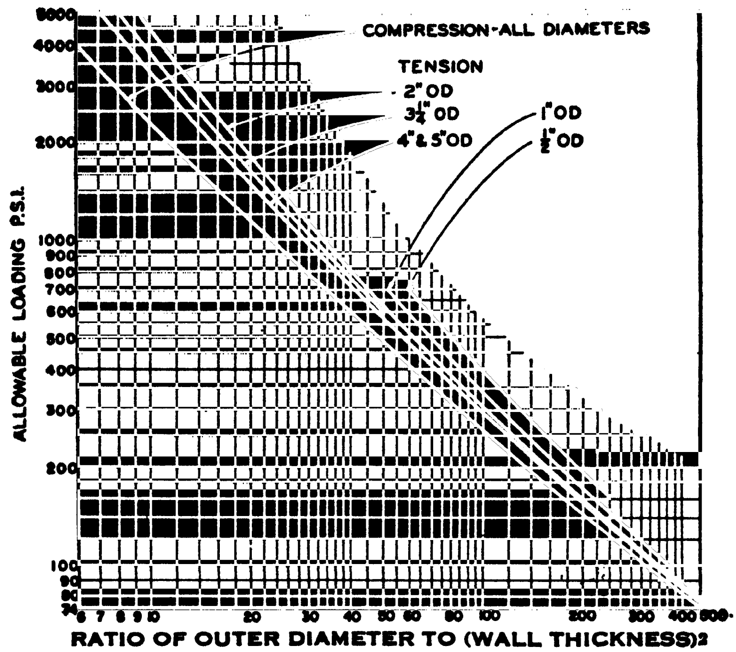


Fig. 9-49. Allowable Loading on Structural Attachments for Tubes.

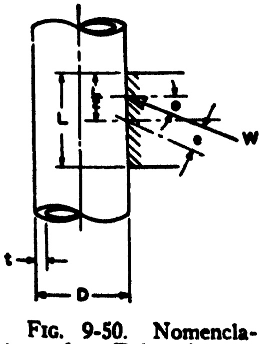


Fig. 9-50. Nomenclature for Tube Attachments and Loads.

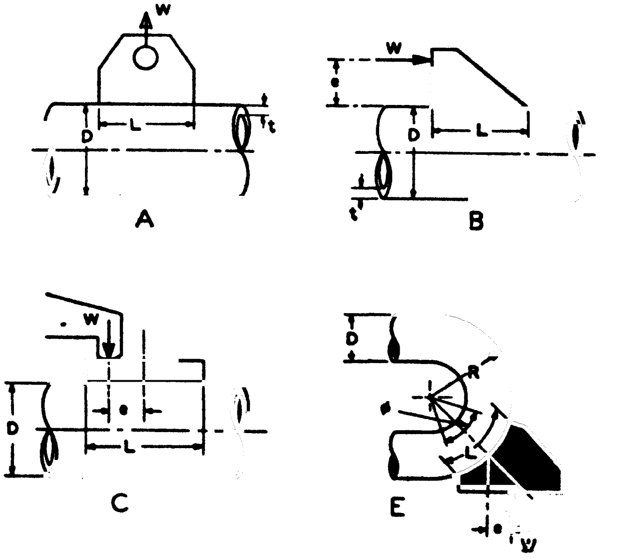


Fig 9-51 Types of Structural Attachments for Tubes.

Solution. The load produces a direct radial force on the tube wall. The tube wall thickness is (4.0 − 3.4)/2, or 0.30 in. The ratio D/t^2 of the outer diameter to the square of the wall thickness is $4/0.30^2$, or 44.4, and the allowable unit load, from Fig. 9-50 is 655 lbs. per in. tension. The load eccentricity is zero, eliminating the second term of Eq. 9-25. The line of action of the load is perpendicular to the tube axis, and angle θ is therefore 0°. Substituting in Eq. 9-25,

$$655 = \frac{1200 \cos 0°}{L}$$

or

$$L = 1.83 \text{ in., approximately}$$

An attachment 2 in. long will serve.

Example 9-7. Check the load on the attachment of Fig. 9-51B if the tube has outer and inner diameters of 3¼ in. and 2½ in. and a reaction of 400 lbs. acts as shown by arrow W. The length of attachment is 2½ in., and the centerline of the reaction is parallel to and 3⅛ in. from the axis of the tube.

Solution. This load induces a pure flexural stress on the tube walls. The line of action of the force is parallel to the tube axis, which makes angle $\theta = 90°$ and eliminates the first term of Eq. 9-25. The load eccentricity e, with respect to the exterior of the tube, is 3.125 − 1.625, or 1.5 in.; the load W and length L are 400 lbs. and 2½ in., respectively; and the actual unit load, from Eq. 9-25, is

$$p = \frac{6 \times 400 \times 1.5}{2.5^2} = 576 \text{ lbs. per in.}$$

The D/t^2 ratio is $4 \times 3.25/(3.25 - 2.50)^2$, or 23.1, and the allowable tensile and compressive unit loads, from Fig. 9-50, are 1300 and 1000 lbs., respectively. The unit load is within these limits, and the attachment design is satisfactory.

To care for expansion and contraction, tubes are sometimes constructed with rubbing strips as shown in Fig. 9-51C. The movement of the tube may cause some eccentricity of the support with respect to the length of the attachment.

Example 9-8. Check the load on the attachment of Fig. 9-51C if the tube has outer and inner diameters of 4 and 3.4 in., and the load is 960 lbs., with a possible maximum eccentricity of ¾ in.

Solution. The D/t^2 ratio is the same as that of Example 9-6 and the allowable unit load p, from Fig. 9-50, is 655 lbs. tension and 555 lbs. compression per inch of length. The length L is 4 in., and the angle θ is 0°; from Eq. 9-25,

$$p = \frac{960 \cos 0°}{4} \pm \frac{6 \times 960 \times 0.750}{4^2} = 240 \pm 270$$

which results in stresses of 510 lbs. per in. compression and 30 lbs. per in. tension. The actual loading, therefore, is within the allowable limits.

The allowable load per inch of length for an attachment on a bent tube is found by increasing the allowable load determined by using the outer diameter by the amount of allowable compression load that would be permitted if the tube had an outside diameter equal to the outer radius R of the bend and were of the same wall thickness.

Example 9-9. A heat exchanger section, illustrated in Fig. 9-51E, is supported by a welded attachment at the tube bend. The outer radius of the bend is 3 in., the tube has a 2-in. diameter and a 0.30-in. wall thickness, and the weight of the section is 1000 lbs. The included angle ϕ is 60°. Check the load on the attachment.

Solution. The length L of the attachment is equal to $6\pi(60°/360°)$ or 3.14 in. The actual applied load, from Eq. 9-25, is

$$p = \frac{1000 \cos 45°}{3.14} \pm \frac{6 \times 1000 \times 0.75}{3.14^2} = 225 \pm 456$$

which results in stresses of 681 lbs. per in. compression and 231 lbs. per in. tension. The D/t^2 ratio for the tube is $2/0.30^2$ or 22.2, and p from Fig. 9-50 is 1050 lbs. compression and 1600 lbs. tension, per in. of length. If the tube diameter is considered equivalent to the outer diameter of the tube bend, the D/t^2 ratio becomes $6/0.30^2$ or 66.6, for which p has a value of 380 lbs. compression and 460 lbs. tension. The total allowable values of p, per inch of length, are $1050 + 380$ or 1430 lbs. compression, and $1600 + 460$ or 2060 lbs. tension, which are appreciably higher than the actual unit loadings on the attachment.

PROBLEMS—CHAPTER 9

1. The vertical axes of three tanks 3 ft. in diameter and 4 ft. high are located on the vertices of an equilateral triangle whose sides are 15 ft. long. The tops of two of the tanks, A and B, are located 14 ft. below the bottom of the third tank C, which serves as a reservoir for A and B. Inlets in tanks A and B are perpendicular to the line of centers of A and B; the outlet in C leading to A and B is parallel to the inlets. The inlets to tanks A and B are ⅜-in. and ½-in. Schedule 40 pipe, respectively, screw fittings; the outlet from C must be of sufficient size to permit a continuous flow into A and B. The outlet and inlets are located 10 in. above the bottoms of the tanks. A supply pipe, of the same size as the outlet, enters C in the center of the bottom, with a check valve to prevent return flow. An arrangement of globe valves is used so that the flow to A or B may be shut off without affecting the flow to B or A. A shut-off globe valve is also used so that the entire system may be shut down by operating a single valve. Overflow pipes, of the same size as the inlets, leave A and B in the center of the top and rise 3 ft. before turning down. Make a freehand orthographic sketch single-line representation of this system, using conventional symbols, and giving pipe sizes, necessary figures, and arrows indicating the directions of flow.

2. Make an isometric single-line representation of the piping system of Problem 1. Show sizes, dimensions, and flow arrows.

3. Make a complete bill of materials for the pipe and fittings required for Problem 1.

4. Find the maximum permissible operating pressure for S-19 Schedule butt-welded pipe, 4-in. diameter, for the maximum operating temperature. Anhydrous ammonia service.

5. Like Problem 4, for S-40, Grade A pipe.

6. Like Problem 4, for oil refinery service.

7. Like Problem 4, for steam service.

8. Find the maximum operating pressure for a standard 8-in. pit cast iron pipe carrying water at atmospheric temperature.

9. A 650-lb., 850° F. superheated steam line has a horizontal run of 60 ft. and a vertical run of 40 ft. to fixed anchor points. The pipe is 10-in. Schedule 80, made of S-18 seamless steel. The bend is effected by a 90° long radius welded elbow. The corrosion allowance is 0.15 in. Determine the resistance of the system to expansion and evaluate the stresses in the pipe.

10. Like Problem 9, but short radius welded elbows are used.

11. A piping system similar in appearance to Case III, Fig. 9-41, has the following dimensions: $A = 5$ ft.; $B = 12$ ft.; $C = 35$ ft.; $G = 12$ ft. The pipe is 8-in. Schedule 160, A-53-36, seamless; the design pressure is 350 psi. at a temperature of 700° F., and the piping is used for oil service. The corners are long radius welding elbows, and all pipe lengths are plain except the upper horizontal section which is corrugated. Determine the resistance to expansion and evaluate the stresses in the pipe system.

12. Like Problem 11, using corrugated long radius elbows and plain pipe.

CHAPTER 10

ATTACHMENTS AND CLOSURES

10-1. Flanged Joints. Connections for piping systems in sizes exceeding 2½ in. are usually effected by means of flanged joints.[52] Integral flanges, Fig. 10-1, are often used for cast or forged vessels. Connections to pipe or to vessels made of plate are made by flanges attached by welded joints, Figs. 10-2, 10-3, and 10-7, or by rivets or pipe threads, Fig. 10-4. Loose-ring slip-on flanges, Figs. 10-5 and 10-6, are employed for lap joint connections where the ends of the pipe are upset to form a collar or lap. The contacting surfaces of any flanged joint are carefully machined and ground or lapped to eliminate minor machining imperfections and to provide a pressure-tight metal-to-metal joint. This process is comparatively expensive. so a sheet or section of some material softer than the flanges, called a gasket, is usually inserted between the contact faces. When the joint is tightened, the softer gasket material flows into the surface imperfections, resulting in a fluid-tight seal.

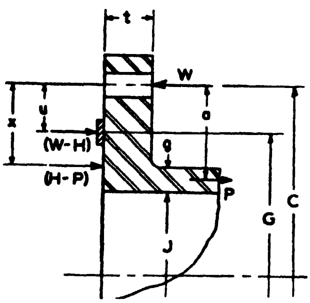


FIG. 10-1. Integral Flanged Fitting.

10-2. Gaskets. Gasket selection is dependent upon the temperature, pressure, and chemical nature of the confined material, and on the relative ease and economy of installation and maintenance. The essential function of a gasket is to seal a joint to prevent leakage, and yet the joint must be easily dismantled and re-assembled. The gasket material should be as serviceable as the materials of the equipment with which the gasket is used, but must not form a permanent bond with the flange faces. Representative gasket materials are listed in Table 10-1; Fig. 10-8 shows some of the more common shapes and forms in which asbestos, metals, and other materials are used in gaskets. (The gasket types referred to in Table 10-1 are those of Fig. 10-8.)

Asbestos is the most common gasket material, and has wide application by itself, or in combination with graphite, cloth, plastics, or metals. Plain asbestos cloth is made from long asbestos fibers, with a twill weave, and is approximately $\frac{3}{32}$ in. thick. Plain asbestos paper is made from short fibers combined with a filler material and a binder. Plain asbestos gaskets are usually of the form shown at *A*, Fig. 10-8. Wire for additional strength is sometimes inserted as strands in the weave of asbestos cloth. Compressed asbestos sheet is made from a mixture consisting of about 75% asbestos fiber, combined

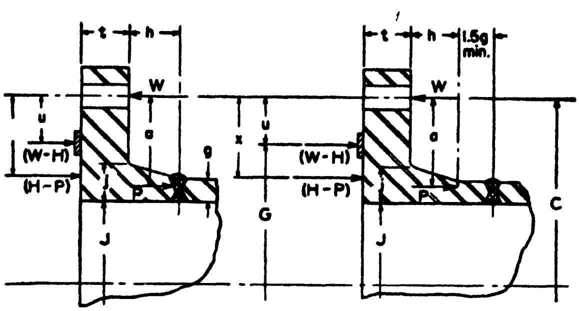


FIG. 10-2. Hubbed or Welding-neck Type Flanged Fittings.

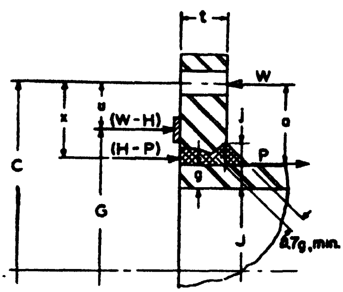


FIG. 10-3. Through-welded Ring Flanged Fitting.

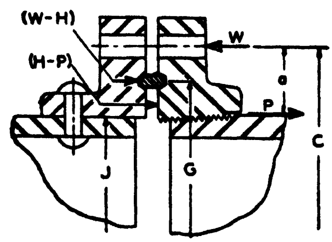


FIG. 10-4. Riveted and Screwed Ring-joint Flanged Fittings.

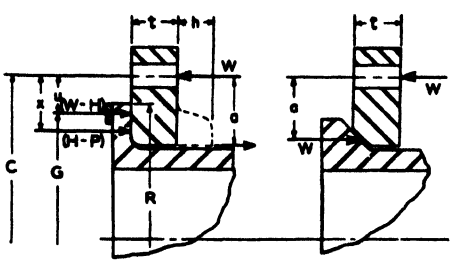


FIG. 10-5. Van Stone or Lap-joint Flanged Fittings.

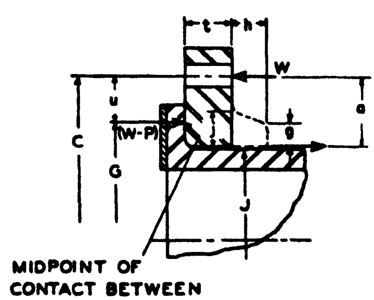


FIG. 10-6. Lap-joint Flanged Fitting with Full-face Gasket.

with rubber, plastic material, graphite, or shredded metal in varying quantities. The mixture is rolled into sheet form and subjected to high pressure to compact and bond the materials. Cotton, hemp, and cork are also used with asbestos. Asbestos or fiber ropes are made by spreading shredded metal throughout the twisted rovings of an asbestos or fiber yarn. Rope forms are also used to make molded shapes similar to *B* or *D*, Fig. 10-8, from mixtures of asbestos, shredded metal, and graphite. Jute, hemp, and other vegetable fiber gaskets are available in sheet form, and are often bound with rubber or plastic materials. Soft rubber compounds are formed into sheets and molded sections of various forms, and are often reinforced by cotton fabric, cloth, or metal wires.

TABLE 10-1.—GASKET MATERIAL DATA

Gasket Material	Represents Fig. 10-8	Gasket Factor m	Yield Point y
Gum rubber sheet	*A, B*	0.50	500
Cloth-inserted soft rubber, or hard rubber sheet.	*A*	0.75	750
Cloth-inserted hard rubber	*A*	1.00	1000
Vegetable fiber sheet (hemp or jute)	*A*	1.50	2000
Compressed asbestos, or asbestos comp.	*A*	2.50	4500
Wire mesh reinforced asbestos		2.50	4500
Corrugated metal, asbestos inserted	*P**	2.50	4500
Asbestos-filled corrugated metal jacket	*Q**	3.00	6000
Corrugated metal, copper	*N**	3.00	6000
Corrugated metal, Monel, iron, soft steel	*N**	3.25	7000
Asbestos-filled flat metal jacket, aluminum, copper	*G, H, J, K, L, R*	3.25	7000
Monel, iron, soft steel		3.50	8000
Chrome (4-6%, 11-13%)		3.75	9000
Solid metal, soft aluminum	*A, B*	4.00	10,000
Soft copper and admiralty		4.75	14,000
Monel, iron, soft steel		5.50	18,000
Chrome (4-6%, 11-13%)		6.00	21,000

* Consider as facing 1, Fig. 10-9, only.

Pure metals may be used for sheet or ring form gaskets. Flat, square, diamond, round, and "obround," or oblong, metal gasket forms are shown in details *A, B, C, D,* and *E,* Fig. 10-8. Gaskets with a so-called "profile" section, as at *F,* are provided with concentric grooves to fit serrations in the mating faces of the flanges. Plain deeply corrugated sheet metal gaskets are shown at *N*;

the detail at *P* shows a similar section in which treated and twisted asbestos cord is cemented in place. Sheet metal gaskets are usually 1/16 in. thick. Sheet and ring gaskets of pure metal are usually heat treated to increase their plasticity, so that excessive pressure will not be required to obtain a tight joint. In

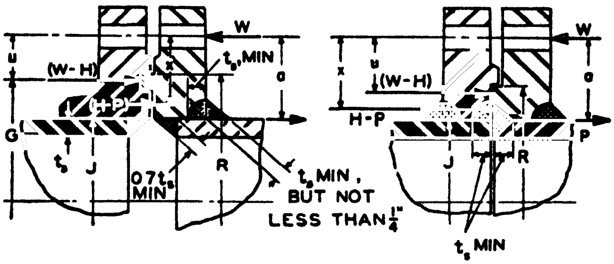


Fig. 10-7. Recessed or Protected Gasket Flanged Fittings.

some instances, solid metal gaskets with regularly spaced perforations are used to reduce the joint-contact-seating pressure, while in other cases solid metal gaskets with raised cross ribs are used.

To provide a high degree of plasticity and to obtain the comparatively long life that pure metal gaskets offer, gaskets are available with partially or wholly

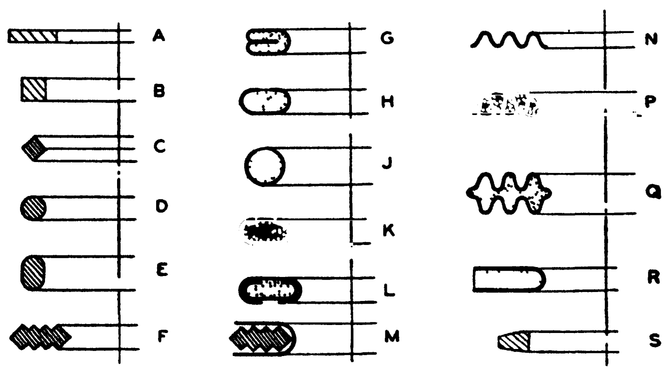


Fig. 10-8. Gasket Types and Shapes.

enclosed asbestos or fiber fillers in sheet metal jackets or retainers. Gaskets of this character are applicable where the joint or fitting is frequently dismantled or re-assembled and where the nature of the connection is such that a high joint pressure is undesirable. Details *H* and *J*, Fig. 10-8, represent single-jacketed gaskets in which a soft filler is partially enclosed by a metal shell. Details *G*, *K*, and *L* show completely enclosed gaskets using a single shell, a single shell and a washer, and a double shell. Detail *R* is termed a "French-Type" gasket;

the shell covers the upper, lower, and inner surfaces of the filler material. The detail at *M* is a French-Type gasket with a profile section filler. The detail at *Q* shows a highly plastic gasket, in which a soft filler is completely enclosed by a corrugated metal jacket. Shredded metal is used both in loose or bulk form and pressed into shape, or used with asbestos, graphite, or other filler for compounds to be enclosed or braced by metal coverings.

10-3. Gasket Application. Several generalities can be made for gasket applications under various pressure and temperature combinations. For temperatures up to 700° F. and pressures up to 300 psi., temperature is the controlling factor in gasket selection. With pressures below 150 psi. and temperatures below 150° F., any of the gasket materials and shapes shown in Table 10-1 and Fig. 10-8 are applicable, although asbestos, fiber, or rubber sheet material is commonly used for such service. For pressures up to 300 psi. and temperatures up to 475° F., the most common materials are compressed asbestos sheet and various metallic reinforced asbestos sheets and cloths. For pressures up to 300 psi. and temperatures up to 700° F., corrugated metal-asbestos gaskets and plain iron, aluminum, copper, and Monel sheet gaskets are used. When pressures exceed 300 psi. and temperatures exceed 700° F., plain metal gaskets are preferred, and the pressure becomes the deciding factor in the gasket choice, although the metals and alloys used must have softening or plastic flow temperatures well above the operating temperature.

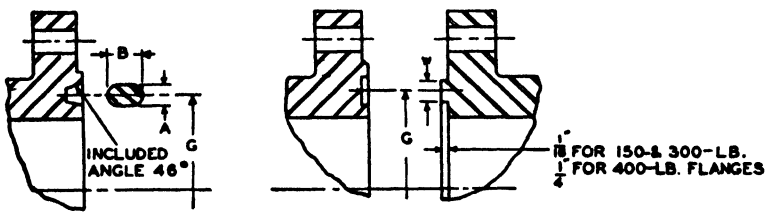
The size and shape of gaskets are dependent on the size and type of flange to be gasketed. Most of the plain gaskets are $\frac{1}{16}$ in. thick or less; enclosed and reinforced gaskets and rings may be of somewhat heavier section. The plain-face flange shown in Fig. 9-9 is used extensively for temperatures up to 475° F. and pressures up to 150 psi. The entire flange face can be used for the gasket contact or seat. A full-face gasket is one which covers the entire flange face; a ring-type gasket covers a portion of the face between the bolt circle and the center opening. The full-face type can be installed more accurately and conveniently since the bolts help to align it. It also provides a wider area of contact, resulting in less chance of flange damage by corrosion and chipping or breaking due to non-uniform tightening of bolts. At higher pressures, however, the gasket pressure required for full-face flanges necessitates a greater bolt stress than is required for a ring-type gasket. Full-face gasket contact surfaces are provided with a serrated finish, consisting of concentric angular grooves, or with the so-called "phonographic finish," in which angular grooves are cut in a continuous spiral across the face of the flange. Such grooves are usually $\frac{1}{64}$ in. deep and $\frac{1}{32}$ in. apart, with a 90° angle. Smooth contact surfaces are most satisfactory for service up to 150 psi.; grooved flanged faces are generally used for higher pressures. Smooth-faced surfaces are always employed for metal-to-metal flange face contact.

Ring-type gaskets are used for the Van Stone or lap joint, Figs. 10-5 and 10-6, the raised face joint, Fig. 10-3, and the male-and-female joint, Fig. 10-7

(right). Gaskets for these types of flanges are usually of the same dimensions as the contact surfaces, although narrower gaskets, as shown in Figs. 10-1 and 10-2, are often employed. In the male-and-female joint of Fig. 10-7, the diameter of the recess is 1/16 in. greater than the dimension R of the male flange, except for very high pressure service. The tongue-and-groove joint, Fig. 10-7 (left), uses a narrow ring gasket, totally enclosed and accurately fitted to the groove. The clearance on each side of the tongue is usually 1/32 in. Male-and-female or tongue-and-groove joints are required where frequent disassembly is indicated.

The metal ring joint, for which dimensions are shown in Table 10-2, requires a solid or totally enclosed gasket, accurately fitted to both grooves.

TABLE 10-2.—JOINT DIMENSIONS



RING JOINT TONGUE AND GROOVE

Nominal Pipe Size		1	1¼	1½	2	2½	3	4	5	6	8	10	12	14	16	18
Ring joint 150-lb.	G	1⅞	2¼	2 9/16	3¼	4	4½	5⅞	6¾	7⅝	9¾	12	15	15⅝	17⅞	20⅜
	A	5/16														
	B	9/16														
Ring joint 300-lb. 400-lb.	G	2	2⅜	2 11/16	3¼	4	4⅞	5⅞	7⅛	8 5/16	10⅝	12¾	15	16½	18½	21
	A	5/16			7/16											
	B	9/16			11/16											
Tongue and groove	G	1¾	2 3/16	2½	3¼	3¾	4⅝	5 11/16	6 13/16	8	10	12	14¼	15½	17⅝	20⅛
	W	¼	5/16	⅜				½			⅝	¾			⅞	

When slight misalignment is likely to be present, flared joints may be used. Such joints are similar to the lap joints shown in Fig. 10-5, but the contact faces of the pipe are shaped to fit the spherical contour of the flared joint ring gasket shown in Fig. 10-8S.

Ease of installation, maintenance and servicing costs, and life of material used must be considered in deciding on the type of flange and gasket. Aside from pressure relief devices, gaskets are generally the weakest structural parts of plant and processing equipment. Their usefulness in preventing leaks is dependent upon the tight fit obtained by the compression of the gasket material

For this reason flanges should be accurately machined and aligned, with perfectly clean faces. Misalignment, or correction of pipe misalignment, by means of the gasket should never be attempted except with the flared-joint type which is specifically designed for this purpose. Since the tightness of the joint depends upon a uniform pressure, the gasket should be as thin as possible. For chemical equipment the only lubricating or coating compound used should be graphite, although special plastic coatings may be occasionally used for specific services. The use of shellac and plumbers "dope" should be avoided.

Gaskets must always be cut accurately, with no frayed, humped, or injured edges. Oversize gaskets, especially in tongue-and-groove flanges, may cause leaks, and cannot be aligned with certainty. Excessive pressure must be avoided on gaskets of all types. The only positive indication that gasket design pressures are not exceeded is to require a hydraulic test of the joint at a pressure approximately twice the intended operating pressure, or to specify the bolt tension by requiring the use of torque indicating wrenches. In any event the flange bolts should never be drawn up to the design limit until the third tightening circuit of the flange.

10-4. Gasket Selection. The ability of a gasket to prevent leakage at a joint is dependent upon the frictional resistance between the flange and gasket contact surfaces. This resistance is in turn dependent upon the total load applied to compress the gasket.

If d represents the inner diameter of a flat gasket, w the gasket width, and p' the pressure required to compress the gasket, then the total area of the gasket subjected to deformation is

$$\pi/4[(d + 2w)^2 - d^2]$$

and the total load required to compress the gasket is

$$\pi/4[(d + 2w)^2 - d^2]p'$$

Expanding $$\pi/4[d^2 + 4wd + 4w^2 - d^2]p'$$

or $$\pi p'(wd + w^2) = \pi p'w(d + w)$$

Substituting the mean diameter G of the gasket for $d + w$, this becomes

$$\pi p'wG$$

For perfectly smooth surfaces the applied force, or joint-contact-surface compression load, should vary directly as the internal pressure and the diameter and thickness of the gasket, and inversely as the coefficient of friction between the surfaces. Experimental data are not, however, in complete accord with theoretical analyses for smooth-surfaced joints, and the determination of apparent coefficients of friction for serrated surfaces, or for corrugated or line-contact gaskets, is extremely difficult. Practical experience dictates the use of empirical formulae, based upon data in the ASME-UPV Code, for the

required joint-contact-surfaces compression load Q necessary to insure a tight joint at operating or working conditions.

If the unit load p' required to compress the gasket is expressed as a product mp, where p is the unit internal operating pressure, and m is an empirical constant dependent upon the gasket material and the contact surfaces, and the actual width w of the gasket is replaced by an effective yield width b, the joint-contact surface compression load becomes

$$Q = 2\pi bGmp \tag{10-1}$$

where b is the effective gasket yield width, G the mean diameter of the gasket, m the gasket factor, and p the internal operating pressure, psi.

The gasket yield width for various types of contact facings is shown in Fig. 10-9, where n represents the actual width of the gasket, and b is the effective yield width based upon the type of surface. Type 1 is representative of smooth-surfaced flange faces; type 2 of tongue-and-grooved joints; types 3 and 4 of flanges with one serrated and one smooth face, or two serrated faces. Type 5 is representative of ring joints, in which obround or circular ring gaskets are fitted into grooves in the flange faces.

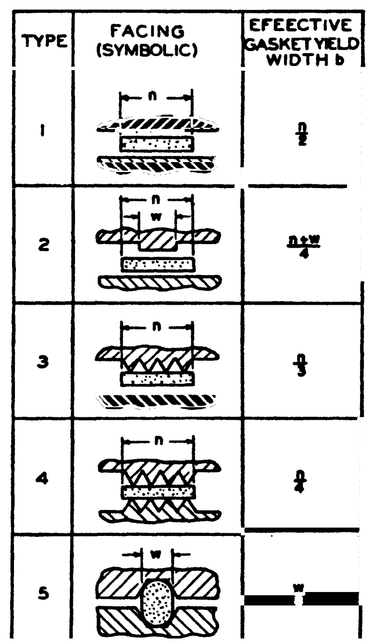


FIG. 10-9. Gasket Contact Surfaces and Effective Widths.

The gasket factor m is dependent upon the frictional resistance of the flange and gasket contact surfaces; representative values of m are listed in Table 10-1. The mean diameter G of the gasket may be obtained from Figs. 10-1 to 10-7; it is probably more nearly correct to refer to G as the mean effective diameter, since it is not always the same as the actual mean diameter (notably in the gasket of Fig. 10-6).

10-5. Flange Bolt Selection. The selection and design of bolts for flanged pipe and other flanged joints fitted with gaskets are closely associated with the selection of the gasket itself, and are based upon two criteria: the load imposed upon the bolts by operating conditions, and the necessity of obtaining and maintaining a sufficiently tight joint to prevent leakage. The bolt capacity must be equal to or greater than the total force represented by the internal pressure, or the load required to seat the joint-contact surfaces at atmospheric temperature conditions without internal pressure.

Under maximum operating or working conditions, the total bolt capacity should be at least sufficient to resist the fluid-static end force exerted by the

internal pressure on the area bounded by the mean diameter of the gasket or joint contact surfaces, and in addition maintain the joint-contact surface compression load Q which from experience will be sufficient to insure a tight joint. This bolt capacity, or minimum bolt load W_m, is found from

$$W_m = H + Q \qquad (10\text{-}2)$$

where H is the total fluid-static end pressure. The fluid-static end pressure is assumed to act over the area of a circle whose effective diameter is the mean diameter G of the gasket or joint-contact surface, and is given by

$$H = \pi G^2 p/4 \qquad (10\text{-}3)$$

The required force for effecting a tight joint at atmospheric temperature, without internal pressure, is called the joint-contact-seating load N, and is given by

$$N = \pi b G y r \qquad (10\text{-}4)$$

where y is the joint-contact-surface unit seating load, psi., or yield point stress of the gasket material, and r is the ratio of the maximum allowable bolt stresses at atmospheric and at operating pressure, psi. The latter value is inserted to permit direct comparison of the operating and initial atmospheric temperature loadings, and thus to facilitate the use of the allowable bolt stress at the operating temperature in subsequent computations. Values of y are obtained from Table 10-1.

The need for providing sufficient load to seat the gasket or joint-contact surfaces, in accordance with Eq. 10-4, will prevail in many low pressure designs, and in cases where the bolt load required for operating conditions, Eq. 10-2, is insufficient to seat the joint initially. For extremely high pressure design, where the bolt load is usually governed by operating conditions, Eq. 10-4 may be transposed and used to assure sufficient gasket or joint-contact area to avoid crushing under the initial bolt seating.

The capacity W_a of the bolts is equal to the product of the root area A_r of all the bolts and the maximum allowable working stress S at the operating temperature, or

$$W_a = SA_r \qquad (10\text{-}5)$$

Bolting is usually selected in conformity with the flange standards listed in Table 10-4; this usually involves some unavoidable excess resulting from the selection of the number of bolts in multiples of four. In other cases, particularly in low pressure designs, excess bolting capacity is provided in order to maintain bolt spacings within reasonable limits to assure uniform loading over the face of the flange. Carbon steel bolts may be stressed to 5500 psi., for temperatures not exceeding 450° F. Permissible stresses for alloy steel bolts at various temperatures are listed in Table 10-3. For temperatures up to 750° F.,

TABLE 10-3.—MAXIMUM ALLOWABLE WORKING STRESSES FOR ALLOY STEEL BOLTING MATERIALS, PSI.

ASTM	ASME	Grade	For Metal Temperatures Not Exceeding Degrees F.											
			—20 to 650	700	750	800	850	900	950	1000	1050	1100	1150	1200
A-96	S-9	A	13,000	11,950	10,900									
A-96	S-9	B	15,000	13,750	12,500									
A-96	S-9	C	16,000	14,700	13,400									
A-193-40T		B4	16,000	16,000	16,000	16,000	13,000	10,000						
A-193-40T		B5	16,000	16,000	16,000	16,000	13,800	11,000	8250	5850	3850	2200		
A-193-40T		B6	16,000	14,700	13,400	11,500	9500	6750						
A-193-40T		B7	16,000	16,000	16,000	16,000	13,000	10,000						
A-193-40T		B7a	16,000	16,000	16,000	16,000	13,800	11,000	8250	5850	3850	2200		
A-193-40T		B8	15,000	15,000	14,600	14,300	14,000	13,400	12,300	10,000	8000	6000	4600	3600

studs and stud bolts can be ordered to ASTM Specification A-96 (ASME S-9), in which the analysis is not defined, but which contains three classifications of physical properties: A, B, and C. Flange bolts and studs over 1-in. diameter are usually of the 8-thread series, as shown in Table 6-1. Nuts should conform to ASTM Specification A-194 (ASME S-51) Class 2 or 2H. ASTM Specification A-193 is also available for a wide selection of materials in various classifications and grades, for temperatures up to 1200° F. For high temperatures, nuts conforming to ASTM A-194, Classes 3 or 4, can be used. Standard nut proportions are given in Table 6-2.

Table 6-1 gives the minimum permissible bolt spacing for socket wrench clearance. In sizes from ½ to 1¼ in., however, a 3-in. pitch is preferred. An approximate expression for the maximum bolt spacing s producing a tight joint is given by:

$$s = 2D + 6t/(m + 0.5) \tag{10-6}$$

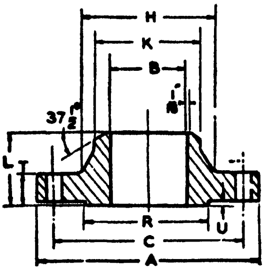
where D is the nominal diameter of the bolt, t the thickness of the flange, and m the gasket factor from Table 10-1.

10-6. Flange Types. Flanges are commercially available in a range varying from ½-in. nominal pipe size up to flanges for 24-in. OD pipe, and in pressure capacities of 150, 300, 400, 600, 900, 1500, and 2500 psi. The dimensions of standard carbon steel flanges for 150-, 300-, and 400-psi. capacities are listed in Table 10-4. Welding neck flanges are designed for butt-welded attachment, as shown in Fig. 10-2; threaded flanges are attached by means of pipe threads, as in Fig. 10-4. Slip-on or lap joint flanges are similar to the threaded flanges shown in Table 10-4, but have through holes to fit the outer diameter of the pipe; slip-on flanges are available in both hubbed and non-hubbed or ring types. Slip-on flanges, with or without hubs, may be used for Van Stone joints, as shown in Figs. 10-5 and 10-6; ring flanges may be welded to the pipe or nozzle by the methods shown in Fig. 10-7, or a through-welded ring, as shown in Fig. 10-3, may be employed. Minimum weld throat specifications are also shown in these illustrations. Blind flanges are used as cover plates for pipe ends, or as closures for manways and other vessel openings. Table 10-5 gives the maximum non-shock service pressure ratings for 150-, 300-, and 400-lb. flanges at various temperatures for water, steam, and oil service. These ratings are based upon standard facings, which include flat and serrated contact surfaces, lap joints, tongue-and-groove joints, and male-and-female joints. Pressure ratings for ring joint facings similar to Fig. 10-4 are slightly higher than these values for temperatures of 500° F. and over.

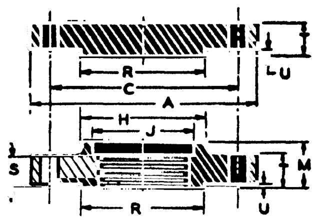
10-7. Flange Selection. For ordinary service, flange selection is based upon the data of Tables 10-4 and 10-5. For integral flanges, or for special conditions of service, an investigation of the stresses caused by bolting loads, gasket loads, and internal pressure may be required. For purposes of stress computation, flanges are classified as loose-type and as integral-type flanges.

Table 10-4.—Flange Dimensions

Welding Neck Flange



Blind Flange



Threaded Flange

Slip-on Flange Is Similar but Has a Hole of Diameter *J*.

K = Outer diameter of welded or seamless steel pipe.
B = Inner diameter of schedule 40 pipe for 150 and 300-lb. fittings, and of schedule 80 pipe for 400-lb. fittings.
N = Number of bolts.
D = Bolt diameter; hole diameter = D + ⅛ in.
U = 1/16 in. for 150 and 300 lb. fittings; *U* = ¼ in. for 400-lb. fittings.

	Nominal Pipe Size	½	¾	1	1¼	1½	2	2½	3	4	5	6	8	10	12	14	16	18
Flange Dimensions	A-150	3½	3⅞	4¼	4⅝	5	6	7	7½	9	10	11	13½	16	19	21	23½	25
	A-300/400	3¾	4⅝	4⅞	5¼	6⅛	6½	7½	8¼	10	11	12½	15	17½	20½	23	25½	28
	T-150	7/16	½	9/16	⅝	11/16	¾	⅞	15/16	15/16	15/16	1	1⅛	1 3/16	1¼	1⅜	1 7/16	1 9/16
	T-300	9/16	⅝	11/16	¾	13/16	⅞	1	1⅛	1¼	1⅜	1 7/16	1⅝	1⅞	2	2⅛	·2¼	2⅜
	T-400	9/16	⅝	11/16	13/16	⅞	1	1⅛	1¼	1⅜	1½	1⅝	1⅞	2⅛	2¼	2⅜	2½	2⅝
	H-300/400	1½	1⅞	2⅛	2½	2¾	3 5/16	3 15/16	4⅝	5¾	7	8⅛	10¼	12⅝	14¾	16¾	19	21
	H-150	1 3/16	1½	1 15/16	2 5/16	2 9/16	3 1/16	3 9/16	4¼	5 5/16	6 7/16	7 9/16	9 11/16	12	14⅜	15¾	18	19⅞
	R	1⅜	1 11/16	2	2½	2⅞	3⅝	4⅛	5	6 3/16	7 5/16	8½	10⅝	12¾	15	16¼	18½	21
Bolting dimensions	*N*-150	4	4	4	4	4	4	4	4	8	8	8	8	12	12	12	16	16
	N-300/400	4	4	4	4	4	8	8	8	8	8	12	12	16	16	20	24	24
	D-150	½	½	½	½	½	⅝	⅝	⅝	⅝	¾	¾	¾	⅞	⅞	1	1	1⅛
	D-300	½	⅝	⅝	⅝	¾	⅝	¾	¾	¾	¾	¾	⅞	1	1⅛	1⅛	1¼	1¼
	D-400	½	⅝	⅝	⅝	¾	⅝	¾	¾	⅞	⅞	⅞	1	1⅛	1¼	1¼	1⅜	1⅜
	C-150	2⅜	2¾	3⅛	3½	3⅞	4¾	5½	6	7½	8½	9½	11¾	14¼	17	18¾	21¼	22¾
	C-300/400	2⅝	3¼	3½	3⅞	4½	5	5⅞	6⅝	7⅞	9¼	10⅝	13	15¼	17¾	20¼	22½	24¾
Misc. dimensions	*L*-150	1⅞	2 1/16	2 3/16	2¼	2 7/16	2½	2¾	2¾	3	3½	3½	4	4	4½	5	5	5½
	L-300	2 1/16	2¼	2 7/16	2 9/16	2 11/16	2¾	3	3⅛	3⅜	3⅞	3⅞	4⅜	4⅝	5⅛	5⅝	5¾	6¼
	L-400	2 1/16	2¼	2 7/16	2⅝	2¾	2⅞	3⅛	3¼	3½	4	4 1/16	4⅝	4⅞	5⅜	5⅞	6	6½
	M-150	⅝	⅝	11/16	13/16	⅞	1	1⅛	1 3/16	1 5/16	1 7/16	1 9/16	1¾	1 15/16	2 3/16	2¼	2½	2 11/16
	M-300	⅞	1	1 1/16	1 1/16	1 3/16	1 5/16	1½	1 11/16	1⅞	2	2 1/16	2 7/16	2⅝	2⅞	3	3¼	3½
	M-400	⅞	1	1 1/16	1⅛	1¼	1 7/16	1⅝	1 13/16	2	2⅛	2¼	2 11/16	2⅞	3⅛	3 5/16	3 11/16	3⅞
	J	0.88	1.09	1.38	1.72	1.97	2.44	2.94	3 56	4.56	5.66	6.72	8.72	10.88	12.88	14.19	16.19	18.19
	S-300/400	⅝	⅝	11/16	13/16	⅞	1⅛	1¼	1¼	1 7/16	1 11/16	1 13/16	2	2 3/16	2⅜	2½	2 11/16	2¾

The flanges shown in Figs. 10-4, 10-5, and 10-6 are considered loose-type flanges; those shown in Figs. 10-1, 10-2, and 10-3 are integral-type flanges. The flanges shown in Fig. 10-7 are considered loose-type flanges if the working pressure is less than 300 psi., the metal temperature less than 700° F., the hub thickness less than 5/8 in., and the ratio of the inner diameter of the flange to the hub thickness less than 300. If any of these limits are exceeded, the flange should be considered an integral-type unit.

TABLE 10-5.—PRESSURE RATINGS FOR CARBON STEEL FLANGES WITH STANDARD FACINGS, FOR WATER, STEAM, AND OIL SERVICE

Service Temperatures Deg. F.	Water and Steam			Oil		
	150	300	400	150	300	400
100	230	500	670	230	500	670
150	220	480	640	220	480	640
200	210	465	620	210	465	620
250	200	450	600	200	450	600
300	190	435	580	190	435	580
350	180	420	560	180	420	560
400	170	405	540	170	405	540
450	160	390	520	160	390	520
500	150	375	500	150	375	500
550	140	360	480	140	360	480
600	130	345	460	130	345	460
650	120	330	440	120	330	440
700	110	315	420	110	315	420
750	100	300	400	100	300	400
800	85	250	335	92	275	370
850	70	200	270	82	245	330
900				70	210	280
950				55	165	280
1000				40	120	160

Integral-type flanges are subjected to three major types of stress: a longitudinal stress in the hub, a radial stress in the flange, and a tangential stress in the flange. These stresses are induced by moments caused by couples composed of the bolt load and the fluid-static pressure on the area of the mean gasket circle.

10-8. Flange Bolt Load. The bolt load W for the investigation of the flange stresses is equal to the average of the minimum required bolt load and the actual bolt capacity, or

$$W = (W_m + SA_r)/2 \tag{10-7}$$

or

$$W = (N + SA_r)/2 \tag{10-8}$$

whichever is greater. In addition to the minimum requirements for safety, these expressions provide a margin against abuse from overbolting of 50%

TABLE 10-6.—MAXIMUM ALLOWABLE WORKING STRESSES FOR STEEL FLANGE MATERIALS, PSI.

ASTM	ASME	Grade	For Metal Temperatures Not Exceeding Degrees F.											
			—20 to 650	700	750	800	850	900	950	1000	1050	1100	1150	1200
A-105	S-8	1	12,000	11,400	10,400	8300	6350	4400	2600					
A-105	S-8	2	14,000	13,300	11,900	8950	6450	4400	2600					
A-105	S-8	1*	12,000	11,400	10,400	9100	7400	5600	3800	2000				
A-105	S-8	2*	14,000	13,300	11,900	10,000	7800	5600	3800	2000				
A-181	S-50	1	12,000	11,400	10,400	8300	6350	4400	2600					
A-181	S-50	2	14,000	13,300	11,900	8950	6450	4400	2600					
A-181	S-50	1	12,000	11,400	10,400	9100	7400	5600	3800	2000				
A-181	S-50	2	14,000	13,300	11,900	10,000	7800	5600	3800	2000				
A-182	S-35	F3	14,000	14,000	14,000	14,000	13,400	11,000	8250	5850	3850	2200		
A-182	S-35	F5	15,000	15,000	15,000	14,000	13,400	11,000	8250	5850	3850	2200		
A-182	S-35	F6	15,000	14,000	13,000	11,500	9500	6750	4000	2400				
A-182	S-35	F7	15,000	15,000	15,000	14,400	12,700	10,400	8000	5000				
A-182	S-35	F8	15,000	15,000	14,600	14,300	14,000	13,400	12,300	10,000	8000	6000	4600	3600

* For 0.10% minimum silicon specification.

of the excess above the required minimum. Since excessive tightening of the bolts may affect the satisfactory operation of the unit, it is assumed that reasonable care will be taken in pulling up the bolts. Where an additional margin of safety is desired, or where it is necessary that the flange withstand the full load that may be placed upon the bolts, the flange is designed on the basis of the actual bolt capacity, or

$$W = SA_r \tag{10-9}$$

10-9. Flange Moments. For integral-type flanges, the total moment M exerted on the flange is equal to

$$M = Pa + u(W - H) + x(H - P) \tag{10-10}$$

where u is the distance from the bolt circle to the mean effective diameter of the gasket, and is equal to $(C - G)/2$, and a is the distance from the bolt circle to the point of application of the fluid-static pressure on the interior of the pipe, and is equal to $(C - J)/2$. When J is less than $20g$, the value $(C - J + g)/2$ may be substituted for distance a, as shown in Fig. 10-1. The distance x is found by:

$$x = \frac{2C - J - G}{4} \tag{10-11}$$

The force P at the hub is found from:

$$P = \pi J^2 p/4 \tag{10-12}$$

For integral-type flanges, and for all hubbed flanges, the longitudinal stress in the hub is obtained from

$$S_h = fM/LJj^2 \tag{10-13}$$

where M is the moment from Eq. 10-10.

This expression for longitudinal stress, and the equations which follow for radial and tangential flange stresses, involve several quantities which are partly empirical and partly theoretical; space does not permit more than a statement of these quantities and the method of their application and evaluation.

10-10. Flange Selection Factors. Factor f is a hub stress correction factor, and is based upon the ratio of the large diameter j to the small diameter g of the flange hub, and the hub length—pipe outer diameter—hub diameter ratio $h/\sqrt{Jg}$. The value of factor f is obtained from Fig. 10-10; the minimum value of f is 1.0; this value of f is also valid for hubs of uniform thickness where j/g equals 1.0, and for loose hubbed flanges.

Factors F and V are hub shape constants, and are obtained from Fig. 10-11 for integral-type flanges, and from Fig. 10-12 for loose-type flanges; both F and V are based upon the ratios j/g and $h/\sqrt{Jg}$.

Factors T, U, Y, and Z are factors involving the ratio of the outer and inner flange diameters A/J, based upon an assumed value of Poisson's ratio of 0.30, and are obtained from Fig. 10-13.

The factor L, in Eq. 10-13, is found from

$$L = \left(\frac{te+1}{T}\right) + \frac{t^3}{d} \tag{10-14}$$

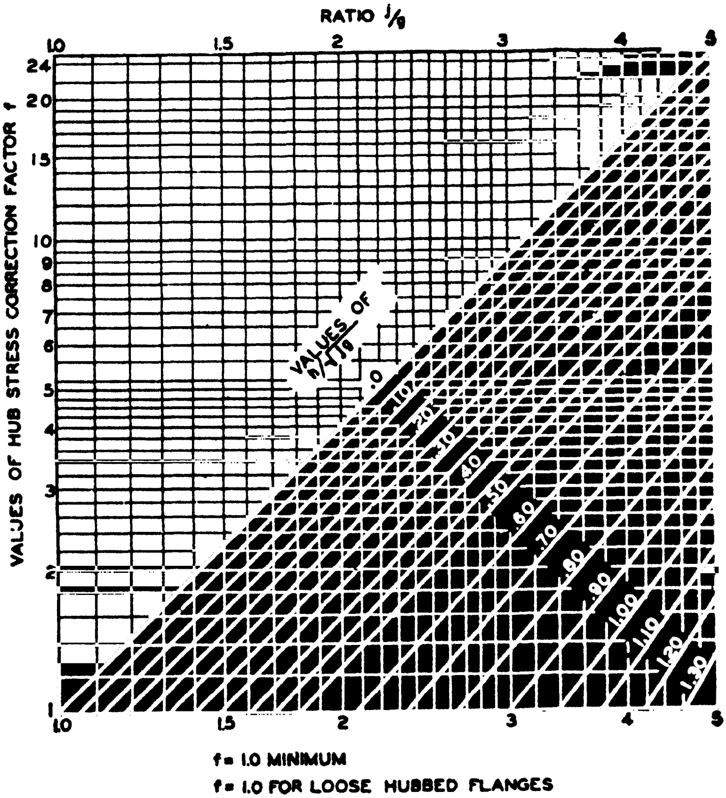


FIG. 10-10. Hub Stress Correction Factor f.

where t is the flange thickness, T is obtained from Fig. 10-13, e is given by

$$e = F/\sqrt{Jg} \tag{10-15}$$

and d is given by

$$d = \frac{Ug^2\sqrt{Jg}}{V} \tag{10-16}$$

For integral-type flanges, and for all hubbed flanges, the radial stress S_r in the flange is obtained from

$$S_r = \frac{(1.33te + 1)M}{Lt^2J} \tag{10-17}$$

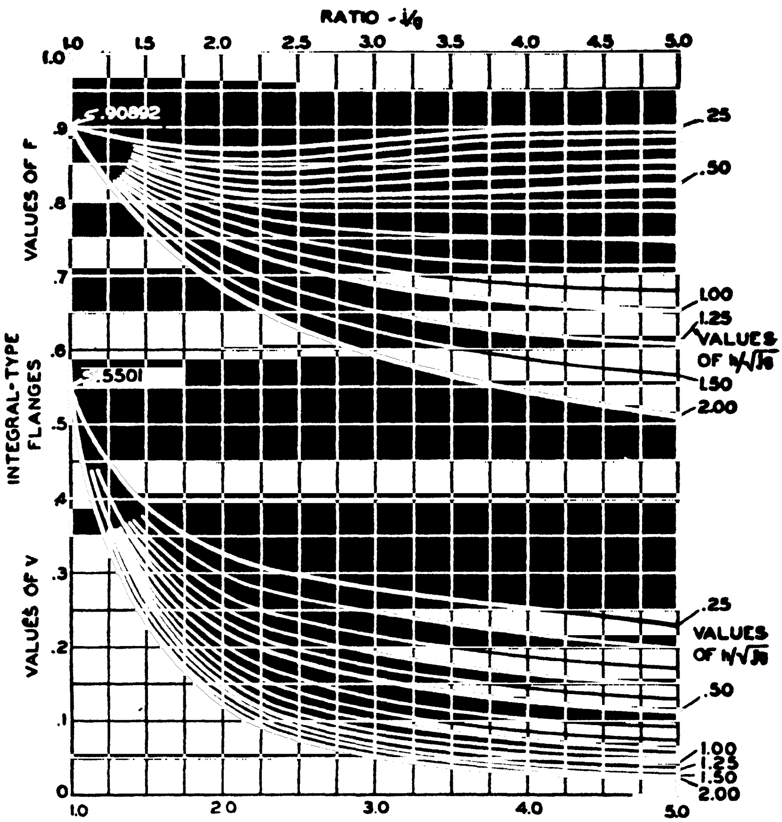


Fig. 10-11. Hub Shape Constants F and V for Integral-type Flanges.

For integral-type flanges, and for all hubbed flanges, the tangential stress S_n in the flange is obtained from

$$S_n = \frac{YM}{t^2J} - ZS_r \tag{10-18}$$

For loose-type flanges with or without hubs, with a gasket only partially covering the face of a lap on the end of the pipe or nozzle neck, as shown in Fig. 10-5 (left), and for loose-type flanges corresponding to those shown in Figs. 10-4 and 10-7, the total moment M is calculated from Eq. 10-10. In the

application of Eq. 10-10 for this purpose, distance J represents the inner diameter of the flange.

For loose-type flanges with full contact with or without a hub, and either with or without a gasket over the entire face of the lap (as in Fig. 10-6), the moment is given by

$$M = Pa + u(W - P) \tag{10-19}$$

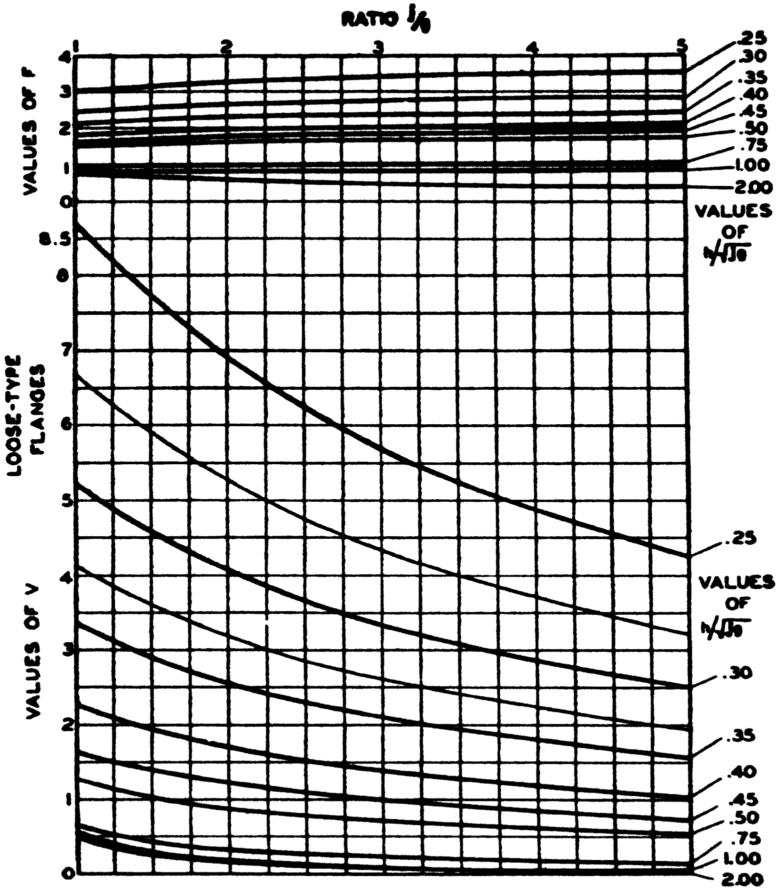


FIG. 10-12. Hub Shape Constants F and V for Loose-type Flanges.

For loose-type flanges with or without hubs, having line contact between the flange and the pipe lap, Fig. 10-5 (right), the moment is given by

$$M = Wa \tag{10-20}$$

Although flanges attached as shown in Figs. 10-4 and 10-7 may be reinforced to a considerable extent by the nozzle neck or pipe wall, no credit is given for such additional strength.

Loose-type ring flanges without hubs are subjected only to a tangential stress S_n, which can be calculated by Eq. 10-18. Since there is no radial stress, the quantity ZS_r is zero for this case. The computed stresses S_h, S_r, and S_t should not exceed the allowable stress S for flanged steels listed in Table 10-4;

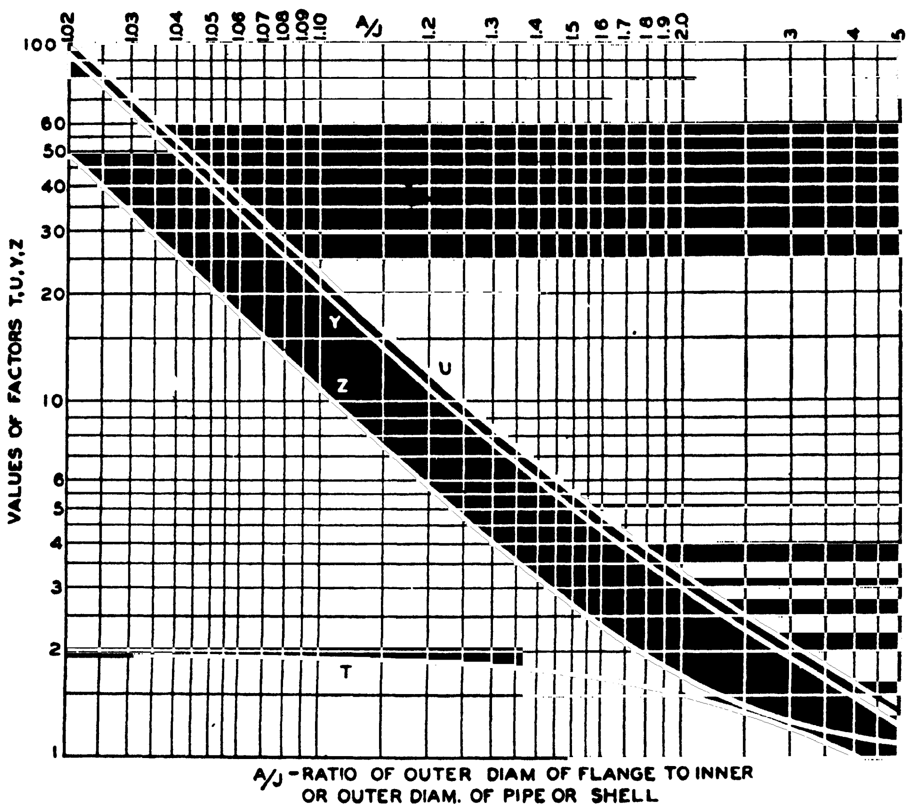


FIG. 10-13. Flange Stress Factors T, U, Y, and Z.

in addition, the average of the longitudinal stress and the radial or tangential stresses should not exceed S. Expressed as an equation:

$$S \geqq S_h, S_r, \text{ or } S_n \tag{10-21}$$

$$S \geqq \frac{S_h + S_r}{2} \text{ or } \frac{S_h + S_n}{2} \tag{10-22}$$

The unit shearing stress in the lap of a Van Stone joint, Fig. 10-5 or 10-6, and the shearing stress across the weld throats in the connections of Fig. 10-7 should not exceed 80% of S. Shearing stresses should be computed on the basis of the contact-surface-compression load Q, Eq. 10-1, or the joint-contact-

seating load N, Eq. 10-4, whichever is greater. For integral flange designs, similar to Fig. 10-1, the radius at the juncture of the flange and vessel or nozzle wall must be equal to at least one fourth of the maximum thickness of the hub, but must not be less than $\frac{3}{16}$ in.

Example 10-1. Select a welding neck flange for a 6-in. pipe subjected to an internal pressure of 160 psi. at a temperature of 400° F. The flange faces are to be tongue-and-groove, similar to Fig. 10-7 (left), and a compressed asbestos gasket is to be used. Investigate the flange and bolt stresses for this service.

Solution. From Table 10-5, the pressure rating of a 150-lb. carbon steel flange at a temperature of 400° F. is 170 psi.; and such a flange will be satisfactory. From Table 10-2, the mean gasket diameter G is 8 in., and the gasket width w is $\frac{1}{2}$ in. From Table 10-1, the gasket compression factor m is 2.5 for a compressed asbestos gasket. The effective gasket yield width b corresponds to Type 2, Fig. 10-9, where w is equal to $\frac{1}{2}$ in., and n is equal to $\frac{1}{2}$ in.; b is equal to $(0.5 + 0.5)/4$, or 0.25. The joint-contact-surface compression load Q, from Eq. 10-1, is

$$Q = 2\pi \times 0.25 \times 8 \times 2.5 \times 160 = 5040 \text{ lbs.}$$

The fluid-static end pressure based upon the mean gasket area, from Eq. 10-3, is

$$H = \pi \times 8^2 \times 160/4 = 8040 \text{ lbs.}$$

The minimum bolt load W_m based upon operating conditions, from Eq. 10-2, is

$$W_m = 8040 + 5040 = 13{,}080 \text{ lbs.}$$

The bolt load required to obtain a tight joint in assembly is found from Eq. 10-4; the joint-contact-surface unit seating load y from Table 10-1 is equal to 4500 psi. for a compressed asbestos gasket; the ratio r of the maximum allowable bolt stresses at atmospheric and at operating pressures is equal to unity, since the operating temperature is within the atmospheric temperature range as indicated in Table 10-3 for bolting materials. Substituting in Eq. 10-4:

$$N = \pi \times 0.25 \times 8 \times 4500 \times 1.0 = 28{,}270 \text{ lbs.}$$

and the bolt design will be based upon the larger of the values W_m and N.

From Table 10-4, it is seen that a 6-in. 150-lb. flange requires eight $\frac{3}{4}$-in. diameter bolts. From Table 6-1, the root area of a $\frac{3}{4}$-in. bolt is 0.302 in., and the total root area A_r of either bolt is 8×0.302, or 2.416 sq. in. If a trial computation be made using carbon steel bolts, for which the allowable stress S is 5500 psi., the bolt capacity (Eq. 10-5) is

$$W_a = 5500 \times 2.416 = 13{,}300 \text{ lbs.}$$

While this capacity is sufficient for operation (load W_m) it is very much too low for initial assembly, and it will be necessary to use ASME-S-9, Grade A bolting material, for which an allowable load of 13,000 psi. (Table 10-3) may be assumed. For this material, the bolt capacity, from Eq. 10-5, is

$$W_a = 13{,}000 \times 2.416 = 31{,}400 \text{ lbs.}$$

Standard flange bolting, as listed in Table 10-4, should be ample to provide a tight joint, but a check of the bolt spacing may be of interest The angle between the centerlines of adjacent bolts is 45°; the bolt circle radius, from Table 10-4, is $4\frac{3}{4}$ in.; the center-to-center distance S of adjacent bolts, which corresponds to the chord of the bolt circle, is

$$S = 2 \times 4\tfrac{3}{4} \times \sin 22^\circ\ 30' = 3.65 \text{ in.}$$

The maximum spacing for a tight joint, from Eq. 10-6, is

$$S = 2 \times 0.75 + [6 \times 0.938/(2.5 + 0.5)] = 3.38 \text{ in.}$$

The standard spacing is sufficiently close to the approximate maximum to insure a tight joint.

For an investigation of the flange stresses, it is necessary to determine the probable bolt load that will be exerted. From Eq. 10-7

$$W = \frac{13,080 + 31,400}{2} = 22,240 \text{ lbs.}$$

From Eq. 10-8

$$W = \frac{28,270 + 31,400}{2} = 29,835 \text{ lbs.}$$

From Eq. 10-9

$$W = 31,400 \text{ lbs.}$$

Since the preceding value of 29,835 lbs. includes an overbolting capacity (section 10-8), it may be used with safety.

The total moment M exerted on the flange is obtained from Eq. 10-10. The diameter J is equivalent to the inner diameter of a Schedule 40 pipe, or 6.065 in.; the distance C from Table 10-4 is equal to 9.50 in. The distance a is equal to $(9.5 - 6.065)/2$, or 1.718 in. (This value of a may be used since the ratio of diameter J and thickness g is 6.065/0.28, or 21.6, which is greater than 20.) The distance u is equal to $(9.5 - 8)/2$, or 0.75; the distance x from Eq. 10-11 is

$$x = \frac{2 \times 9.5 - 6.065 - 8}{4} = 1.234 \text{ in.}$$

The force at the hub, from Eq. 10-12, is

$$P = \pi \times 6.065^2 \times 160/4 = 4640 \text{ lbs.}$$

Substituting in Eq. 10-10,

$$M = 4640 \times 1.718 + 0.75(29,835 - 8040) + 1.234(8040 - 4640) = 28,520 \text{ in.-lbs.}$$

The longitudinal stress in the hub is obtained from Eq. 10-13. The minimum hub thickness g is equal to one half of the difference between diameters K and B, Table 10-4, which correspond to the outer and inner diameters of Schedule 40 pipe; the thickness g is given by $(6.625 - 6.065)/2$, or 0.280 in. The maximum hub thickness j is equal to one half of the difference between diameters H and B, Table 10-4, or $(7.563 - 6.065)/2$, or 0.749 in. The value of the ratio A/j is obtained by substituting values from Table 10-4, and is 11/6.065, or 1.820. The ratio j/g is equal to 0.749/0.280, or 2.67; the value $\sqrt{Jg}$ is equal to $\sqrt{6.065 \times 0.280}$, or 1.305. The value of the hub length h is obtained from Fig. 10-2 (right) and Table 10-4; in the latter, the difference between lengths L and T is equal to $3.5 - 1.0$, or 2.5 in.; if the straight portion of the hub has a length equal to $1.5g$, as indicated in Fig. 10-2, the length h is equal to $[2.5 - (1.5 \times 0.280)]$, or 2.08 in.

The factor d is calculated from Eq. 10-16, and is dependent upon the values of factors V and U from Figs. 10-11 and 10-13. In Fig. 10-11, the value of V depends upon the quantity $h/\sqrt{Jg}$ and j/g, which are respectively equal to 2.08/1.305, or 1.594, and 2.67; V is therefore equal to 0.07. In Fig. 10-13, the value of U depends upon the ratio A/J, equal to 1.820, and is equal to 3.9. Substituting in Eq. 10-16,

$$d = \frac{3.9 \times 0.280^2 \times 1.305}{0.07} = 5.67$$

The factor e is calculated from Eq. 10-15; factor F from Fig. 10-11, and depends upon values of j/g and $h/\sqrt{Jg}$ which are equal to 2.67 and 1.594; F then has a value of 0.630. The value of e is:

$$e = \frac{0.630}{1.305} = 0.483$$

The factor L is obtained from Eq. 10-14; factor T from Fig. 10-13 depends upon the ratio A/J or 1.820, and has a value of 1.6. Substituting

$$L = \frac{0.938 \times 0.483 + 1}{1.60} + \frac{0.938^3}{5.67} = 1.055$$

The factor f is obtained from Fig. 10-10, and is dependent upon $h/\sqrt{Jg}$ and j/g, and has a value of unity since the value of 1.594 for $h/\sqrt{Jg}$ does not appear on the chart.

From these values, the longitudinal stress is found by Eq. 10-13 to be

$$S_h = \frac{1.0 \times 28{,}520}{1.055 \times 6.065 \times 0.749^2} = 7940 \text{ psi.}$$

The radial stress in the flange is found from Eq. 10-17 to be

$$S_r = \frac{[1.33 \times 0.938 \times 0.483 + 1](2850)}{1.055 \times 0.938^2 \times 6.065} = 8140 \text{ psi.}$$

The tangential stress in the flange is found from Eq. 10-18; this is dependent upon factors Y and Z which are found from Fig. 10-13 to be equal to 3.5 and 1.9. Substituting,

$$S_t = \frac{3.5 \times 28{,}520}{0.938^2 \times 6.065} - (1.9 \times 8140) = 3290 \text{ psi.}$$

The allowable stress S in the flange must be equal to or greater than any of these induced stresses, or greater than the average of the longitudinal and either the radial or tangential stresses. Substituting in Eq. 10-22,

$$S = \frac{7940 + 8140}{2} = 8040 \text{ psi.}$$

and

$$S = \frac{7940 + 3290}{2} = 5615 \text{ psi.}$$

The radial stress of 8140 psi. is then the controlling stress; by reference to Table 10-6, it is clear that any of the listed materials are satisfactory for this service.

10-11. Fittings for Pressure Vessels. Openings in pressure vessels are necessary to make connections to other vessels or to piping systems, or for inspection or access purposes. The strength of the vessel will of course be reduced to some extent by the opening in the vessel wall, and since it is standard procedure to reinforce the adjacent metal rather than to design for increased strength in the whole of the vessel, some type of connecting medium between the shell and the external part is usually added. Flanges and nozzles are in common use for this purpose, and may logically be considered in three classifications: integral, fabricated, and formed.

Integral flanges and nozzles are formed from a portion of the shell or head material itself, and are used to some extent in such applications as that of Fig. 3-12. From a manufacturing viewpoint, the forming operation is expensive and limited in application; it should be noted also, that a manway flange adds 15% or more to the thickness (and consequently to the cost) of the head itself. Fabricated flanges and nozzles are constructed from pipe, tubes, and plates. Representative acceptable nozzle constructions are shown in Figs.

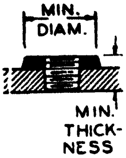


FIG. 10-14. Threaded Opening with Welded Pad.

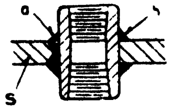


FIG. 10-15. Use of Standard Pipe Coupling for Threaded Opening.

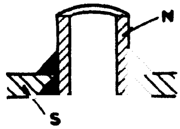


FIG. 10-16. Unreinforced Welded Nozzle.

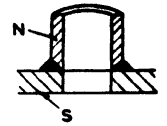


FIG. 10-17. Unreinforced Welded Nozzle.

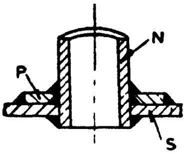


FIG. 10-18. Welded Nozzle with Outer Reinforcing Plate.

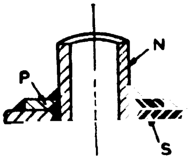


FIG. 10-19. Welded Nozzle with Outer Reinforcing Plate.

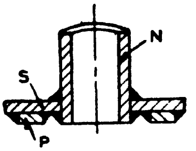


FIG. 10-20. Welded Nozzle with Inner-reinforcing Plate.

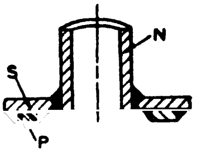


FIG. 10-21. Welded Nozzle with Inner-reinforcing Plate.

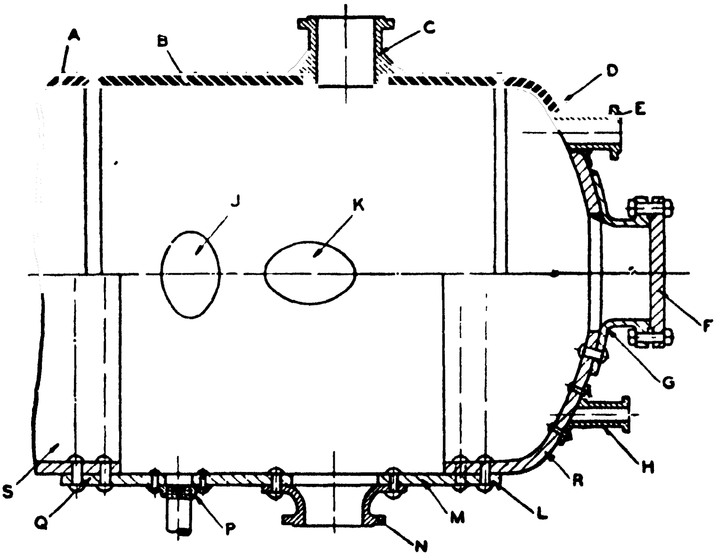


FIG. 10-22. Representative Welded and Riveted Fittings.

10-14 to 10-21. Fabricated nozzles and connections are employed to a considerable extent in low pressure applications, and in designs where the opening is relatively small, but have been largely replaced by formed nozzles and flanges in medium and high pressure designs.

Several types of formed nozzles for both welded and riveted application are shown in Fig. 10-22. This illustration shows a cylindrical pressure vessel with a dished head; the details above the horizontal axis are representative of welded joint practice, those below of riveted joint practice. *A* and *B* are successive courses, with the head *R* attached to *B* by a double-welded butt joint. *Q* and *S* are successive riveted courses, with double-riveted chain lap joints at the girth joint, or juncture of *Q* and *R*. *J* and *K* represent elliptical manway openings; the regular-type *J* has the minor diameter of the ellipse in alignment with the vessel axis and is referred to as "long way on sweep"; the irregular-type *K* is referred to as "short way on sweep."

Welding-type nozzles, shown at *C*, Fig. 10-22, and in Fig. 10-24, are available in 150, 300, 400, 600, 900, and 1500 psi. pressure capacities, and in size ranges as indicated in Table 10-7. Twenty and 24-in. sizes are also obtainable. Welding-type nozzles in 150- and 300-lb. ratings are made of ASTM-181 steel, corresponding to ASME-S-50 carbon steel, in two grades; 400-lb. and higher rating nozzles are made of ASTM-105 steel, corresponding to ASME-S-8 carbon steel. Adjusted allowable pressures at various temperatures are given in Table 10-5. The nozzles may be either the extended "neck" type shown in Fig. 10-22, or with a plain surface at the region of attachment as shown in Table 10-7. Plain-type nozzles must be fillet welded inside and outside; extended-neck type nozzles can be butt welded from the inside and fillet welded around the exterior, as illustrated in Fig. 10-22. Welding-type nozzles are usually of sufficient cross-sectional area to provide all necessary reinforcement or replacement metal to compensate for the shell opening. In order to provide adequate reinforcement area, the hub dimensions are varied to suit the shell thickness.

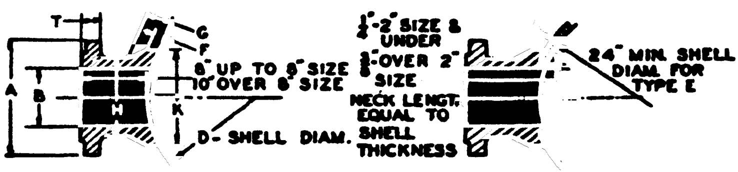
E, Fig. 10-22, represents a forged steel welding neck, which is inserted in a hole in the shell or head and butt welded in place. Welding necks are provided with a reinforcing plate *D*, as illustrated, to provide replacement metal for the opening. Welding necks can be obtained commercially in the same capacities and sizes as welding-type nozzles; a representative selection is listed in Table 10-8.

Straight-neck nozzles, shown at *G*, Fig. 10-22, and in Fig. 10-28, may be adapted either to riveting or welding, and are commercially obtainable in 150, 300, 400, and 600 psi. pressure capacities, and in sizes up to 24 in. Two varieties, low-type and high-type, are available; the low-type nozzle is shown in Figs. 10-22 and 10-28; the high-type nozzle is approximately twice as long as the low-type nozzle. These nozzles can be obtained with curved flanges for offset location, as shown at *H*, with conical or spherically faced flanges for head

attachment, as at *G*, or with cylindrical curving for attachment to the vessel shell.

The straight-neck nozzle at *G* is used as a manway, with a blind flange *F* serving as a manway cover. Several manufacturers also list special nozzles of this character, expressly adapted for manways, in 100, 150, and 200 psi. pressure capacities, and in 16, 18, 20, 24, 30, and 36 in. diameters. Dished, bolted-on covers of corresponding size are also obtainable.

TABLE 10-7.—WELDING NOZZLE DATA



Type P, Plain Type E, Extended Neck

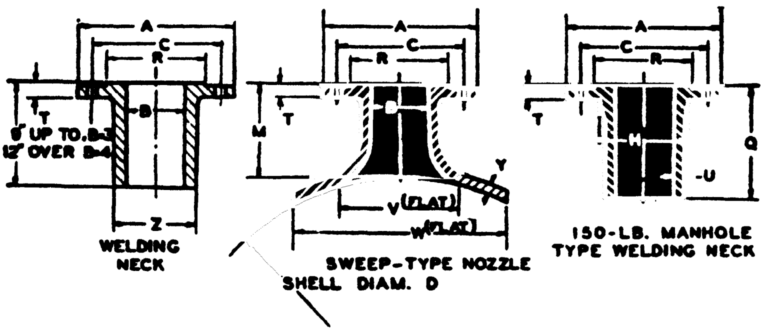
Size B	H Standard Pressure		DIMENSIONS—VARIOUS SHELL THICKNESSES t																		Recommended Minimum D	Minimum D
			F						G					K							
	150	300 400	¾	1	1¼	1½	2	2½	¾	1	1¼, 1½	2	2½	¾	1	1¼	1½	2	2½		
1	2	2½							1¼		4			3			3				
1½	2⅝	2⅞							1 5/16					3½			4				8
2	3⅛	3⅛							1⅝					4			5				
2½	3¾	3 11/16			¾						2½			4¾			5½			12	
3	4⅛	4⅝							1¼					5⅛			6½				9
4	5½	5¾											3½	6½			8			15	12
5	6½	7										3½		7½			9			18	15
6	7¾	8⅛							1⅝	3½				9		10			11	30	
8	9¾	10⅛						1¼	1⅞				4⅛	12		12½		14		42	18
10	12	12⅝			1		1¼		2¾		3¾		4⅛	14½		16		17			20
12	14⅝	14¾		1								4	4½	17½	18	18½	19		21	60	24
14	16	16⅜				1½			3					20		21		22½	24		30
16	18	19	1	1¼				2	3½				3¾	23	24		25	26	28	66	36
18	20	21					1¾				3¾	4		26		27		28½	30½	72	42

Note: Bolting Data and Flange Dimensions from Table 10-4

Flared or sweep-type nozzles, shown at *N*, Fig. 10-22, are available in riveting or welding types. These can be used to advantage on all pressure vessels where a smooth orifice is required, since the design minimizes flow turbulence, erosion, and corrosion that may result from sharp-cornered outlet construction. Representative sizes are given in Table 10-8.

Forged boiler flanges *P*, Fig. 10-22, are available with plain or standard taper pipe threads in three pressure capacities—standard, extra-heavy, and heavy marine types—corresponding to Schedule 40, 80, and 160 pipe. These flanges are usually riveted in place, and can be curved to suit offset or dished head application. A range of sizes and weights is given in Table 10-9.

TABLE 10-8.—NECK AND NOZZLE DATA



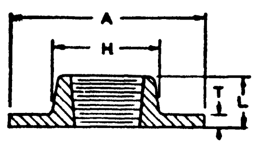
Data and Dimensions A, C, R, and T from Flange Tables

Size *B*	1	1¼	1½	2	2½	3	4	5	6	8	10	12	16	18
Z-150-lb.	2	2⅜	2⅝	3¼	3¾	4¼	5½	6½	7¾	9¾	12	14⅜	18	20
Z-300-lb. 400-lb.	2⅛	2½	2¾	3¼	3 15/16	4⅝	5¾	7	8⅛	10¼	12⅝	14¾	19	21
W-150-lb.				10	11½		13½	14½	15½	18	20½	24		
V-150-lb.				2¼	6	6¼	7	7½	8¾	11⅝	12	14		
Y-150-lb.				½	7/16		½			9/16	¾			
M-150-lb.				4			5				6½			
W-300-lb.			10	11	13		15½	16	18	20½	24	27		
V-300-lb.			2	2½	6	6¼	7¼	9	10	12	14	16		
Y-300-lb.			⅝					11/16	¾	13/16		1		
M-300-lb.			5				6		7					
U													¾	
H													18	20
Q													10 or 12	

10-12. Design of Attachments API-ASME Code. Although the underlying principles are the same, the design of vessel attachments differs slightly in the API-ASME and the ASME-UPV Codes; each will be considered separately. Figs. 10-14 and 10-17 represent typical connecting media specified by

the API-ASME Code as satisfactory for openings up to 2-in. diameter, for which no additional reinforcement is required. Plugs, pipe and seamless tubing up to 4-in. nominal diameter can be screwed directly into the vessel shell, if the limitations as to plate thickness, etc., set forth in Table 10-10 are observed. A pad of weld metal, as illustrated in Fig. 10-14, may be used to build up the necessary thickness. It should be noted that threaded nozzles 2- to 4-in. nominal size, inclusive, cannot be used for vessels with shell thicknesses over 1 1/4 in., or for operating temperatures over 450° F. The 4-in. size is employed as a plug closure only. The attachment shown in Fig. 10-15 must be either an extra-heavy steel pipe coupling or a tapped seamless steel tube welded into the wall; either method of attachment, *a* or *b*, is satisfactory. The maximum opening in this type of attachment is that corresponding to 1 1/2-in. nominal pipe size.

TABLE 10-9.—BOILER FLANGES



Pipe Size	Standard				Extra-Heavy				Heavy Marine			
	A	*T*	*L*	*H*	*A*	*T*	*L*	*H*	*A*	*T*	*L*	*H*
3/4	6	5/16	1	1 7/8					6	7/16	1 1/8	1 7/8
1				2 1/16	6	3/8	1 1/16	2 1/16				2 1/16
1 1/4	6 1/2			2 1/2	6 1/2			2 1/2	6 1/2		1 1/4	2 1/2
1 1/2	7	3/8	1 1/4	2 13/16	7	1/2	1 3/8	2 13/16	7	1/2	1 3/8	2 13/16
2	8			3 5/16	8			3 5/16	8		1 1/2	3 5/16
2 1/2	8 1/2		1 1/2	3 7/8	8 1/2		1 5/8	3 7/8	8 1/2		1 5/8	3 7/8
3	9			4 5/8	9			4 5/8	9	9/16		4 5/8
4	10	7/16	1 3/4	5 9/16	10	9/16	1 7/8	5 9/16	11		2 1/4	5 7/8
5	11 1/2	1/2	2	6 11/16	11 1/2	5/8	2 1/8	6 11/16	12 1/2	3/4	2 3/4	7
6	12 1/2			7 3/4	12 1/2			7 3/4	13 1/2		3	8 1/8
8	15	5/8	2 1/2	9 7/8	15	3/4	2 5/8	9 7/8				
10	17 1/2	3/4		12 3/16	17 1/2	7/8		12 3/16				

The API-ASME Code requires reinforcement for all openings greater than 2-in. diameter; for openings of any diameter if the distance between the centers of adjacent openings is less than the sum of the diameters of the openings; and for openings of any size if rapid fluctuations in pressure within the vessel are anticipated.

Figs. 10-18 and 10-23 show a fabricated nozzle *N* made of a seamless steel tube held in the vessel wall *S* by welds *W*. If the inner diameter *d* of the nozzle is greater than 2 in., a reinforcing plate *P*, welded in place, is required to compensate for the reduction in strength caused by the opening in the shell. In accordance with the API-ASME Code, computation of required and available reinforcement is based upon cross-sectional areas lying in a plane passing through the axis of the opening and nozzle; in other words, projected areas rather than volumes of parts are considered.

TABLE 10-10.—REQUIREMENTS FOR SCREWED FITTINGS DIRECTLY ATTACHED TO PRESSURE VESSELS

Pipe Size Nominal	ASME-API		ASME-UPV		Minimum Diameter of Pad
	Min. Plate Thickness	Temp. and Plate Th. Limitation	Min. Plate Thickness	Pressure Limitation	
½	0.45	None			1½
¾	0.45	"			1⅞
1	0.55	"	0.348	None	2$\frac{7}{16}$
1¼	0.55	"	0.348	"	3⅛
1½	0.60	"	0.435	"	3⅝
2	1.10	450° 1¼	0.435	"	4½
2½	1.20	"	0.875	"	5⅜
3	1.20	"	0.875	"	6$\frac{11}{16}$
*†4	1.40	"	0.875	125 psi.	8$\frac{11}{16}$
†5			1.00	"	
†6			1.00	"	
†8			1.25	"	
†10			1.625	"	
†12					

* For plug closure only, ASME-API Code.
† No limitation on inspection openings and plug closures—ASME-UPV Code.

Fig. 10-23 also shows an outline drawing of the nozzle of Fig. 10-18; in this illustration, *n* represents the actual thickness of the nozzle, *t* the actual thickness of the shell, *d* the diameter of hole in the nozzle, *f* the theoretical thickness of the nozzle, *h* the theoretical thickness of the shell considering the longitudinal seam or joint of the vessel, and *e* the efficiency of the longitudinal seam or joint of the vessel expressed as a decimal fraction.

To be effective, any reinforcement that is provided should be closely adjacent to the opening. The API-ASME Code specifies that the permissible limit within which reinforcement may be credited is a rectangular area B equal to the product of distance GJ and twice distance GH, Fig. 10-23. Distance GH is equal to the diameter d of the nozzle hole; distance GJ is equal to $t + 2a$ or $t + a + b$, whichever is smaller. As a is equal to $2.5t$ and b to $2.5n$, distance GJ may be expressed as the smaller of the two quantities $6t$ or $3.5t + 2.5n$.

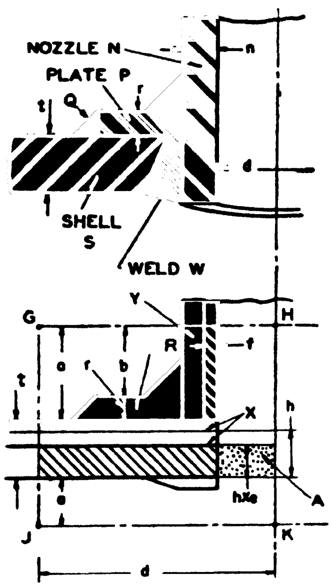


FIG. 10-23. Fabricated Nozzle.

The actual projected area of the plate displaced by the nozzle opening is equal to the product of the diameter d of the nozzle hole and the thickness t. The plate thickness t, however, is usually larger than the required theoretical thickness h to permit the use of standard gages. The theoretical thickness h is based upon the shell thickness required with a longitudinal seam of an efficiency e; if no portion of the nozzle passes through such a seam (and the vessel should be constructed so that such coincidence does not occur), the actual shell thickness necessary to withstand the internal pressure for which the vessel is designed is he. The actual area A for which replacement must be provided is therefore

$$A = dhe \qquad (10\text{-}23)$$

Within the limits of area B, there is excess replacement area equivalent to the difference between the actual and the theoretical seamless plate thicknesses. This area X is considered to be integral reinforcement, and will reduce the amount of external reinforcement required.

$$X = d(t - he) \qquad (10\text{-}24)$$

The nozzle wall is usually thicker than the theoretical value required, leaving an area Y for reinforcement:

$$Y = 2a(n - f) = 5t(n - f) \qquad (10\text{-}25)$$

or

$$Y = 2(b + r)(n - f) = (5n + 2r)(n - f) \qquad (10\text{-}26)$$

(either applies, depending upon whether a or b is the controlling dimension for distance GJ). The choice of equations for Y will depend upon whether a or b is used as the controlling dimension for the rectangular area of reinforcement. If the area to be added to provide the necessary reinforcement is R, then

$$R = A - X - Y \qquad (10\text{-}27)$$

In Fig. 10-23 one half of the nozzle, and consequently one half of the various areas, is shown. The area for which replacement is required is stippled, areas which are available for reinforcement are left blank, and areas not available for reinforcement are cross-hatched. The available area for reinforcement inherent in formed fittings is given for 150- and 300-lb. sweep-type nozzles in Table 10-11, and for 300- and 400-lb. welding-type nozzles in Table 10-12. It should be noted that these areas vary with the nozzles designed for a given

TABLE 10-11.—AVAILABLE REINFORCING AREAS IN SWEEP-TYPE NOZZLES

150-lb. Standard

Nozzle Size Inches	Limiting Dimension of Reinforcing Area in Saddle	Shell Thickness						
		½	⅝	¾	⅞	1	1⅛	1¼
2	10	4.89	5.20	5.39	5.39	5.39	5.39	5.39
2½	11½	3.87	4.26	4.26	4.26	4.26	4.26	4.26
3	11½	3.51	3.72	3.72	3.72	3.72	3.72	3.72
4	13½	4.47	4.64	4.78	4.78	4.78	4.78	4.78
5	14½	4.72	4.89	5.03	5.03	5.03	5.03	5.03
6	15½	4.81	5.08	5.78	5.78	5.78	5.78	5.78
8	18	5.25	5.77	6.24	6.74	6.74	6.74	6.74
10	20½	8.08	8.71	9.30	9.85	9.85	9.85	9.85
12	24	10.01	10.71	11.34	11.97	12.51	13.05	13.38

300-lb. Standard

Nozzle Size Inches	Limiting Dimension of Reinforcing Area in Saddle	½	⅝	¾	⅞	1	1⅛	1¼
1½	10	5.61	6.00	6.39	6.78	6.78	6.78	6.78
2	11	6.87	7.24	7.81	8.16	8.16	8.16	8.16
2½	13	5.74	6.13	6.52	6.52	6.52	6.52	6.52
3	13	5.74	6.13	6.48	6.48	6.48	6.48	6.48
4	15½	6.69	7.12	7.51	7.90	7.90	7.90	7.90
5	16	6.54	7.01	7.36	7.71	7.71	7.71	7.71
6	18	7.32	7.79	8.26	8.69	9.08	9.08	9.08
8	20½	8.61	9.24	9.83	10.38	11.13	11.13	11.13
10	24	10.32	11.02	11.65	12.28	12.82	13.36	13.69
12	27	12.81	13.75	14.53	15.23	15.86	16.44	16.98

plate thickness. In Table 10-11, the limiting length of the saddle, which in some cases is less than the outer diameter of the saddle, gives the limit of the length of the reinforcing area. The data in Table 10-12 are also used for approximate computation for 150-lb. nozzles, but for final results these values should be checked by reference to a manufacturer's catalog.

Table 10-12.—Available Reinforcing Areas for 300- and 400-Lb. Welding-Type Nozzles (Table 10-7)

Nozzle Size Inches	Shell Plate Thickness					
	¾	1	1¼	1½	2	2½
1	1.87	2 50	3.13	3 75	5.00	5.40
1½	2.62	3.75	4.68	5.59	7.19	7.89
2	3.04	4.72	5 52	6.28	6.67	6.67
2½	3.52	5.62	6 52	7.42	8.21	8.21
3	3.66	6.60	7.64	8.63	9.90	9.90
4	4.02	8.03	9 12	10.22	11.86	15.02
5	4.24	8.25	9 50	10.75	14.25	17.35
6	4.94	9 53	10 54	11 95	15.08	20.11
8	6.52	10 05	12 18	13.78	20.26	24.03
10	7 84	13 14	15 66	17 86	24 61	28.99
12	9.44	15 00	17 68	23 44	25 44	35 94
14	10 64	13 52	18 90	21.47	29.56	35 94
16	12.52	18.28	21.12	26 25	33.38	42.75
18	14.22	18 28	23.94	27 00	36 56	44.25

Note: Available reinforcing areas for 150-lb. welding-type nozzles are somewhat smaller, being about 0.50 less in the larger sizes.

Seamless fabricated nozzles may be made from any standard grade of steel; if cast steel is used, the minimum nozzle thickness, regardless of pressure, is ⅜ in.

Example 10-2. The stabilizer tower of Example 4-7 is equipped with a 6-in. inside diameter nozzle, made of cast steel with an ultimate tensile strength of 55,000 psi. Design the nozzle and any necessary reinforcement.

Solution Steel castings fall in group C classification, and the material factor F_m has a value of 0.92. From Table 4-1, a factor F_s with a value of 0.167 should be applied to the ultimate strength. The allowable design stress S_t is

$$S_t = 55{,}000 \times 0.92 \times 0.167 = 8450 \text{ psi.}$$

Since the nozzle is seamless the efficiency is 1.0, and the thickness from Eq. 4-3 is

$$t = \frac{6 \times 300}{2 \times 8450 \times 1.0 - 300} = 0.1085 \text{ in.}$$

Since a cast steel nozzle is used, the minimum wall thickness is ⅜ in.; the actual stress in the nozzle is therefore very low. In Fig. 10-23, n is equal to ⅜ in., t to 13/16 in., d to 6 in., h to 0.77 in., f to 0.109 in., and e to 0.80 in. The area of reinforcement required, from Eq. 10-23, is

$$A = 6 \times 0.77 \times 0.80 = 3.7 \text{ sq. in.}$$

Assume a reinforcing area without any consideration of a reinforcing plate. Then distance GH equals 6 in., and distance GJ equals 6 × 13/16, or 4⅞ in. The area X available for reinforcement in the plate, from Eq. 10-24, is

$$X = 6[0.813 - (0.77 \times 0.80)] = 1.182 \text{ sq. in.}$$

The area Y available for reinforcement in the nozzle, from Eq. 10-25, is

$$Y = 5 \times 0.813(0.375 - 0.109) = 1.08 \text{ sq. in.}$$

The allowable unit stress in the nozzle wall is 8450 psi., while the allowable stress in the shell, from Example 4-1, is 14,560 psi. The available reinforcing area in the nozzle must therefore be multiplied by the factor 8450/14,560 × 1.08, or 0.627 sq. in. The reinforcement area R required, from Eq. 10-27, is

$$R = 3.7 - 1.182 - 0.627 = 1.891 \text{ sq. in.}$$

This reinforcement may be obtained by employing a circular plate P, with an inner diameter to fit the outer surface of the nozzle, an outer diameter of 10 in., which brings the plate extremities with the limits of the reinforcing area, and a tentative thickness of ⅝ in. The available area in this plate is

$$10 - 6\tfrac{3}{4} \times \tfrac{5}{8} = 2.03 \text{ sq. in.}$$

which is slightly greater than the required area R.

It will be advisable to recalculate the reinforcing area of the excess material in the nozzle wall, because the presence of a reinforcing plate reduces the limit of area GJ. From Eq. 10-26, we have

$$Y = [(5 \times 0.375) + (2 \times 0.625)](0.375 - 0.109) = 0.831 \text{ sq. in.}$$

Reducing Y to an equivalent strength area,

$$Y = 8450/14,560 \times 0.831 = 0.483 \text{ sq. in.}$$

which gives a value of R of

$$R = 3.7 - 1.182 - 0.483 = 2.035 \text{ sq. in.}$$

which is satisfactory. A more rigorous analysis might also include the areas of the welds within the limits of the reinforcing area and, if necessary, the projection of the nozzle inside the vessel. The foregoing analysis, however, is on the safe side and is computed more easily.

Example 10-3. Select a suitable welding-type nozzle for the 6-in. opening in the stabilizer tower of Example 4-7.

Solution. The internal pressure of 300 psi. at a temperature of 250° F., with a shell thickness of 13/16 in., requires a 6-in. 300-lb. nozzle, with hub dimensions for a shell, from Table 10-7. The nozzle flange includes a considerable area of reinforcement, which is found to be 9.53 sq. in. from Table 10-12. From this area the necessary thickness of nozzle wall for pressure resistance must be deducted. The allowable design stress, for

S-50 grade I carbon steel at a temperature of 250°, from Table 10-6, is 12,000 psi., and the theoretical thickness of the nozzle wall, from Eq. 4-3, is

$$t = \frac{6 \times 300}{2 \times 12{,}000 - 300} = 0.077 \text{ in.} = f$$

The reinforcing area, as illustrated in Fig. 10-24, has a height outside the shell plate of either $2.5t$ or $2.5n$ plus the height of the nozzle hub. This height is equal to 2.5×0.813, or 2.032, which is less than the hub height alone, as given in Table 10-7, and is therefore the limiting value. The area required for pressure resistance in the nozzle wall is then $2.032 \times 2 \times 0.077$, or 0.313 sq. in.

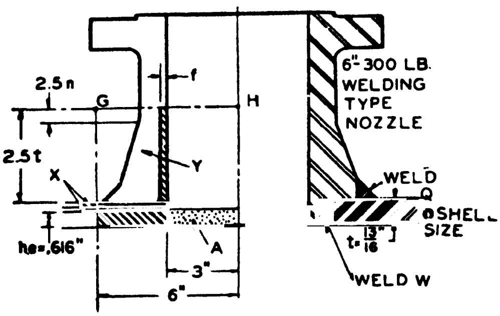


Fig. 10-24. Welding Type Nozzle.

The area A for which replacement is required, from Eq. 10-23, is $6 \times 0.77 \times 0.80$, or 3.70 sq. in. The net area in the nozzle is $8.69 - 0.313$, or 8.37 sq. in., which is greater than the area to be replaced, even though the reinforcement provided by the shell is neglected. In general, the reinforcement furnished by the nozzle is so much in excess of that required that computation is unnecessary for any except special conditions.

It is interesting to note that if the operating temperature of the stabilizer tower were 900° F., as in Example 4-2, a welding-type nozzle designed for a primary service pressure of 600 lbs. would have been required. The 300- and 400-lb. nozzles listed in Table 10-7 are suitable for only 210- and 280-lb. pressures at this temperature (Table 10-5).

10-13. Strength of Welded Joints for Attachments. The stresses induced in the welded joints of attachments and nozzles should be investigated for all fabricated attachments, and should often be checked for special or highly loaded formed attachments. The allowable unit stress S in a welded joint used for an attachment is found from

$$S = S_u \times F_s \times F_t \times F_r \times e \times F_m \times P \qquad (10\text{-}28)$$

Here, S_u represents the minimum ultimate strength of the material, psi., and F_s is the allowable percentage, expressed as a decimal, of S_u that is used for design. Values of F_s at various temperatures are gotten from Table 4-1. F_t is a stress-type factor, having a value of 1.0 for tension and 0.80 for shear. F_r is the stress-relief factor 1.06 described in section 4-10, and may be used if the entire vessel is stress relieved. For vessels not subjected to stress relieving, S_r should be taken as 1.0. E is the efficiency of the joint; values of e, from Chap. 4, for various types of joints, are as follows:

Double-welded butt joint	0.80
Single-welded butt joint	0.70
Double-welded lap joint	0.65
Single-welded lap joint	0.55

F_m is the material factor (section 4-10) and is equal to 1.00 for group A, 0.97 for group B, and 0.92 for group C steels. P is a weld position factor, and is equal

to 1.00 for loads perpendicular to the weld, 0.750 for loads parallel to the weld, and 0.875 for combined loadings around openings. To illustrate, suppose it is desired to find the allowable shearing stress in a single-welded butt joint between a nozzle and a shell, made of material having a minimum ultimate tensile strength of 50,000 psi., group C. The entire vessel is stress relieved, and its operating temperature is 800° F. From Eq. 10-28

$$S = 50{,}000 \times 0.18 \times 0.8 \times 1.06 \times 0.70 \times 0.92 \times 0.875 = 4300 \text{ psi.}$$

Acceptable proportions for nozzle joints are shown in Fig. 10-25. The weld throat dimensions at t and t_2 should have the following proportions: For t or n equal to or less than ¾ in., with t greater than n, $t_1 + t_2$ should have a minimum value of $1.25n + 0.10t$. For t or n greater than ¾ in., with t greater than n, $t_1 + t_2$ should exceed 1 in. The minimum value of t_1 or t_2 is ¼ in. When n is greater than t, the values of n and t should be interchanged in the above expressions. Nozzles or tubes attached by single-welded lap joints are not permitted by the API-ASME or ASME-UPV Code.

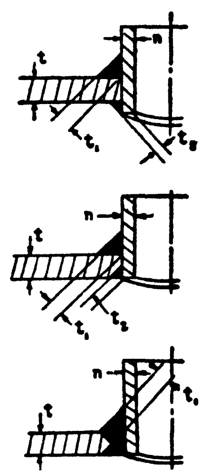


FIG. 10-25. Nozzle Weld Proportions.

10-14. Weld Analysis API-ASME Code. The strength analysis of welded joints for attachments designed and constructed on the basis of the API-ASME Code is partly theoretical and partly empirical. The theoretical basis for such an analysis is predicated upon the possibility of failure by splitting the nozzle, blowing or tearing off the nozzle or reinforcing plates, or tearing the nozzle. Since these modes of failure may occur in combination, an empirical basis for determining the required strength is used; this basis is essentially one in which the total strength of the attachment is made equal to or greater than the strength of the material removed from the shell by the opening. Since the reinforcing plate, and in many instances some portion of the nozzle wall, serves to reinforce the opening in the vessel wall, it is essential that the welds joining the plate to the shell (or the plate to the nozzle) develop sufficient strength to permit the transmission of the increment loads carried by the separate units. The strength analysis of attachment welding is illustrated by a problem.

Example 10-4. Analyze the stresses in the welds used for attaching the nozzle of the stabilizer tower of Example 4-7.

Solution. The load which will be carried by the reinforcement will be equal to the displaced area given by Eq. 10-23, minus the integral reinforcement provided by the shell, and given by Eq. 10-24. (It should be noted that the excess area in the nozzle is not considered, since it is necessary to attach the nozzle to the vessel by the welds.) From Example 10-2, $A - X$ is equal to 3.7 — 1.182, or 2.518 sq. in.

From Example 4-7, the allowable gross design stress is 14,560 psi. The load to be carried by the attachment welding and the nozzle wall is $2.518 \times 14{,}560$, or 36,300 lbs. This load must be carried into the nozzle wall by shear and tension in the attachment welding.

According to the code, the principal possibility of failure is that encountered in splitting the nozzle and tearing out the shell, by the forces acting on the plane passing through the axis of the nozzle. Such tendency to failure develops shear resistance at the throat of the butt weld W surrounding the nozzle, and in the throat of the fillet weld Q which holds the plate P to the shell. The excess area within the nozzle may also be considered to develop shear resistance in this regard, since it projects through and is securely welded to the shell. The allowable unit shearing stresses in welds W and Q are calculated from Eq. 10-28. Here S_u is equal to 60,000 psi. and F_s to 0.25, from Table 4-1; the stress values based upon the plate material stress values, rather than the nozzle material stress values, are used because the possibility of failure by tearing out of the plate is under consideration. Since the shearing stress is desired, F_t is taken as 0.80. Stress relieving is not involved and the factor F_r has a value of unity in this case. The welded joint efficiency E is taken as 0.80 for the double-welded butt joint W, and as 0.65 for the lap joint Q; although the latter is a single-welded lap joint, the higher efficiency is justified in this case because of the stiffening effect of weld W. The material factor F_m is taken from Example 4-1 as 0.97. The position factor P is taken as 0.875 because both W and Q are all-around welds. Substituting in Eq. 10-28, the allowable unit shear in the weld W is given by

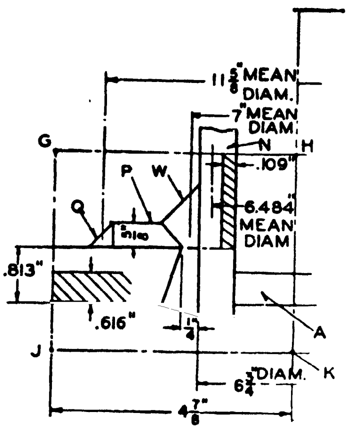


FIG. 10-26. Fabricated Nozzle Analysis.

$$S = 60{,}000 \times 0.25 \times 0.80 \times 0.80 \times 0.97 \times 0.875 = 8150 \text{ psi.}$$

and in the weld Q by

$$S = 60{,}000 \times 0.25 \times 0.80 \times 0.65 \times 0.97 \times 0.875 = 6610 \text{ psi.}$$

The allowable unit shear in the nozzle wall is 80% of 8450, or 6760 psi.

The butt weld W resists horizontal shear along its ¼-in. throat, which extends for a distance equal to one half the mean circumference of the weld; the mean diameter of the weld is 7 in., as shown in Fig. 10-26, and the shearing resistance of the inner weld is

$$7 \times \frac{\pi}{2} \times 0.25 \times 8150 = 22{,}400 \text{ lbs.}$$

The outer fillet weld resists shear across the diagonal throat, and its length is equal to one half the indicated mean circumference, Fig. 10-26; the mean diameter is 11⅝ in.; the throat depth is 0.707 × 0.625, or 0.442 in.; and the shear resistance is

$$11{,}625 \times \frac{\pi}{2} \times 0.442 \times 6610 = 53{,}300 \text{ lbs.}$$

The shear resistance afforded by the nozzle wall is based upon the excess area in the nozzle, over and above that required to withstand the internal pressure, or hoop stress. The theoretical thickness of the nozzle is 0.109, and the mean diameter of the excess area is $[6 + (2 \times 0.109) + (0.375 - 0.109)]$, or 6.484 in. The shear resistance afforded by the nozzle wall is

$$6.484 \times \frac{\pi}{2} (0.375 - 0.109) 6760 = 18{,}300 \text{ lbs.}$$

The total strength of the nozzle wall and the inner weld is 18,300 plus 22,400, or 40,700 lbs.; this in itself is greater than the load of 36,300 lbs. In addition, the outer weld has a strength of 53,300 lbs.; the entire attachment welding is therefore amply strong.

It is necessary to check the weld between the shell and the nozzle wall to ascertain whether it has sufficient strength to develop the shearing resistance of 40,700 lbs. carried by the nozzle and the inner weld. The inner weld is equal to the product of the shell thickness

and one half the outer circumference of the nozzle. The stress in the weld is based upon the nozzle material, and the allowable unit tensile stress, from Eq. 10-28, is

$$S = 55{,}000 \times 0.167 \times 1.0 \times 0.80 \times 0.92 \times 0.875 = 5900 \text{ psi.}$$

The outer diameter of the nozzle is 6¾ in., and the allowable load that the weld will carry is

$$6.75 \times \frac{\pi}{2} \times 0.813 \times 5900 = 51{,}000 \text{ lbs.}$$

An analogous check of the weld between the reinforcing plate and the nozzle wall should also be made. This load is

$$6.75 \times \frac{\pi}{2} \times 0.625 \times 5900 = 39{,}200 \text{ lbs.}$$

This value is slightly less than the shear strength of the inner weld and nozzle wall, but it is greater than the total load of 36,300 lbs., so the weld is satisfactory.

The excess area in the nozzle wall that serves as reinforcement, from Example 10-2, is 0.831 sq. in., and the total load this area will carry is 0.831 × 8450, or 7030 lbs. The weld strength of 39,200 lbs. is sufficient to tranmit this load from the reinforcing plate to the nozzle wall.

It is also necessary to check the outer weld between the reinforcing plate and the shell to determine if it will carry the load from the shell to the plate. The area of the plate is 2.66 sq. in., and the load carried is 2.66 × 14,560, or 38,750 lbs. The strength of this weld is 53,300 lbs., so it is amply safe.

Example 10-5. Analyze the stresses in the welds employed to attach the welding-type nozzle selected in Example 10-4 to the shell.

Solution. From Example 10-4, the load to be carried by the attachment welding to the shell is 36,300 lbs. Fig. 10-24 indicates that a butt weld W is used between the extended neck of the nozzle and the shell, and a fillet weld Q is used at the juncture of the hub and shell. The hub height F, from Table 10-7, is ¾ in. for this size of nozzle permitting a ¾-in. fillet weld Q; the weld W has a width of ¾ in.

The possibility of failure by splitting the nozzle and tearing out the shell, caused by the forces acting on a plane passing through the axis of the nozzle, is counteracted by shear developed in the nozzle welds and shear in the nozzle wall itself. The allowable unit shearing stress in the nozzle welds is the smaller of two values: the shear based upon the nozzle material, and the shear based upon the shell material. The allowable unit shear based upon the nozzle material is

$$S = 60{,}000 \times 0.20 \times 0.80 \times e \times 1.00 \times 0.875 = 8400 \text{ psi.}$$

(The value of 60,000 psi. is the minimum ultimate tensile strength of the nozzle material; the factor F_s is obtained from the allowable value of 12,000 psi. given for this material in Table 10-6.

The efficiency e for single-welded butt joints is 0.70, and the double-welded lap joints 0.65; the allowable unit stress in the butt joint is 8400 × 0.70, or 6080 psi., and in the lap joint is 8400 × 0.65, or 5460 psi. These values are less than the respective values of 8150 and 6610 psi. for the shell given in Example 10-4, and are the ones used.

The area in shear is equal to the product of the weld throat distance and one half the mean circumference of the weld. Assuming a ¼-in. neck thickness, and a consequent diameter of 6½ in., the allowable load on the butt weld is

$$(6.50 + 0.75)\frac{\pi}{2} \times 0.75 \times 6080 = 52{,}000 \text{ lbs.}$$

The outer diameter of the nozzle is 10 in., giving a mean circumference, with a ¾-in weld, of 10¾ in. The throat width of the weld is 0.750 × 0.707, or 0.53 in. The allowable load in shear is then

$$10.75 \times \frac{\pi}{2} \times 0.53 \times 6080 = 54{,}400 \text{ lbs.}$$

Either weld is amply strong to resist the load of 36,300 lbs.

10-15. Design of Attachments—ASME-UPV Code. The essential principles governing the design of attachments in accordance with the ASME-UPV Code are similar to those controlling designs based upon the API-ASME Code, but they differ in many of the details, and separate treatment of the provisions of the ASME-UPV Code is of importance.

Plain unreinforced holes up to 8-in. diameter, such as threaded openings tapped directly into the shell, drilled holes for tube connections, and studded connections, are permitted if the restrictions indicated in Fig. 10-27 are com-

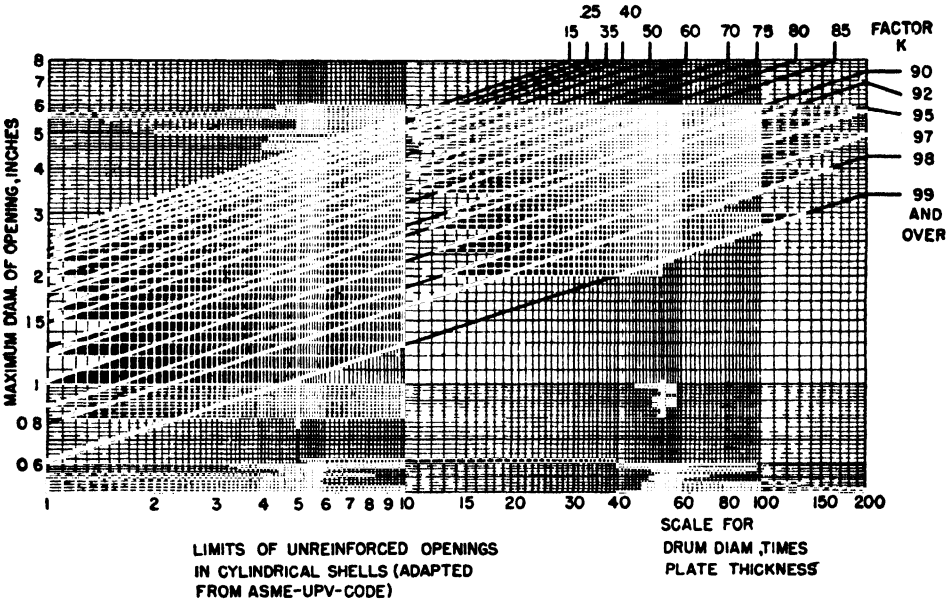


Fig. 10-27. Limits of Unreinforced Openings in Cylindrical Shells.

plied with. The maximum diameter of an unreinforced opening is found by locating the product of the shell diameter and thickness on the horizontal scale, finding the intersection of the vertical ordinate with the curve representing the proper K factor, and locating the maximum diameter of the opening on the scale at the left. The factor K is given by

$$K = PD/2tS \qquad (10\text{-}29)$$

where P is the working pressure, and S the working stress, psi, and D the outer diameter and t the actual thickness of the shell, in inches. The diameter of a threaded opening should be taken as the root diameter of the thread.

Example 10-6. Find the maximum diameter of an unreinforced opening in the pressure vessel of Example 3-1.

Solution. In the original problem, P is equal to 200 psi., D to 73.563 in., t to 0.781 in., and S to 11,000 psi. From Eq. 10-29, the factor K is

$$K = 200 \times 73.563/2 \times 0.781 \times 11{,}000 = 0.856$$

The product Dt is equal to 73.563×0.781, or 57.5. Entering the chart at 57.5 on the horizontal scale, and finding the intersection with the curve $K = 0.85$, the maximum hole diameter, on the vertical scale at the left is 5.7 in.

10-16. Threaded connections must conform to the data in Table 10-10; if the shell thickness is not sufficiently great to permit the minimum length of engagement, a welded pad can be used as illustrated in Fig. 10-14. When the working pressure exceeds 125 psi., threaded joints for nipple and pipe connections shall not be used at either the shell or terminating end of such connections, except for inspection openings and end closures.

Any opening greater than that permitted by Fig. 10-27 must be reinforced. For circular or elliptical openings, the reinforcement area required must be at least equal to an area equal to the product of twice the diameter of the opening minus 2 in. and the theoretical seamless thickness T, as indicated in Fig. 10-28. This thickness is equal to

$$T = \frac{PD_v}{1.8S} \tag{10-30}$$

and the required reinforcing area is

$$A = (2D-2)T = (2D-2)(PD_v/1.8S) \tag{10-31}$$

where D_v is the inner diameter of the vessel, P is the working pressure, and S is the allowable unit stress. This area is equal to twice the area of the rectangle $BEFC$. It should be noted that D is the diameter of the hole in the shell, which is often made larger than the diameter of the hole in the nozzle when calking or seal welding is permitted.

Any reinforcement that is credited must lie within a rectangle whose length is equal to twice the diameter D of the opening, and whose height is equal to $2m + t$; Fig. 10-28 shows rectangle $HGJK$, which represents one half of this area. The distance m is equal to the smaller of the quantities $2.5t$, $2.5n + f$, or $2.5n + f_1$, where t is the actual shell thickness, n is the actual nozzle thickness. f is the thickness of the nozzle flange, and f_1 is the thickness of an interior reinforcing ring. No credit can be given for additional strength of material having a higher tensile strength than that of the vessel wall to be reinforced. If the reinforcing material, however, is of lesser tensile strength than the vessel wall, the actual area of the part added for reinforcement shall be computed on a reduced basis equal to the product of the actual area and the ratio of the strength of the reinforcement and the vessel wall materials.

10-17. Riveted Connections ASME-UPV Code. Forged and cast steel nozzles are used for both welded and riveted attachments. Cast iron nozzles and fittings with riveted connections may be used if the pressure and tem-

perature do not exceed 160 psi. and 450° F. Riveted cast iron attachments can be considered as reinforcement provided the minimum thickness of the parts is ⅝ in. and the total area of the cast iron reinforcement is at least twice that required for steel.

Riveted connections must develop sufficient resistance in tension to guard against failure induced by internal pressure acting within the area of the calking circle. If the attachment is calked on the inside, the area is that of the inside calking circle; if it is calked on the outside, or on *both* inside and outside, the area of the outside calking circle is used. The shearing strength of the rivets on

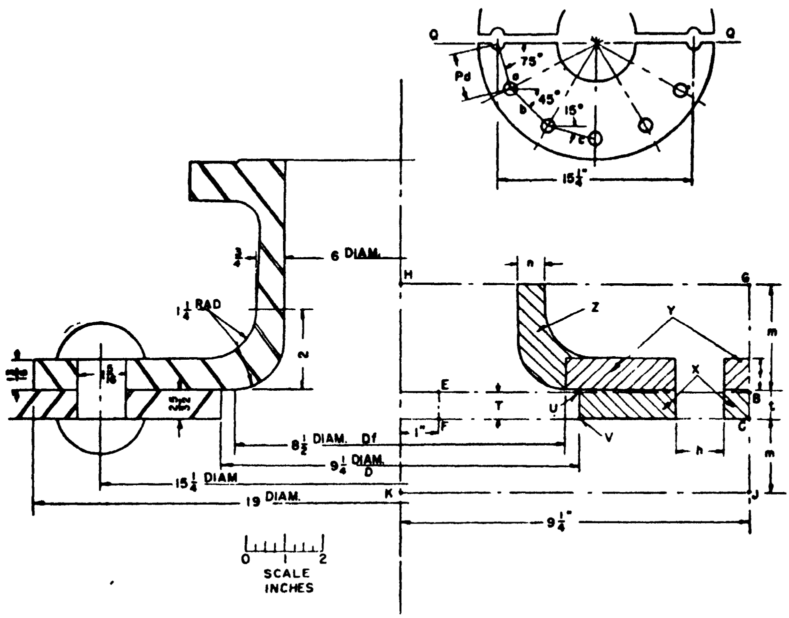


Fig. 10-28. Straight-neck Nozzle-low Type.

one side of the centerline of the attachment must be equal to the tensile strength of the cross section of the reinforcement within the entire limiting rectangle (twice *GHKJ*) or the tensile strength *R* represented by twice the area *UEFV*, Fig. 10-28, whichever is smaller. Twice the area *UEFV* may be considered to have a tensile strength $R = S(D-2)T$; by substituting in Eq. 10-30, the tensile strength *R* may be expressed as

$$R = \frac{PD_v(D-2)}{1.8} \tag{10-32}$$

where D_v is the vessel diameter and D the hole diameter, in inches, and P is the internal pressure, psi. In addition to the consideration of shear failure, the rivet bearing area must be adequate.

The shell plate may fail by tearing around the rivet holes. The efficiency of a riveted joint, considering failure across the net section of one line of rivets, is $(P - D)/P$, where P is the pitch of the rivets, and D is the diameter of the rivet hole. A joint subjected to direct tension has its greatest stress existing on a right section. For a diagonal ligament or section this stress can be resolved into a direct tension perpendicular to, and a shearing stress parallel to, the centerline of the holes. For unsymmetrically spaced holes the series of diagonal ligaments a, b, and c, as illustrated in Fig. 10-29, is referred to the longitudinal projection of the rivet circle on a diameter, and the efficiency e of this longitudinal projection, termed the ligament efficiency, is obtained from the following:

$$e = \frac{D_v - 2(L - a_2 - b_2 - c_2)}{D_v} \tag{10-33}$$

$$e = \frac{D_v - 4(L - a_2 - b_2 - c_2)}{0.8\,D_v} \tag{10-34}$$

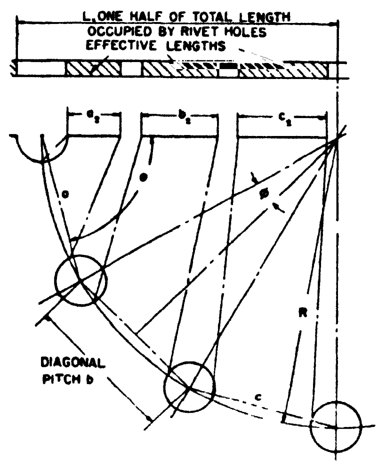


FIG. 10-29. Determination of Ligament Efficiency.

These expressions are empirical, and are derived from the ASME Power Boiler Code, 1940. To avoid failure, the lesser of the values of e obtained from Eqs. 10-33 and 10-34 must not be less than the efficiency of the longitudinal joint of the vessel. If the diameter D_v of the shell exceeds 60 in., D_v shall be used as 60. The projected lengths a_2, b_2, c_2, etc., are to be obtained by computing the projected longitudinal components a_1, b_1, and c_1 of distances a, b, c, etc., and multiplying these by the efficiency referred to the longitudinal joint obtained from Fig. 10-30. To use this chart, find the ratio between the diagonal pitch a and the rivet hole diameter D, enter the chart along the ordinate representing the angle between the diagonal and longitudinal centerlines, find its intersection with the curve representing the P/D ratio, and read the referred efficiency on the vertical scale.

Example 10-7. The vessel of Example 3-4 is to be equipped with a 6-in. diameter low-type straight neck nozzle riveted in place. Select a suitable nozzle from a manufacturer's catalog, and design the rivet arrangement and any necessary reinforcement.

Solution. Straight neck nozzles are commercially obtainable in 150-, 300-, 400-, and 600-lb. pressure capacities; from Table 10-5, a vessel with a working pressure of 150 psi. at a temperature of 125° F. will require a nozzle with a 150-lb. primary service capacity having an allowable working pressure of 220 lbs. at 150° F. This nozzle is delineated in Fig. 10-28, necessary dimensions having been obtained from a manufacturer's catalog;

1⅝-in. diameter rivets will be used corresponding to the size used for the joints in the vessel. For riveted construction the opening in the shell should be equal to the diameter D_f at the points of tangency of the nozzle neck and flange, plus an allowance of ¾ in. to permit inside calking. The diameter D_f is equal to 8½ in., from catalog data; diameter D is therefore 9¼ in. As the maximum diameter of an unreinforced opening in this vessel, from Example 10-6, is 5.7 in., suitable reinforcement is required.

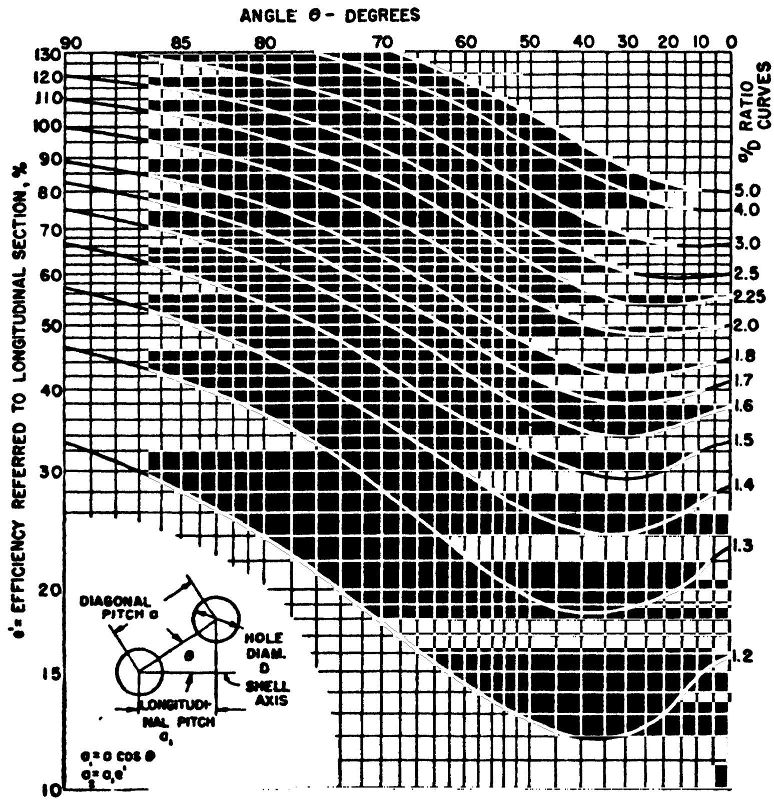


Fig. 10-30. Referred Efficiency of Diagonal Ligaments.

The area for which reinforcement must be provided is twice the area of rectangle $EBCF$, which, from Eq. 10-31, is equal to

$$A = (2 \times 9.25 - 2)(200 \times 72/1.8 \times 11{,}000) = 12 \text{ sq. in.}$$

The area within which reinforcement may be credited has a length equal to $2D$, or 18.5 in. The shell thickness t is 0.781 in., the nozzle wall thickness is 0.75 in., and the nozzle flange thickness is 0.813 in. The height m above or below the shell plate, $2.5t$, which equals 2.5×0.781, or 1.95 in., is the controlling height since the distance $2.5n + f$ which equals $2.5 \times 0.75 + 0.813$, or 2.688 in., is greater.

The area available for reinforcement in the shell is represented by X, Fig. 10-28, and is given by $(2D - D - 2h)t$, or $(D - 2h)t$, where h represents the rivet hole diameter. This area is equal to $[9.25 - (2 \times 1.313)]0.781$, or 5.18 sq. in. Note that the projected area

of the rivets, represented by the quantity $2ht$, is deducted from the shell area available for reinforcement.

The areas available for reinforcement in the nozzle are the net flange area Y, and the corner area Z, shown cross-hatched in Fig. 10-28. The flange area is equal to the product of the length $2D - D_f - 2h$ and the thickness f of the flange, which equals $[18.5 - 8.5 - (2 \times 1.313)]0.813$, or 6 sq. in.

The corner area Z is estimated as twice the area of a rectangle of a height m and a breadth of 1 in., or $2 \times 1.95 \times 1$, or 3.9 sq. in. The total area available for reinforcement is thus $X + Y + Z$, or $5.18 + 6 + 3.9$ which is equal to 15.1 sq. in., and is ample.

If a reinforcing plate or ring had been required, it should have been riveted to the inside of the shell. The minimum thickness of such a reinforcing ring can be obtained from Table 10-12; the maximum outer diameter should be equal to the diameter of the flange of the nozzle; the inner diameter should be somewhat greater than the hole D in the shell, but the inner circumference should be at least 2 in. from the centerline of the rivet hole to provide a sufficient marginal distance to prevent failure by tearing out the hole.

No consideration has been given to the area required to resist the tensile stress in the nozzle wall, because the smallest part of the hole in the nozzle is entirely outside the area of reinforcement.

As a tentative design for the rivets required to attach the nozzle to the shell, assume twelve 1⁵⁄₁₆-in. rivets equally spaced on a 15½-in. diameter circle. The rivet pitch is approximately $15.25 \times \pi/12$, or 4 in., which is about three times the diameter; the edge distance is $(19 - 15.25)/2$, or 1.88 in., which is about one and one half times the rivet diameter. These proportions give sufficient clearness between heads to permit riveting, and the marginal distance is small enough to permit effective outside calking, and large enough to avoid failure by tearing out at the edge.

The allowable unit tensile stress permitted by the ASME-UPV Code for steel rivets is 11,000 psi. The strength of the rivets in tension is thus $11{,}000 \times 12 \times \pi \times 1.313^2/4$, or 178,500 lbs. If we employ outside calking for the nozzle, the total tensile force in the rivets is equal to the product of the area of the calking circle and the unit pressure within the vessel, and is equal to $200 \times \pi \times 19^2/4$, or 56,700 lbs.; this value is less than one third of the allowable load. Obviously, the rivet load induced by inside calking will be considerably smaller.

Failure may occur by separation along section QQ as illustrated in the reduced scale detail in Fig. 10-28. Two types of failure are possible: splitting the nozzle, as illustrated, or shearing the five rivets in one half of the nozzle flange.

The force exerted on one half of the nozzle section is equal to the tensile strength represented by twice the rectangle $EUVF$; the strength R represented by this area is found from Eq. 10-32 to be equal to $200 \times 72(9.25 - 2)/1.8$, or 58,000 lbs. This strength value R is used instead of considering the strength of the entire reinforcement, which would include the sum of areas X, Y, and Z, and which would give the greater value of $15.1 \times 11{,}000$, or 166,000 lbs. The resistance to splitting the nozzle along section QQ, which is equal to the product of the nozzle areas Y and Z and the allowable unit tensile stress, is $(6 + 3.9)11{,}000$, or 109,000 lbs. Since this value is greater than the force of 58,000 lbs., there is no danger of failure by splitting. The shearing resistance of the rivets on one side of section QQ is equal to the product of the rivet areas in single shear and the allowable rivet shear stress, or $(\pi \times 1.313^2/4)5 \times 8800$, equal to 59,500 lbs., which is slightly greater than the force of 58,000 lbs. Had the shearing resistance of the rivets been appreciably less than the force acting on them, a nozzle with a flange of greater diameter, to permit the use of more rivets, might be advisable. The bearing resistance of these rivets is $\pi \times 1.313 \times 0.781 \times 5 \times 19{,}000$, or 306,500 lbs., which is greater than the shearing strength.

The possibility of failure by tearing out the shell plate between the rivet holes should also be considered. There are twelve rivet holes, and the angle ϕ, representing one half of the included angle between the centers of adjacent holes, is equal to $360/2 \times 12$, or 15°. The diagonal pitch a, b, or c, Fig. 10-29, is equal to $2R \sin \phi$, and for a 15½-in. diameter rivet circle this expression is equal to $15.5 \sin 15°$, or 3.947 in. The projected length of

each diagonal ligament upon the diameter of the rivet circle is a_1, b_1, or c_1, etc., and from Fig. 10-30,

$$a_1 = a \cos 75° = 3.947 \cos 75° = 1.022$$

$$b_1 = b \cos 45° = 3.947 \cos 45° = 2.791$$

$$c_1 = c \cos 15° = 3.947 \cos 15° = 3.813$$

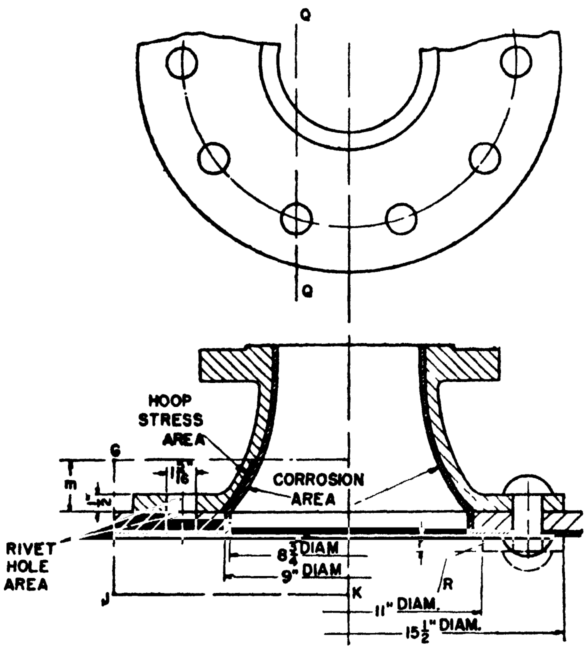


FIG. 10-31. Nozzle with Corrosion Allowance.

The pitch/diameter ratio a/D is equal to 3.947/1.313, or 3.0. From Fig. 10-31, the referred efficiencies for the various values of angle θ are

$$\theta = 75°;\ e' = 1.12$$

$$\theta = 45°;\ e' = 0.77$$

$$\theta = 15°;\ e' = 0.665$$

The effective lengths a_2, b_2, and c_2 are given by the product of a_1e', b_1e', c_1e', as

$$a_2 = 1.022 \times 1.12 = 1.146$$

$$b_2 = 2.791 \times 0.77 = 2.150$$

$$c_2 = 3.813 \times 0.665 = 2.530$$

The vessel diameter is 72 in. and the maximum value of 60 for D_v will control the ligament efficiency. The projected length L is equal to one half of the sum of the bolt circle

diameter and the rivet diameter, which is equal to $(15.25 + 1.313)/2$, or 8.281 in. The ligament efficiency from Eq. 10-33 or 10-34 is

$$e = \frac{60 - 2(8.281 - 1.146 - 2.15 - 2.53)}{60} = 0.935$$

$$e = \frac{60 - 4(8.281 - 1.146 - 2.15 - 2.53)}{0.8 \times 60} = 1.042$$

Since these efficiencies are greater than the efficiency of 84.6% or 0.846 for the longitudinal joint, from Example 3-1, there is no probability of failure by tearing out the shell plate.

10-18. Unreinforced holes must not be located in the longitudinal or circumferential joint of a pressure vessel constructed under the provisions of the ASME-UPV Code. The minimum distance between a welded joint and the edge of an unreinforced hole is 1 in. for plate thicknesses less than 1 in., 2 in. for plate thickness greater than 2 in., and is equal to the plate thickness for plates between 1 and 2 in.

The allowable stresses for welded joints for nozzles and reinforced openings must correspond to the allowable tensile stresses given in Chap. 4, for the particular classification of vessel. The unit working shear stress should be taken as 80% of these values.

10-19. Fabricated Nozzle Selection. Fabricated nozzles are illustrated in Fig. 10-26. The welds in the upper and central constructions shown in this figure should have a value of $t_1 + t_2$ not less than 1.25 t; the weld in the lower construction may have a minimum throat depth t_1 of 0.75 t, but must be welded from both sides. The above data hold when t is the thickness of the shell, if it is equal to ¾ in. or less, and if t is less than n. If n is less than t, substitute n for t. If the thickness of the shell is greater than ¾ in., the minimum weld size $t_1 + t_2$ for the upper and central figures is 1 in.; the weld throat distance t_1 of the lower figure is 9/16 in. By the ASME-UPV Code, nozzles fitted into the shell may be fillet welded from the exterior only, if the nozzle diameter is limited to 4-in. nominal pipe size for shell diameters greater than 24 in. In such cases, the throat dimension of the weld has a minimum value equal to 1.25 t for shell thicknesses up to ¾ in., and a minimum value of 1 in. for greater thicknesses. Other approved nozzle constructions are shown in Figs. 10-14 to 10-21.

10-20. Corrosion. Corrosion allowance cannot be credited as reinforcing area in the analysis of openings; when reinforcement areas are given for nozzles, as in Tables 10-11 and 10-12, the material that may be removed by corrosive action must be deducted from the reinforcement area given.

Example 10-8. Select a 6-in. sweep-type nozzle for the pressure vessel of Example 3-4, using a plate thickness of 25/32 in., and check the opening reinforcement, making an allowance of ⅛ in. for possible corrosive action.

Solution. The actual thickness of the plate, considering corrosion, will be 25/32 + ⅛, or 29/32 in. The nozzle is selected from Table 10-8, based upon a shell thickness of ⅞ in.; from Table 10-5, a 6-in. 150-lb. nozzle will serve for a pressure of 200 psi. at 125°, since its allowable working pressure is 220 psi. at 150° F. The nozzle is delineated in Fig. 10-31. The opening in the shell should be at least equal to the largest diameter V of the nozzle

hole, or 8¾ in. (from Table 10-8), and with ⅛-in. corrosion allowance, the shell opening diameter should be taken as 9 in. The length JK of the reinforcing limit rectangle is therefore 9 in.; the height above and below the exterior and interior surfaces of the shell is 2.5 and 1.95 in. respectively.

The area for which reinforcement must be provided, from Eq. 10-31, is

$$A = (2 \times 9 - 2)(200 \times 72) \div (1.8 \times 11{,}000) = 11.6 \text{ sq. in.}$$

The area X available for reinforcement in the shell is $18 - 9 - 2 \times 1.313 \times 0.781$, or 4.98 sq. in. From Table 10-11, the gross area available for reinforcement in the nozzle is 5.78 sq. in. Since the saddle thickness is ½ in., the area deductible for two rivet holes is equal to $2 \times 1.313 \times 0.5$, or 1.313 sq. in. The theoretical thickness f of the nozzle, for a 9-in. diameter hole, from Eq. 3-3, is

$$f = 200 \times 4.5/11{,}000 = 0.082 \text{ in.}$$

The corrosion thickness is ⅛ in., and the length of the nozzle wall within the reinforcing area is approximately 2 in.; the deductible area is therefore twice $2(0.083 + 0.125)$, or 0.83 sq. in. The net area in the nozzle available for reinforcement is $5.78 - 1.313 - 0.83$, or 3.64 sq. in. The total available area within the limits of the reinforcement rectangle are $3.64 + 4.98$, or 8.62 sq. in., which is approximately 3 sq. in. less than is required. If a reinforcing ring R is used, as indicated by the dot-and-dash lines in Fig. 10-31, with an outer diameter equal to the flange of the nozzle, or 15½ in., and an inner diameter of 11 in., the required thickness r of the ring will be approximately

$$r = 3/[(15.5 - 0.25) - (11 + 0.25)] = 0.75 \text{ in.}$$

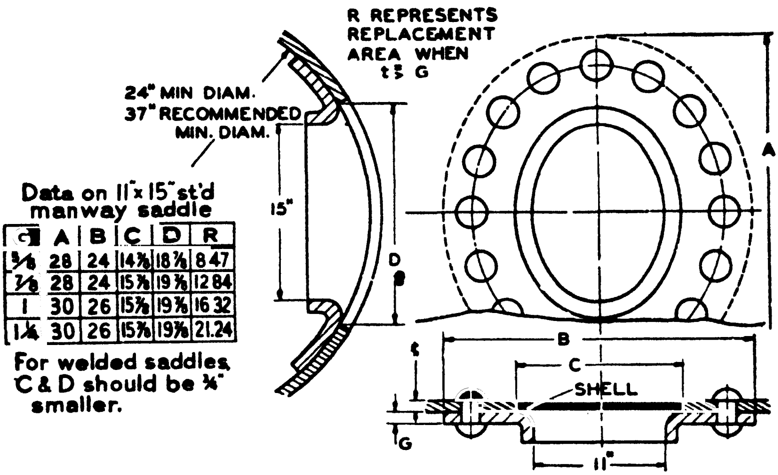
Note that corrosion allowance has been considered at the inner and outer peripheries of the ring. The thickness must be increased ⅛ in. for the same reason, so that the ring will actually be made of ⅞-in. plate. It may be noted that the minimum ring thickness, from Table 10-13, is ⅝ in.

In computing the stresses in the rivets the minimum section QQ, rather than the axial plane, should be considered.

Table 10-13.—Minimum Thickness of Independent Riveted Reinforcing Rings or Flanges, ASME-UPV Code

Thickness of Shell Plate	Thickness of Reinforcing Ring or Flange
⅛	⅛
3/16	3/16
¼ to 11/32	¼
⅜ to 13/32	5/16
7/16 to 15/32	⅜
½ to 19/32	7/16
⅝ to 27/32	½
⅞ to 31/32	⅝
1 to 1 3/32	11/16
1⅛ to 2	¾
Over 2	1

10-21. Covers. A variety of cover plates and other forms of closures are used for manway, handhole, and nozzle openings. A standardized manway cover is shown in Fig. 3-12, for use with a flanged-in head. Manways are also attached to the shell itself, and may be located either "short way" or "long way" "on sweep," as shown at *J* and *K*, Fig. 10-22. Shell manways are usually fitted with a manway saddle, in preference to flanging-in the shell plate; data on saddles for an 11 × 15-in. manway are given in Fig. 10-32. In this illustration, *R* represents the effective replacement area for reinforcement when the plate thickness *t* is greater than the saddle thickness *G*. It should be noted that computation of required reinforcement, under the ASME-UPV Code, should be based upon Eq. 10-31; the size of the opening *D* must be taken as



G	A	B	C	D	R
5/8	28	24	14 7/8	18 7/8	8 47
7/8	28	24	15 3/8	19 3/8	12 84
1	30	26	15 5/8	19 5/8	16 32
1¼	30	26	15 5/8	19 5/8	21.24

FIG. 10-32. 11 in. × 15 in. Standard Manway Saddles.

dimension *C* from Fig. 10-32 when the manway is placed "long way on sweep," and as dimension *D* when placed "short way on sweep."

For low pressure vessels, elliptical rings of rectangular sections are sometimes used instead of saddles. For 11 × 15-in. manways, these are available with an 11⅛ × 15⅛-in. opening and with section thicknesses and depths of ½, ¾ and 1 in., and 2, 3, 4 and 6 in.; they are welded in place, employing full-fillet lap welds on both sides of the plate. Reinforcing areas are equal to the product of twice the thickness and the depth of the section of the ring, if within the limits of the reinforcing area; a ring ¾ in. thick and 3 in. deep consequently has an available reinforcing area of 2 × 0.75 × 3, or 4.5 sq. in., for plate thicknesses equal to or greater than ½ in. For a plate thickness of ⅜ in., the total height of the reinforcement area rectangle is equal to the sum of twice 2.5 *t* and *t*, and is equal to 6 *t* or 6 × 0.375, or 2.25 in. Despite the fact that the actual ring depth is 3 in., only 2¼ in. of this depth is within the permissible

limits of reinforcement, and the available reinforcing area is 2 × 20.75 × 2.25, or 3.375 sq. in.

Manways may be circular as well as elliptical; circular manways should have a diameter not less than 15 in. Manway covers similar to that shown in Fig. 10-33 are used when frequent access to the opening is desired. The edges of the cover and vessel flanges are slotted so that the eyebolts may be swung out of the way after the nuts have been loosened. These holes are a source of weakness as far as the flange stresses are concerned; to compute flange stresses the diameter *d* to the inside of the slotted holes should be taken as the dimension *A* in finding factors *T, U, Y,* and *Z* from Fig. 10-13.

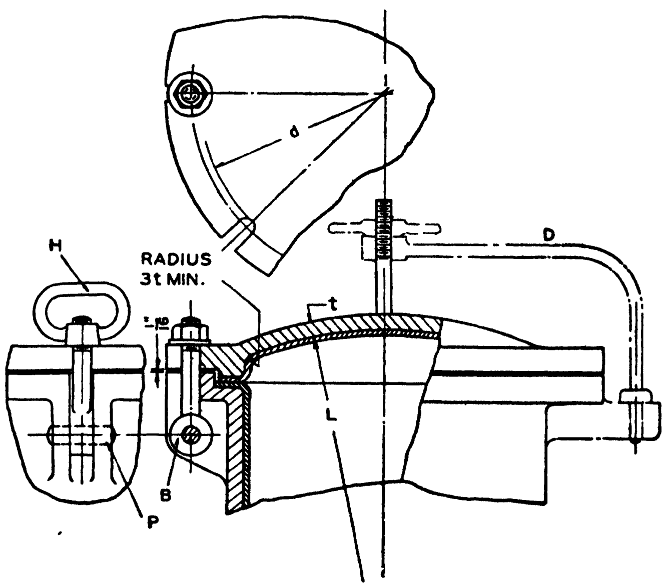


FIG. 10-33. Swing-type Cover.

For openings in a vertical plane, the cover is often equipped with a hinge at one point in its periphery; for openings in a horizontal plane, a rod *R* may be welded to the cover so that it can be lifted and swung to one side by means of the davit *D* shown in Fig. 10-33. The detail at *H* shows an alternate form of nut.

The protection of gasket edges is considered mandatory in access openings which are in sufficiently frequent use to justify swing cover construction. Either male-and-female joints, as illustrated, or tongue-and-groove facings should be used for this purpose.

The cover shown in Fig. 10-33 is made of cast iron, with a lead lining; the lining extends across the contact surfaces of the flanges, and further gasketing

is not required. The thickness t of cast iron spherically dished heads is found from the following, where p is the internal pressure, L the radius of the dished head, and S the allowable flexural stress.

$$t = \frac{0.6\,pL}{S} \tag{10-35}$$

The radius L should not exceed the inside diameter of the shell to which the head is attached. The strength of the flanges must not be less than that of the several American Standards for cast iron fittings of the same sizes. For ellipsoidal heads, where the head depth is not less than one fourth the vessel diameter, the head thickness must be equal to the thickness required for a cast iron cylinder of the same diameter.

10-22. Blind Flange Cover Plates. Blind flanges, as shown in Fig. 10-22-F, are often used as cover plates for circular manways. Two types of such closures are shown in Fig. 10-34; the type shown at B has a flat or serrated contact surface, with a full-face gasket over the entire area. The necessary thickness t of the blind flange is given by

$$t = C\sqrt{(0.162)p/S} \tag{10-36}$$

where C is the bolt circle diameter, as indicated in Fig. 10-34-B, p the internal pressure, and S the allowable design stress, psi. For blind flange closures as shown at A, Fig. 10-34, and for tongue-and-groove and ring-joint gasket fittings as in Figs. 10-7 and 10-4, where the bolt setting tends to dish the cover plate, the necessary thickness t is given by

$$t = G\sqrt{[0.3\,pGH + 0.7\,pW(C - G)]/SGH} \tag{10-37}$$

where G is the mean gasket or contact surface diameter, as shown in Fig. 10-34 and in Figs. 10-1 to 10-7 inclusive, H is the fluid-static end force on the mean gasket area from Eq. 10-3, W is the design bolt load from Eq. 10-7 or 10-8, and p and S are the internal pressure and the allowable design stress, psi. Standard blind flanges from Table 10-3 can often be used for these closures.

Equations 10-36 and 10-37 have been devised to give satisfactory results based upon stress conditions. In special cases, greater thicknesses may be necessary to maintain tight joints. In a bolted cover plate, the deflection of the plate under pressure may relieve the gasket contact pressure sufficiently to cause leakage. Further tightening of the bolts will tend to correct this condition. If a cover plate is bolted to the channel casting of a multiple-pass heat exchanger, to make the seal with the partitions in the channel castings separating the different passes, its deflection under bolt tension or pressure, or both, may be sufficient to break its contact with the partitions and short-circuit the various passes. For such conditions further tightening of the bolts will tend to aggravate the possibility of leakage, and it becomes necessary to use a considerably heavier plate to avoid deflection.

The fittings *E* and *F*, Fig. 10-34, are known as studding outlets or rings; ring *E* is designed for welded and *F* for riveted fastening. These outlets are available in a range of sizes from 1½- to 12-in. diameter, and in 150- and 300-lb. service capacities. The bolting and contact face specifications are in accordance with the data of Table 10-4.

10-23. Fabricated cover plates or closures of spherically dished or ellipsoidal form can be used as removable heads for cylindrical vessels, or for manways or other access openings. Several representative constructions are shown in Fig. 10-35. Type A shows a flanged dished head welded to a ring flange; type B shows a welding neck-type flange butt welded to a flanged dished

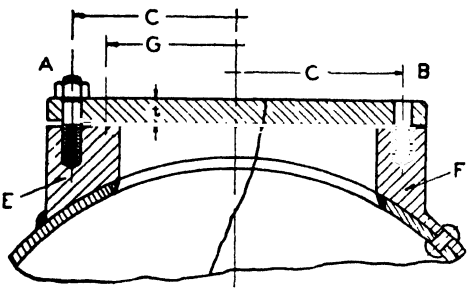


FIG. 10-34. Removable Flat Heads and Blind Flanges.

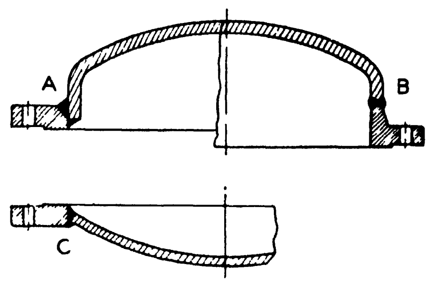


FIG. 10-35. Fabricated Dished Covers and Closures.

head. The construction at C is suggested as a fabricated replacement for the cast dished head of Fig. 10-33; it consists of a ring flange welded to a "dished-only" head. For special applications and conditions where the number of heads required does not warrant the cost of a pattern for a casting, such fabricated heads will prove very satisfactory if a careful analysis is made of the flange stresses. Of the three types shown in Fig. 10-35, that of C is probably the least expensive.

PROBLEMS—CHAPTER 10

1. A cylindrical vessel 5 ft. in diameter has dished heads made of S-2 grade B steel and is used for storing oil at a pressure 150 psi., at atmospheric temperature. A flanged nozzle, welded to one of the heads, is used as a manhole and is closed by a blind flange. Determine the thickness of the vessel head and select the smallest possible size nozzle for the manhole

2. Check the stresses in the attachment welds for the nozzle in Problem 1.

3. Evaluate the question of opening reinforcement for the nozzle of Problem 2.

4. Select a suitable blind flange for the nozzle of Problem 1.

5. Select a suitable gasket for the flange of Problem 4 and check the flange and bolt stresses.

6. Like Problems 1 to 5, for a sweep-type nozzle.

7. Design a cover plate, similar to construction B, Fig. 10-34, for the cover of Problem 4. Check the stresses in the head and flanges and select a suitable gasket.

8. Find the maximum diameter of an unreinforced opening for the vessel of Problem 11, Chapter 3.

9. Like Problem 8, for the vessel of Problem 13, Chapter 3.

10. The vessel of Problem 11, Chapter 3, is equipped with an 8-in. diameter nozzle attachment made of cast steel. Select the nozzle, compute the wild stresses, and investigate the reinforcement.

11. The vessel of Problem 13, Chapter 3, is equipped with a 10-in. nozzle riveted in place. Select the nozzle and design the attachment and any necessary reinforcement.

CHAPTER 11

NON-FERROUS CONSTRUCTION

11-1. Non-ferrous metals are used for the construction of equipment and for structural members and machine parts largely because of specific advantages in resistance to corrosion and in lightness of weight. Electrical or thermal conductivity, ductility, and ease of fabrication as compared to iron or steel parts may also influence the selection of a non-ferrous in preference to a ferrous metal. The primary consideration, however, is usually one of chemical resistance. After such selection has been made, it is necessary to consider the strength and structural suitability of the non-ferrous material, followed by the economic factors involved in forming and fabricating. To illustrate, aluminum and magnesium are usually selected for their light weight and resistance to corrosion. Both are relatively easy to fabricate, and can be cast, drawn, and machined. Lead is a heavy material that possesses good corrosion resistance, but is so weak structurally that it generally requires reinforcement or "backing-up," and is usually used as a lining for vessels or containers. Copper and nickel, in either pure or alloy form, have a high degree of chemical resistance and possess sufficient strength and rigidity to permit their use in structural members

Application of Non-Ferrous Metals to Containers and Pressure Vessels

11-2. Stresses in Non-Ferrous Construction. The theoretical and empirical formulae developed and presented in Chaps. 3, 4, 9, and 18 for ferrous material construction may be applied to non-ferrous equipment without essential modification other than the selection of suitable working stresses. Permissible stresses for non-ferrous material design in accordance with the ASME-UPV Code (Spec. U20) are summarized in Table 9-5, and may be used for the design of pressure vessels, piping, and tubes covered by the Code. In many instances, however, materials not included in the ASME-UPV Code may be selected for equipment; in other cases, cold-worked, age-hardened, or annealed materials and alloys with ultimate strengths differing to some extent from those listed in the Code may be found suitable. Table 11-1 is recommended as a guide for the selection of suitable working stresses for such materials, and gives the permissible percentage of the ultimate strength that may be used in design.

Fabrication methods affect the selection of suitable working stresses for non-ferrous metals to about the same degree as in iron and steel. Non-ferrous pressure vessels may be cast, drawn, spun, or formed and joined by mechanical

TABLE 11-1 —PERMISSIBLF PERCENTAGE OF MINIMUM ULTIMATE TENSILE STRENGTH FOR NON-FERROUS METAL DESIGN

Material	ASME Code Specifications	For Temperatures Not Exceeding Degrees F							
		Sub zero to 150	250	350	400	450	500	550	700
Copper plates Copper tubes Copper pipe	S 20 S-22 S-23	20.0	14.2	12.5	11.1				
Muntz metal and high brass tubing condenser tubes and plates	S 24 S 47 S 59	11.1	9.1	5.6					
Red brass tubes	S 24	16.7	15.4	12.5	11.1				
Admiralty tubes	S 24	15.4	14.2	13.3	12.5	10.0			
Admiralty condenser tubes	S 47								
Copper silicon	S 36	16.7	16.7	8.3					
Alloy plates, tubes and other shapes	S 37	14.2	14.2	7.1					
Steam bronze	S 41	18.2	16.7	15.4	15.4	14.2	11.8	9.1	
Steam bronze	S 46	18.2	16.7	15.4	11.8				
Cupro nickel	S 47	20.0	16.7	15.4	14.2	12.5	10.5	8.3	
Monel Nickel sheet Nickel tubing	70,000 tensile * rolled and annealed 110,000 tensile 90,000 tensile	20.0	20.0	20.0	20.0	18.2	18.2	16.7	16.7
Aluminum	to 40,000 tensile	20.0	14.2	10.0					

* For thickness of 3/16 in and above lower percentages can be used

or fusion processes. Vessel wall thicknesses for cast construction should be sufficiently greater than the theoretical to provide for variation in size caused by coring and molding; one-piece drawn vessels and heads and covers shaped by metal-spinning or other cold-working processes should be designed so that the unavoidable reduction at the corners or edges, incident to such processes, does not affect the stress capacity of the vessel. A common specification for corners and knuckle radii of vessels or dished heads is that the reduction in thickness in that region does not exceed 10% of the original flat plate thickness.

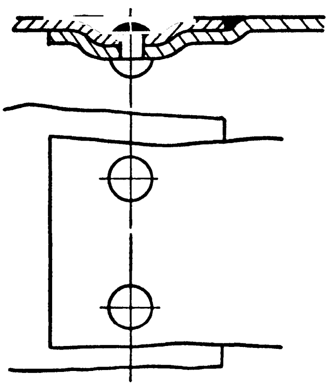


Fig. 11-1. Sealed Rivet Construction.

11-3. Joints. Non-ferrous metals are joined by methods similar to those used for iron and steel. Efficiencies for riveted, welded, brazed and soldered joints can be determined as in the case of joints in steel plates. Riveted joint design is similar to that described in Chap. 3, but rivet dimensions and shapes are somewhat different than in the case of steel construction. Riveted joints are sometimes made as shown in Fig. 11-1, which indicates how the rivet head may be seated in a depression in the sheet metal, and covered with solder or other metal to form a continuous inside surface. Such construction is often desirable to eliminate small crevices which would either induce corrosion or permit the retention of small foreign particles. Riveted seams are often sweated to add additional joint strength and provide almost perfect closure and leak prevention. Fig. 11-2 shows riveted joints for head connections in cylindrical

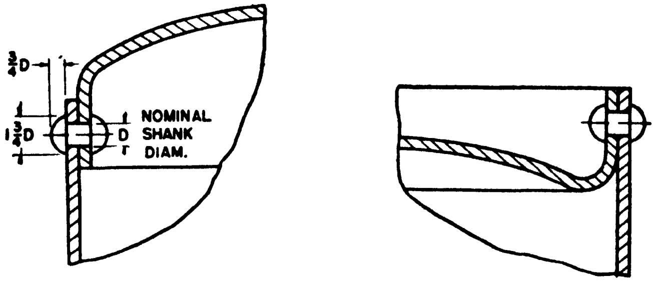


Fig. 11-2. Pressure Head Attachment for Non-ferrous Vessels.

vessels of non-ferrous construction. These joints are similar to the head connections shown in Chap. 3.

Fig. 11-3 shows a lapped joint frequently used in non-ferrous construction. The edges of both plates are beveled, and the edge of the plate at the right of the figure is slit and bent so that alternate tongues of metal lie on either side

of the left-hand plate. The tongues are seated together, but solder or brazing metal is applied so as to fill up the joint discontinuities. Non-ferrous parts that are to be joined by welding are finished along the edges for V- or U-shaped welds, in a manner similar to that shown in Fig. 4-1 The size of the openings and angles may differ to some extent from those shown in this illustration.

Fig. 11-4 is a composite illustration in which various forms of seams and seam connections for non-ferrous construction are shown. The double-locked seam construction shown at *B* is commonly employed for the circumferential seams of cylindrical vessels or tubes. The joint is usually sealed or fastened by brazing or other bonding media. A similar seam, for attaching the bottom of a vessel to the cylindrical portion, is shown at *C*. A somewhat simpler form

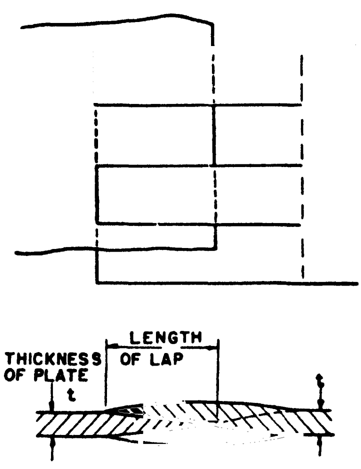


FIG. 11-3. Split and Lapped Joint for Non-Ferrous Vessel Joints.

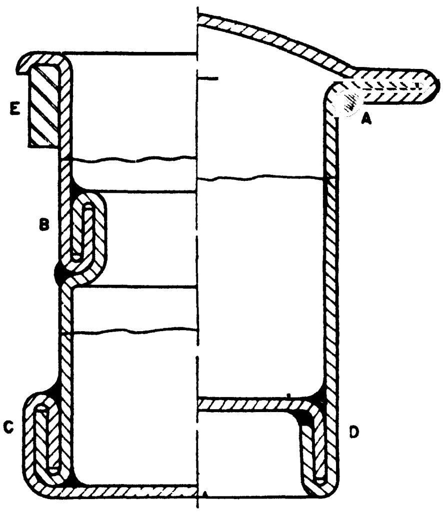


FIG. 11 4. Lock Seam Construction and Applications.

of connection, shown at *D*, is used when a flat head is fastened in place by rolling or spinning the cylindrical wall over the head flange. The detail at *A* shows a type of closure in which only one welded or brazed seam is required. Since this seam may be fabricated from the exterior, the cover can be permanently fastened to the vessel without requiring a manhole. The detail at *E* shows how a steel ring can be used to reinforce the edge or flange of a vessel made of soft metal.

Double-locked seams, if properly fabricated, are quite resistant to leakage; even single-locked seams, as shown at *A* and *D*, are considerably more effective in this regard than riveted joints. Some form of metal to metal bonding, such as brazing or welding, is essential if the container is to be pressure-tight. The ease of formation of locked seams depends upon the complexity of the seam and the malleability of the material. Rolled sheets of non-ferrous metals often

require tempering or annealing treatment before forming; a hardness or temper specification may be of utility in many cases. Comparatively thin steel plate, if properly annealed, can also be bent or formed to some degree.

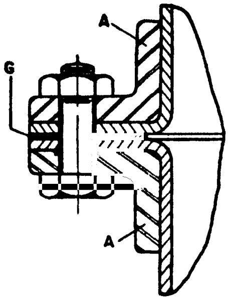


FIG. 11-5. Reinforcing Flanges for Cover Plates.

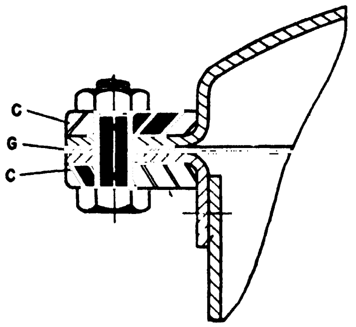


FIG. 11-6. Reinforcing Flanges for Cover Plates.

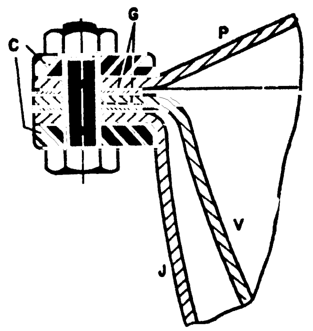


FIG. 11-7. Closure Details and Jacketed Vessel Construction.

Figs. 11-5 and 11-6 show two forms of circular reinforcing flanges for designs in which removable cover plates or manhole closures are required. The construction shown in Fig. 11-6 is less expensive, although not as rigid as that

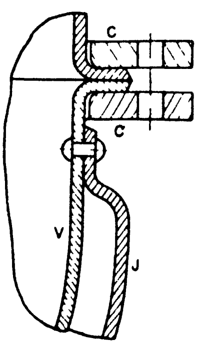


FIG 11-8 Closure Details and Jacketed Vessel Construction.

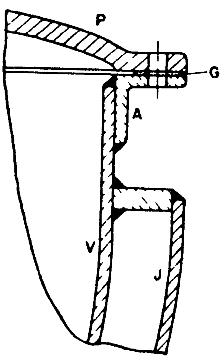


FIG 11-9. Closure Details and Jacketed Vessel Foundation.

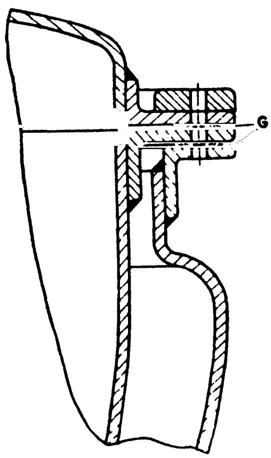


FIG. 11-10. Closure Details and Jacketed Vessel Construction.

of Fig. 11-5, which consists of two rings *A* formed of structural angles. Both types of reinforcing rings may be welded or brazed to the vessel and head, or they may be employed as loose rings. The former construction is obviously more effective than the latter. Figs. 11-7, 11-8, 11-9, and 11-10 show a variety

of closure details for jacketed vessels and kettles. In the construction shown in Fig. 11-8, the jacket is riveted to the vessel shell *V*, and closure is effected by two clamp rings *C* similar to those of Fig. 11-6, making a direct metal to metal seal. In Fig. 11-9, the jacket shell *J* is welded or brazed to the vessel shell *V*; a structural angle *A*, bent to circular form, is welded to the upper edge of the vessel shell. The reinforcing ring is drilled for bolt holes, so that the cast or forged cover plate *P* can be attached by through bolts. Figs. 11-7 and 11-10 are examples of construction in which it is possible to disassemble the head, jacket, and vessel. The design of Fig. 11-10 is more complicated than that of Fig. 11-7, but is considerably more rigid, requires fewer forming operations on the jacket and vessel sheets, and is used for medium and large sized pieces of equipment. It may be noted that only one gasket *G* is required for the design of Fig. 11-9, while two are necessary in those of Figs. 11-7 and 11-10, where the jacket is removable.

The joints and closures described in the preceding paragraphs should only be considered typical of modern practice, since many other types of connections are in present-day use. Various modifications are dictated by the properties of the materials used, the soldering or other bonding methods employed, and the required permanency of the joint. Joints which are frequently disconnected, and those where gaskets are used, are often fabricated in slightly different ways from similar joints in iron vessels or structures.

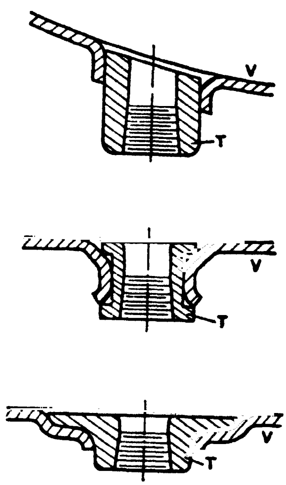


FIG. 11-11. Connections for Threaded Pipe in Non-ferrous Vessels.

11-4. Attachments. Because most vessels made of non-ferrous metals are constructed of comparatively thin sheets, connections for threaded pipe and other fittings are usually made as illustrated in Fig. 11-11. In the three designs shown, the threaded connection *T* is held in place by crimping or rolling the vessel sheet *V* around the fitting, and soldering or brazing the joints to act as a seal and to make a smooth, continuous joint, particularly on the interior of the vessel. In cases where the shell thickness is of sufficient size, it is possible to deposit a pad of brazing or welding metal at the region where the opening is to be placed, and drill and tap the pad for the threaded connection, in a manner similar to that used for iron or steel construction illustrated in Fig. 10-4.

The design of drainage or "draw-off" connections for jacketed vessels is more complicated than that for single-shell vessels, particularly where jacket expansion and contraction must be provided for. Fig. 11-12 shows a comparatively simple connection, in which a nozzle *N* is brazed or soldered to the vessel shell *V*, with the jacket shell *J* subsequently bonded to the fitting. Figs. 11-13

and 11-14 show three forms of draw-off connections in which it is possible to disassemble the jacket *J* without destruction of the joint connection. In Fig. 11-13, a stuffing box with a packing gland is used to permit expansion and

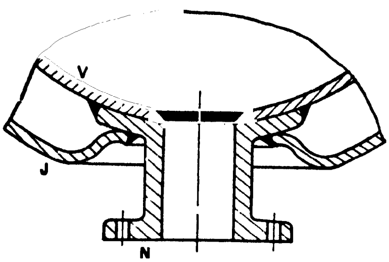


FIG. 11-12. Drainage Connection for Jacketed Vessel.

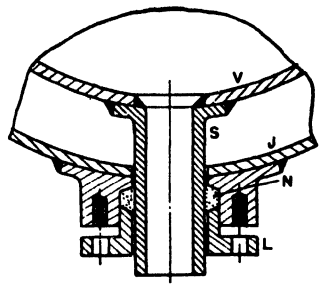


FIG. 11-13. Expansive Drainage Connections for Jacketed Vessel.

contraction of the jacket without cracking or straining the shell or the seam. The sleeve *S* is brazed to the vessel shell; the packing *N* is compressed by the gland *L,* and provides an effective seal for the jacket interior. Fig. 11-14 shows double- and single-gasketed removable connections; the vessel wall *V* is rolled or crimped around the fitting *T,* which is then sealed in place by solder.

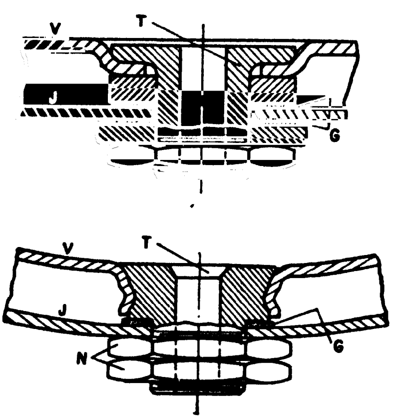


FIG. 11-14. Removable Drainage Connections for Jacketed Vessel.

When seal brazing or welding is not feasible, riveted joints for non-ferrous construction can be calked in the same manner as that used in steel construction. For medium or high pressure service, openings and connections in non-ferrous construction may require reinforcement. The design of suitable reinforcement is handled in a manner similar to that described in Chap. 10.

11-5. Soldering and Brazing. Soldered joints may be made by causing molten solder to flow between the parts, by wiping, or by sweating. The strength of the joint depends largely upon the area of contact between the parts and upon the composition of the solder. Ordinary solder is composed of equal parts of lead and tin; a high tin content (up to 63%) makes for stronger solder. In general, thin sheets soldered together will have a joint strength equal to that of the base metals, provided the soldered area is equal to 20 times the cross-sectional area of the sheets to be joined.

Hard soldering or brazing requires the use of brazing rods or powder composed of tin, zinc, and copper alloys, in many grades and varieties, for use with different base metals. Silver soldering or brazing is frequently used for low emperature copper equipment, and for nickel and nickel alloy parts; the brazing alloy is composed of a high tin content solder to which some silver has been added. Brazing is usually limited to plate thicknesses less than ½ in., plates thicker than $\frac{1}{16}$ in. may be beveled or machined, in the same way that plates are prepared for welding, to facilitate making the joint.

Many of the non-ferrous metals and alloys can be welded if the proper techniques are employed, although the process is attended with some difficulty. Some of the factors preventing good results are: presence and difficulty of removal of oxidizing film; high thermal conductivity and expansion; weakening of the base metal caused by incidental heat treatment in the vicinity of the weld; lack of strength at elevated temperatures; and the inferior corrosion resistance that the weld offers. Some or all of these difficulties may be eliminated by a proper selection of a welding rod. Copper, silicon, tin and phosphorus alloy rods are extensively used for welding pure copper, copper alloy, pure nickel, and nickel alloy. For aluminum welding, the rods are similar in composition to the base metal, with about 5% silicon added. Practically all non-ferrous parts are welded by using either the electric resistance process, or by gas welding. In the latter process, the oxyhydrogen torch is preferred to the oxyacetylene type. Electric-arc welding processes are under development at the present time, and it is probable that shielded-arc processes will be used in the near future.

11-6. Light-weight Metals and Alloys. Aluminum and magnesium in the pure and alloyed forms are the most important of the light-weight metals. These metals are used principally because of their low density, although aluminum is often used for its corrosion resistant properties. Table 11-2 gives data for a comparison of the strengths and densities of these alloys and some of the commoner steels. The best method of visualizing the application of the light-weight alloys to processing equipment and structural parts is to compare them with similar equipment made of steel. In general it can be said that for equivalent load-carrying capacity a mild steel beam weighing one unit is equivalent to an alloy steel beam of one-half the weight, an aluminum alloy beam of one-third the weight, and a magnesium alloy beam of one-fourth the weight. The modulus of elasticity of aluminum and magnesium alloys is appreciably lower than that of steels, and light metal beams must thus be larger in cross-sectional area to provide the same stiffness and resistance to deformation as steel beams. On the other hand, the lower elastic modulii of the light metals are an indication that they have a higher capacity for absorbing energy resulting from shock or vibration. Aluminum alloys have notable hardening characteristics. Age hardening, and aging at elevated temperatures, are phenomena peculiar to aluminum alloys, and are used to obtain increased hardness.

Since the light metals are quite plastic, the softer grades are sometimes very difficult to machine, although magnesium alloys can be easily cut without lubricants if properly ground cutting tools are used. Because of the high thermal expansion of the light metals, it is necessary to allow for the relatively large contraction that results when the machining is completed and the part has cooled to room temperature.

TABLE 11-2.—COMPARISON OF ALLOYS FOR STRUCTURAL PURPOSES

Material	Specific Gravity	Yield Strength psi. (set = 0.2%)	Tensile Strength psi.	Modulus of Elasticity psi.	Ratio of Yield Strength ÷ (S.G. × 44.3)
Mild steel	7.9	35,000	55,000	29,000,000	100
Low-alloy steel	7.9	55,000	75,000	29,000,000	180
Alloy steel	7.9	140,000	180,000	29,000,000	400
Magnesium alloy	1.8	30,000	42,000	6,500,000	377
Aluminum alloy	2.8	40,000	60,000	10,300,000	323

Aluminum alloys are designated by the letter S, preceded by a number which indicates the chemical composition, such as 3S or 17S. In one group of alloys, hardness and strength are provided by cold-working the material after final annealing. In these, the fully annealed material is designated by O, as 3S-O, while the fully hardened material is designated by H, as 3S-H. Intermediate degrees of hardness are designated by fractions, as 52S-½ H. In a second group of alloys, final hardness and strength are primarily provided by heat treatment. The annealed alloy is designated by O, the fully heat-treated and aged alloy by T, as 17S-O and 17S-T. Intermediate tempers, after heat treatment but before aging, are designated by W, as 53S-W.

11-7. Copper and Its Alloys. Relatively pure copper is used for the construction of chemical processing equipment, but the material is more often used in alloyed form. Copper alloys have been in use for thousands of years; zinc and tin are the principal alloying elements in the ordinary bronzes and brasses, but many other substances, such as nickel, lead, aluminum, beryllium, and silicon, are also employed. One very important application of pure copper is as an electrical conductor, since practically all alloying elements result in a drastic reduction in the electrical conductivity of the material. For example, one per cent of aluminum, silicon, iron, manganese, or phosphorus will reduce the conductivity of copper to less than one half of its normal value. Pure copper can be obtained in sheet, plate, bar, tubing, and angle form. Copper alloys in these and in sand-cast and die-cast form are also available.

TABLE 11-3.—PROPERTIES OF TYPICAL ALUMINUM ALLOYS

Alloy	Tension		Compression	Shear		Typical Applications
	Yield Strength psi.	Ultimate Strength psi.	Yield Strength psi.	Yield Strength psi.	Ultimate Strength psi.	
3S-O	6000	16,000	6000	4000	11,000	Sheet metal work; tank plates of intermediate strength
3S-H	25,000	29,000	25,000	14,000	16,000	
4S-O	10,000	26,000	10,000	6000	16,000	
4S-H	34,000	40,000	34,000	18,000	21,000	
14S-T	58,000	68,000	58,000	38,000	45,000	Forgings
17S-T	37,000	60,000	37,000	22,000	36,000	General structural applications, rivets, bolts and structural sections
A51S-T	40,000	48,000	40,000	26,000	32,000	Forgings
52S-O	14,000	29,000	14,000	9000	18,000	Sheet metal work; tank plates of higher strength
52S-½H	29,000	37,000	29,000	16,000	21,000	
52S-H	36,000	41,000	36,000	20,000	24,000	
53S-W	20,000	33,000	20,000	12,000	20,000	Structural applications requiring high corrosion resistance
53S-T	33,000	39,000	33,000	20,000	24,000	

Weight—about 0.10 lb. per cu. in.
Modulus of elasticity approximately 10,300,000 psi.

TABLE 11-4.—PROPERTIES OF TYPICAL MAGNFSIUM ALLOYS

Designation		Condition	Yield Strength psi.	Tensile Strength psi	Elongation in 2 in. %	Brinell Hardness	Shear Strength psi.	Endurance Limit psi.
Dow Chemical Co.	American Magnesium Corp.							
A	AM241-T4	Solution heat treatment	11,000	33,000	10.0	48	——	7500
G	AM240-T4	Solution heat treatment	12,000	35,000	9.0	52	——	11,000
G	AM240-T61	Heat treated and aged	19,000	36,000	1.0	69	——	8000
B	AM246-T6	Heat treated and aged	20,000	32,000	0.5	85	——	7000
H	AM265-T4	Solution heat treatment	12,000	35,000	9.0	51	——	11,000
M	AM3S	Extruded and stretched	26,000	35,000	7	40	16,000	——
O	AM58S	Extruded and stretched	36,000	47,000	13	60	21,500	16,000
M	AM3S-T	Soft (heat treated)	15,000	32,000	16	40	17,500	8000
F	AM53S-O	Annealed sheet	22,000	36,000	18	50	——	——
F	AM53S-H	Hard rolled sheet	35,000	44,000	10	60	——	——
O	AM58S	Press forging	30,000	45,000	6	60	22,000	15,000

Copper and most of its alloys can be fabricated by rolling, drawing, spinning, stamping, or casting; joints can be made by riveting, soldering, brazing, or welding. Pure copper is sometimes used as a complete body or as a lining for vessels made of other metals. Due to its plasticity, copper or copper alloy vessels used to withstand relatively high stresses are fabricated or backed-up as shown in Fig. 10-32.

The strength of copper and its alloys is dependent to a considerable extent upon the method of fabrication of the base materials, that is, whether the material is cast, rolled, or hammered, worked cold or hot, and whether it is annealed or not. For example, the tensile strength of copper as cast ranges between 20,000 and 30,000 psi.; annealed copper has a tensile strength range of from 30,000 to 40,000 psi.; cold rolled or drawn copper has a tensile strength range from 50,000 to 70,000 psi. The yield point of copper is in the neighborhood of 12,000 psi.; the modulus of elasticity of annealed copper is 15×10^6 psi. In designing copper vessels it is essential that the tensile strengths used are compatible with the methods of fabrication employed.

Copper vessels should never be used for temperatures exceeding 450° F. The usual percentages of the ultimate strength of copper and copper alloys for use in computing stresses in pressure vessels, operating at various temperatures, are shown in Table 11-1. The allowable working stresses for copper may also be obtained from the ASME-UPV Code, an abstract of which is given in Table 9-5. This Code calls for slight modifications in the weld thickness of copper vessels as compared to those computed by the usual stress equations. In forming operations for copper vessels any reduction in material thickness produced by the fabrication operation should not exceed 10% of the calculated required thickness.

TABLE 11-5.—TEMPER DESIGNATIONS FOR BRASS SHEETS

Temper	Reduction in Thickness Caused by Rolling	Tensile Strength psi.	Rockwell "B" (samples over 0.040" thick)
¼ Hard	0.289	46,000-56,000	30-60
½ Hard	0.258	53,500-63,500	50-73
¾ Hard	0.229	61,000-71,000	70.80
Hard	0.204	68,000-78,000	78-85
Extra hard	0.162	79,000-88,500	85-89
Spring	0.129	86,000-95,000	88-92
Extra spring	0.102	89,500-98,500	89-93

The design of copper vessels under external pressure differs somewhat from steel vessel analysis. Since the stress-strain curve for copper is straight in the

lower regions, but changes slope rather rapidly at relatively low stresses, the modulus of elasticity is constant only at comparatively low stresses. It is usually necessary to assume a definite thickness of metal and compute the stresses developed on this basis, and a series of approximations may be required to arrive at the proper thickness to prevent collapse.

Of the many useful copper alloys, brasses are of prime importance. The physical properties of the many types of brasses can only be determined by direct test and from manufacturers' data. The curves shown in Figs. 11-15 and 11-16 illustrate the type of available data and the common method of its presentation. It can be seen from these curves that the tensile strength is affected by the amount of working and by the annealing of the material. A change in area accompanies a change in tensile strength and elongation, and the actual grain

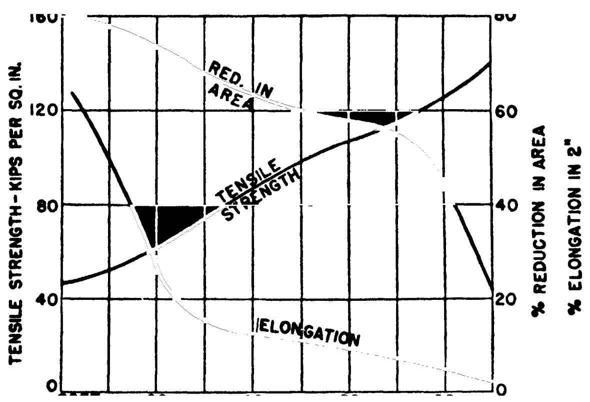


FIG. 11-15. Cold Drawing Effect on High Brass (65 to 68% Cu.)

size of the constituents of the alloy affect the physical properties when different annealing processes are used. It is possible, therefore, to obtain brasses of varying tensile strength depending upon the way they are worked and the temperatures at which they are annealed. Standard specifications have been developed by the ASTM for the control and specification of various heat treatments for brass in sheet and other forms. A typical set of designations for brass sheets is shown in Table 11-5. This table also illustrates the change in Rockwell hardness, thickness, and tensile strength, as a result of heat treatment and cold working operations. Table 11-1 gives typical generalized data applicable where tensile strengths are known from such data as in Figs. 11-15 and 11-16. The physical properties of copper alloys should be obtained from current manufacturers' bulletins, from Metals' Handbook,[7] and from ASTM and other code specifications.

Muntz metal is a brass containing zinc in amounts from 39 to 41%. Muntz metal can be extruded, rolled, or forged while hot and then finished by cold

rolling or drawing. It is a very useful form of bronze since it is rather stiff and easily fabricated.

Manganese bronze contains less than 1% manganese and small amounts of other alloying elements together with the zinc, and forms rather high tensile strength alloys. It is possible to obtain tensile strengths of approximately 90,000 psi. in manganese bronze castings.

Copper-nickel alloys are useful in the chemical industries, and the commonest one, Monel metal (a natural alloy), is usually composed of 67% nickel, 30% copper and small amounts of other elements. Aluminum is often added, which produces very hard alloys (up to 350 Brinell). The tensile strength of Monel metal may run as high as 100,000 to 150,000 psi. in cold drawn rods.

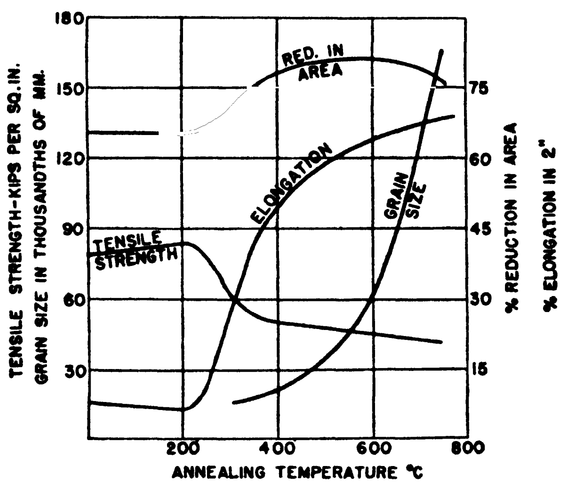


FIG. 11-16. Annealing Temperature Effect on High Brass.

Monel metal is rather easy to fabricate and machine and has excellent corrosion resistance.

Everdur is a copper alloy containing from 1 to 4% silicon and is very resistant to corrosive gases and liquids. It has a tensile strength approximately equal to steel; allowable working stresses are completely specified by the codes of practice. Everdur is very easily welded; fabrication and construction possibilities are similar to those for iron and steel.

11-8. Nickel. Commercial nickel of about 99.5% purity is used as a corrosion resistant material, but numerous nickel alloys are also extensively employed. These materials are available in cast form, and as drawn and rolled shapes, and can be fabricated like copper alloys. Thick sections of nickel or nickel alloy parts can be welded in a manner similar to that used for iron or steel, but thin sections should be bonded by using silver solder. Since nickel

is a comparatively expensive material, reaction vessels requiring nickel surfaces are usually constructed with a thin nickel shell, supported by steel, instead of using solid nickel castings. For thin-walled vessels, heavier nickel castings are used to support the more pliable shell metal at flanged joints and other strained regions.

Allowable working stresses for nickel up to approximately 20% of the tensile strength of the metal may be used (Table 11-1). The tensile strength of nickel can be increased by mechanical working. Nickel is very useful in relatively high temperature applications; at 700° F., for example, an allowable working stress as high as 15,000 psi. can be used. The modulus of elasticity of nickel is 30×10^6 psi. and its yield strength is often as high as 32,000 psi. Since nickel is an expensive material, the permissible percentage of the ultimate strength is often adjusted to some extent, depending upon the thickness of material. A factor of safety of 3.5 is generally considered safe for vessel thicknesses of ½ in. and over.

11-9. Lead. Lead is used for its special acid resistant properties. Dilute sulphuric acid is handled commercially in either glass or lead equipment and sulphurous gases can be handled in lead pipes and equipment provided the temperature is low. Pure lead is not often used; antimony is generally alloyed with the lead and serves as a hardening agent. Tin and copper may also be added, but the common chemical lead contains about 90% lead and 10% antimony. Lead is most frequently used in sheet form and as tubes or castings. Evaporator parts and bodies, lead coils for heat transfer, and lead gaskets are used either for their corrosion resistant properties or because of their ductility. Lead is so easily shaped that it is often used as a lining in tanks and ducts, as shown in Fig. 10-32, and as a covering for various types of stirrers and braces which may be used in solution or evaporation tanks.

Since the tensile strength of lead is quite low and its density is high, lead construction must be adequately braced and backed by stiffer materials. Lead linings may be attached by clamping them to the metal or wood container, but the material is more often welded in place; the welding process is called "lead burning." Fig. 11-18 shows a method of lining cast iron vessels used for plating and pickling operations. The lead sheet is cut into "gores" or meridian sections, hammered to fit the curvature of the interior of the vessel, and sealed by "burning" the joints. One of the oldest uses of lead for chemical resistance is the familiar lead-lined chamber for sulphuric acid manufacture. Typical construction details used to support the weight of the sheet lead in vertical and horizontal position are shown in Fig. 11-17. Large diameter lead pipe is usually constructed where required by forming the sheet lead and burning the edges. Large diameter pipes (15 to 30 in.) are often used for handling corrosive gases at low temperatures; the pipe must be supported at very frequent intervals, however, and considerable maintenance is necessary to keep it in condition. Smaller pipes, made of hard alloy lead, can be obtained in ap-

proximately nominal iron pipe sizes. Stiff lead castings made from high antimony lead alloys are used for tubes, sheets, covers, and similar elements.

The tensile strength of lead and its alloys is very low. The yield point of a 90% lead, 10% antimony alloy is 2800 psi. at 70° F. and 1250 psi. at 212° F. The Brinell hardness of this alloy is 14.5 at 70° F. and 6.5 at 212° F., with a melting point of 473° F. and a specific gravity of 10.7. It is evident from these figures that lead in itself is not a satisfactory structural material. Lead behaves like most thermoplastic material, and in many instances will not support even its own weight when formed into useful equipment.

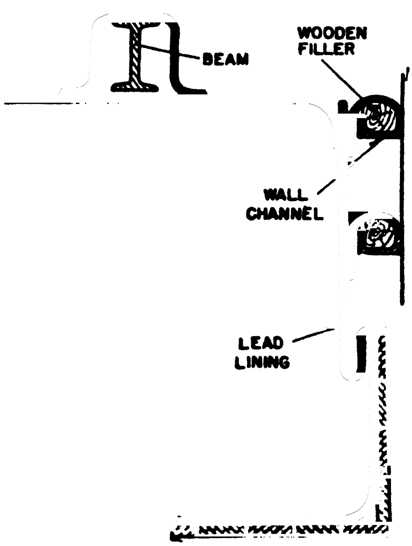


FIG. 11-17. Application of Lead Lining to a Reaction Chamber.

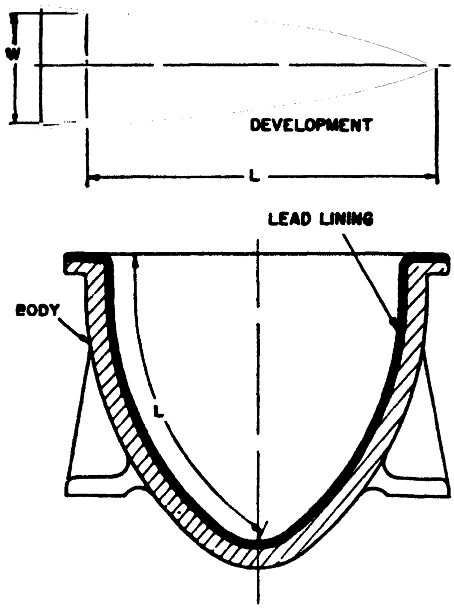


FIG. 11-18. Lead Lined Pickling Tank.

11-10. Alloy Clad Steel. Alloy clad steels consist of a relatively thin layer of corrosion resistant metal or alloy thoroughly bonded to a thick layer of steel. Commercially available "clads" include stainless, copper, and nickel steels. The thickness of the corrosion resistant material is usually about 10 to 15% of the total plate thickness. The obvious reason for using clad steels is that of economy since the material combines the high corrosion resistance of the clad and the high strength of the steel backing. The bond between the two metals is so thorough that clad steels can be bent and fabricated to shape without rupture at the interface. Care should be exercised, however, in fabricating operations so that excessive strains and cracks do not occur at sharp bends. Stress release due to difference in expansion between the clad material and the steel should also be considered. Nickel has approximately the

same thermal coefficient of expansion as steel and it is thus a very easy metal to handle in clad form. Clad steel shapes are now obtainable for pressure vessels of all types and covers; clad steel heads and flanges for pipe connections are commercially available.

While the clad material has some inherent stress capacity, it is customary to disregard the thickness of the clad material layer in stress calculation. The steel used for this purpose is usually a mild steel but various forms can be obtained. Vessels and other clad steel parts can be designed for strength on the basis of the steel in the same way that ordinary steel vessels are designed. Corrosion allowance, however, need not be added to the thickness of the vessel when clad steel is used, since the clad material prevents corrosion of the backing steel.

Riveting can be used to join clad steel, but it is necessary to calk the ends of the overlapping sheets to prevent any exposure of the steel backing. Since this process is laborious and expensive, welded joints are preferred in alloy clad construction. Nickel and stainless clad steels can be welded readily but it is necessary to weld the clad side with the clad material and to weld the steel side with steel. (It is customary to weld the clad side first and to finish with the steel side.) Methods of welding are practically identical with those described in Chap. 4, but the welds must be composed either of wholly clad material (as on the inside of a vessel) or part of the weld must be of alloy material and the remainder of the usual steel or iron welding rod. Flanged connections are usually welded in place and are made up of either a clad material or a pure alloy fitting.

Structural Application of Aluminum Alloys

11-11. Aluminum alloy parts are extensively used in airplane structures, for automotive engine parts, such as piston and connecting rods, and in many other instances where it is desirable to combine comparatively high strength and light weight. Aluminum alloys are also used to some extent at the present time in building construction, and the chemical process industries will probably use more light alloy construction as its advantages become more appreciated. A very important application of this character occurs when additions or revisions to existing structures are to be made, as exemplified in the structure described in Chap. 8, in which supporting beams and cradles for holding two storage tanks are attached to existing roof trusses. A reference to section 8-20 will show that each of the supporting beams weigh approximately 250 lbs., and it is practically impossible for two men to handle and support the beam while it is being bolted in place. If an aluminum beam of the same strength and stiffness is substituted for the steel beam, its weight will not exceed 100 lbs., and the member could be handled with relative ease by two men. Similarly, the cradles and possibly even the storage tanks might be made of aluminum alloy plate, thus considerably reducing the weight of the entire assembly, and thereby facilitating erection

to such a degree that the somewhat higher material cost would be more than compensated for by the reduction in assembling and erecting time. Steel and aluminum forms are widely used in building erection, and as scaffoiding in the erection of fractionating towers and absorbers. (Scaffolds are indeterminate structures, and their design should be handled by references 41 and 58.) It is quite within the range of possibility that such structures as roof trusses, supporting brackets for pipe, and building columns made of aluminum alloy will replace similar steel structures in the near future. The recent expansion of production facilities for aluminum alloy shapes and sections will probably result in material cost reductions which will enable this metal to compete with structural steel on an overall cost basis.

Structural aluminum alloy shapes are available in the form of channels, I, H. and Z beam sections, and equal and unequal leg angles. The dimensions and elements of these sections correspond to the data given in Tables 7-4, 7-2, 7-6, and 7-5. At the present time, channel and I beam sections with a depth of 12 in., and angle leg lengths of 6 in., are the maximum sizes available. (Table 11-6 shows a few of the available aluminum alloy I beam sections.) Angles as small as $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{16}$ in., however, are available in structural aluminum alloy. Round and rectangular bar stock, seamless tubing, and a broad range of sheet and plate widths and thicknesses are also available.

In contrast to steel, aluminum alloy sections and structural shapes may be produced by extrusion, as well as rolling, and a far greater variety of shapes for special purposes may be obtained by making use of the extrusion process. Data on standard and special shapes and their dimensions and specifications may be obtained from the Structural Aluminum Handbook.[2]

The theory and the major features of structural design for aluminum alloys are essentially the same as for steel construction, and the data of Chaps. 5 and 7 may be used for the broader aspects of construction. In considering design details, however, particularly when empirical formulae and data are applicable, some modification in analysis and procedure is required.

11-12. Working Stresses for Structural Aluminum Alloys. The selection of suitable working stress for structural aluminum alloys is dependent upon many of the factors discussed in Chap. 2. In contrast to steel construction, aluminum alloy construction is not codified at the present time, and the choice of working stresses for a particular material depends to a considerable extent upon the judgment and previous experience of the designer, who must rely upon the literature of the subject to some degree.

The permissible working stress for members subjected to simple stresses may be based upon the ultimate strength, the typical yield point strength, or the guaranteed yield point strength of the material in question. The ultimate and typical yield strengths may be obtained from Table 11-3; Table 11-7 gives a few representative values of the guaranteed yield and ultimate strengths for 17S-T alloy. In the absence of previous experience, or of more pertinent

experimental data, the allowable working stress for members subjected to tension should not exceed one half the yield point strength, nor one third the ultimate strength of the material. From Table 11-3, the typical yield and ultimate strengths of 17S-T alloy are 37,000 and 60,000 psi. Using the preceding percentages, the working stress should not exceed the higher of $0.50 \times 37{,}000$ equaling 18,500 psi. and $0.33 \times 60{,}000$ equaling 20,000 psi. Since from Table 11-7 the guaranteed yield and tensile strengths of this alloy are 30,000 and 55,000 psi., respectively, a permissible stress in tension of about 15,000 psi. will be satisfactory for the usual structural applications encountered by the processing engineer or the occasional designer of equipment.

TABLE 11-6.—17S-T ALUMINUM ALLOY AMERICAN STANDARD OR I BEAM SECTIONS

Nominal Size	Area of Section	Torsion Factor J
12 × 5¼	14.57 11.84	2.85 1.78
12 × 5	10.20	1.10
10 × 4¾	8.75 7.38	0.86 0.62
8 × 4	7.43 5.97	0.75 0.42
7 × 3¾	5.83 4.43	0.46 0.25
6 × 3⅜	4.29 3.61	0.24 0.17
5 × 3	2.87	0.12

See Table 7-2 for other dimensional data.

TABLE 11-7.—MINIMUM OR GUARANTEED STRENGTHS FOR 17S-T ALLOY

Form	Thickness	Tensile Strength psi.	Yield Strength psi.
Sheet and plate	0.01 -1.50 1.501-2.000 2.001-3.000	55,000 53,000 50,000	32,000 32,000 32,000
Rolled rounds, squares, and hexagons	0.125-0.750 0.751-3.000 3.001-8.000	55,000 53,000 50,000	30,000 30,000 30,000
Rectangular bars	Up to 0.750 0.751-3.000	53.000 50,000	30,000 30,000
Rolled structural shapes		50,000	30,000
Extruded structural shapes		50,000	35,000

In common with other ductile materials, aluminum alloys do not possess a definite ultimate compressive strength. Since the compressive yield strength is equal to the tensile yield strength, the allowable stress in compression is usually taken as equal to the allowable tensile stress. The allowable stress in shear for structural members is usually equal to 60% of the allowable tensile stress, since the ultimate and yield points in shear are about three fifths of those in tension.

Aluminum alloy rivet strengths are not necessarily as high as the parent alloy from which they are made, since the method of driving and heating often results in a reduction in both shearing and bearing strengths. For this reason ultimate shearing and bearing stresses based upon actual test results of joints and rivets are preferred to the typical values given in Table 11-3. It has also been found that the stress at which bearing failures occur is a function of the edge distance in the direction of the application of the load. When the distance from the center of the hole to the edge of the plate is equal to or greater than twice the hole diameter, the ultimate bearing strength is equal to 180% of the tensile strength. The ultimate bearing strength is proportionately reduced for smaller edge distances. Comparative data on aluminum alloy and steel rivets are shown in Table 11-8. Immediately after driving, the ultimate shearing strengths of aluminum alloy rivets are about 75% of those listed in Table 11-8. After standing at ordinary temperatures for about four days, the rivets usually harden to full strength.

TABLE 11-8.—ALUMINUM ALLOY RIVET STRENGTHS

Rivet Material	Temperature and Method of Driving	Ultimate Shear Strength psi.	Ultimate Bearing Strength psi.	Bearing Yield Strength psi.
17S-T	Cold	35,000	105,000	60,000
17S-T	At 950° F.	34,000	102,000	58,000
53S-W	Cold	25,000	59,000	33,000
53S	At 950° F.	18,000	53,000	28,000
Steel	Hot or cold	45,000	135,000	

As a guide to safe practice, it may be recalled that the permissible shearing and bearing stresses for steel rivets in structural fabrication are equal to about one third the ultimate shearing and bearing stresses. Based upon this practice, values of 10,000 psi. in shear and 25,000 psi. in bearing may be used for occasional design for 17S-T alloy rivets driven hot or cold in structures. These values correspond to apparent factors of safety of about 3 and 2 based upon the ultimate and yield strengths of the materials. For pressure vessel joints,

steel rivets are stressed to one fifth their average ultimate strengths, which by correspondence would call for permissible shearing and bearing stresses of 6000 psi. and 15,000 psi. for aluminum alloy rivets in pressure vessels.

For purposes of design, the diameter of a driven aluminum alloy rivet is usually assumed as 1.05 *D*, where *D* is the nominal diameter of the rivet. The rivet hole must be at least $\frac{1}{16}$ in. larger than the nominal diameter of the rivet, and the full size of the hole should be deducted in computing the net section of tension members.

If members and connecting rivets are made of dissimilar materials, the bearing resistance of the fastening should be based upon the weaker material. For 17S-T alloy structural members and rivets, the allowable bearing stresses given for the rivets will control the design, but if steel bolts are used to connect members, as is often the case where additions to an existing structure are made, the bearing strength of the member itself should govern. For 17S-T alloy, an allowable bearing stress of 25,000 psi. is considered satisfactory.

In the foregoing discussion, mention has been made of "structural applications encountered by the processing engineer" and "occasional design." The stresses given above are satisfactory for design work of this character, where the aluminum alloys are used because of their resistance to corrosion or their lack of weight, and where the additional cost of excess material is not too important. In major design problems, however, such as of trusses, the manufacturer of the alloy should be consulted to determine stresses more closely, so that the material may be used in the most economical manner.

11-13. Design of Compression Members. Column design for structural aluminum (17S-T) is based upon two formulae: a straight-line expression for slenderness ratios L/k up to 83, and an adaptation of Euler's formula for higher slenderness ratios. The critical load P for these values of L/k is given by

$$P = (43{,}800 - 350\,L/k)A \tag{11-1}$$

and

$$P = \frac{102{,}000{,}000\,A}{(L/k)^2} \tag{11-2}$$

where L is the effective length of the column, k is the least radius of gyration, and A the area of the column cross section. The effective length L is taken as equal to the actual length for round-end or pin-connected columns; for columns whose ends are rigidly fixed the effective length L may be taken as eight tenths to one half the actual length. The allowable load for structural columns composed of H beams, or built-up channel or angle columns, should not exceed from one third to two fifths the ultimate load obtained from the above expressions. Structural columns are usually considered pin-end members, unless the column section is very light with very rigid end supports.

When a flat plate is used as the component part of a compression member, it may buckle locally under edge compression at stresses below the compressive

yield strength of the material. Such buckling occurs in the form of wrinkles, is practically independent of the length of the member, and may be treated as a local column failure, by substituting an equivalent slenderness ratio L/k based upon the unsupported width b and thickness t of the plate in Eqs. 11-1 and 11-2.

For simply supported edges, similar to the web of an H beam with relatively thin flanges, the equivalent slenderness ratio is given by

$$L/k = 1.5\ b/t \tag{11-3}$$

For conditions in which one edge is fixed or built in and the other edge is free, similar to the outstanding leg of an angle riveted or welded to heavier members,

$$L/k = 3\ b/t \tag{11-4}$$

For conditions in which one edge is simply supported, and the other edge is free, similar to the longest outstanding leg of a single angle strut,

$$L/k = 5\ b/t \tag{11-5}$$

In cases where both edges of a flat plate are built into or supported by heavier members, as in Fig. 11-19, the load carrying area is considered equal to the sum of the areas of the supporting members and those portions of the plate that are assumed to be effectively supported by the reinforcement. In the figure, w represents the supported or effective length at each edge, and b is the plate length between the edge reinforcement. The magnitude of w for any aluminum alloy is given by

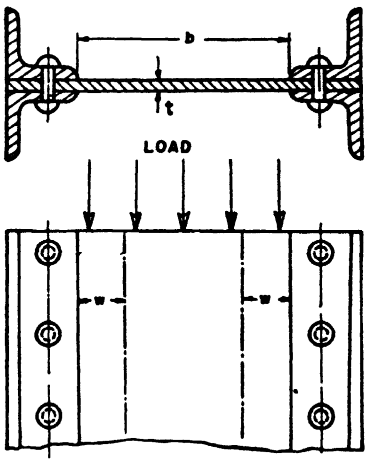


FIG. 11-19. Structural Aluminum Alloy Column Section.

$$w = \frac{2700\ t}{\sqrt{S_y}} \tag{11-6}$$

where t is the plate thickness and S_y is the yield strength. For 17S-T aluminum, the yield strength, from Table 11-3, is 37,000 psi. Substituting,

$$w = \frac{2700\ t}{\sqrt{37{,}000}} = 14\ t \tag{11-7}$$

Obviously, w can never be greater than one half b, no matter what the plate thickness is.

Example 11-1. Find the allowable load that can be carried by the member of Fig. 7-12 if it serves as a main member column whose effective length is 14 ft. Consider the gusset plate to be 1 in. thick instead of ¼ in. as indicated, with 17S-T alloy angles.

Solution. The least radius of gyration of this column, from Example 7-5, is about axis *xx*, and is equal to 1.58 in. The L/k ratio is 106, which is greater than 83, so Eq. 11-2 applies. The area A of the two angles, from Table 7-5, is 8 sq. in. Substituting in Eq. 11-2,

$$p = \frac{102{,}000{,}000 \times 8}{106^2} = 73{,}500 \text{ lbs.}$$

Since the allowable load should not exceed from one third to two fifths the critical load obtained from the above, the allowable load may range from 24,500 to 29,400 lbs. Note that this load is slightly less than one third the load that can be carried by a steel of similar proportions.

It is advisable to investigate the possibility of local buckling. From Fig. 7-12, it is seen that the longest legs of the angles are attached to the gusset, and that the condition of Eq. 11-4 applies, in which one leg of an angle is fixed to a heavier member. Substituting,

$$L/K = \frac{3 \times 3.5}{0.5} = 21$$

Since this L/k ratio is far less than the ratio used in Eq. 11-2, in the preceding solution, the column will not fail by local buckling. If the column length had been 30 in., however, instead of 14 ft., the possibility of local buckling would control the design, since the L/k ratio from Eq. 11-4 would have been less than the L/k ratio for the column as a whole.

The stresses on the rivets may be found from Eqs. 7-1 and 7-2. The rivets in the connection shown in Fig. 7-12 are in double shear; the diameter of the driven rivet is 1.05×0.875, or 0.92 in. The unit shear stress, from Eq. 7-1, is

$$S_s = \frac{4F_s}{8\pi D^2} = \frac{4 \times 29{,}400}{8\pi \times 0.92^2} = 5560 \text{ psi.}$$

The thickness of the gusset plate is equal to the sum of the leg thickness of the angles, or 1 in., and the unit bearing stress, from Eq. 7-2, is

$$S_b = \frac{F_b}{4Dt} = \frac{29{,}400}{4 \times 0.92 \times 1} = 8000 \text{ psi.}$$

By comparison, it is seen that the possibility of shear failure controls, since the actual shearing stress is slightly greater than one half the allowable, while the actual bearing stress is one third the allowable.

Curved plates subjected to edge compression resist local buckling more effectively than flat plates, and the critical stresses are usually higher for a given plate thickness. The critical loads may be obtained from Eq. 11-1 or 11-2, by substituting an equivalent slenderness ratio L/k from the following:

For built-up complete tubular members, or for adequately stiffened portions of cylindrical plates

$$L/k = 7.1\sqrt{R/t} \qquad (11\text{-}8)$$

For seamless tubes

$$L/k = 4.7\sqrt{R/t} \qquad (11\text{-}9)$$

where R is the mean radius of curvature and t the thickness of the plate. Longitudinal stiffeners increase the buckling resistance of curved plates, if the spacing h is less than R. Since no general formulae for the effect of stiffeners have been devised, it is customary to compute the buckling resistance of the member by considering it as a flat sheet supported between stiffeners. The

actual critical stress is somewhat higher than this computed value because of the stiffening effect of the curvature.

11-14. Beam Selection and Design. Aluminum alloy structural members used as beams and girders are designed on the basis of the flexure formula, Eq. 5-13. The allowable stress in flexure for 17S-T alloy is usually taken as 15,000 psi. This stress is applicable to the net section of the tension flange, and the gross section of the compression flange, but the computation is usually based upon the net section of both flanges, as discussed in section 7-13 and Example 7-9, solution *b*. Structural aluminum members subject to both flexural and compressive stresses, or to eccentrically applied loads, should be checked both for the individual flexural and compressive stresses, and by the limiting expression, Eq. 7-10, for combined axial and flexural stresses.

In beams with flanges for which there is little or no lateral support, failure may occur by sidewise buckling of the compression flange. The critical stress at which such failures can occur may be predicted by using Eqs. 11-1 and 11-2, where the slenderness ratio L/k is equal to the quotient of the effective unsupported lateral length L, and k is the equivalent radius of gyration of the compression flange. The value of k is obtained from:

$$k = \sqrt{\frac{0.2}{Z}\sqrt{I_{yy}[(JL)^2 + 13.1\,I_f d^2]}} \qquad (11\text{-}10)$$

where Z is the section modulus, in in.^{3}, of the beam section about an axis normal to the web; I_{yy} is the moment of inertia, in in.^{4}, for the beam section about the centroidal axis parallel to the web; I_f is the moment of inertia, in in.^{4}, of the compression flange of the beam about the centroidal axis parallel to the web (for symmetrical sections such as standard I beams, I_f may be taken as equal to one half I_{yy}); d is the depth and L the span of the beam, in.; and J is a torsion factor. The value of J for aluminum alloy I beams may be obtained from Table 11-6, and for angles and other structural shapes from tables of elements of sections for structural aluminum. For built-up members, J may be approximated by considering the area of the section divided into a series of rectangles of length b and thickness t, and using the following summation

$$J = \Sigma\, bt^3/3 \qquad (11\text{-}11)$$

The foregoing discussion of lateral stability applies principally to I and H beam sections, and to built-up members of I form. It may be applied, however, to channel-shaped members if they are adequately supported against twisting at the point of application of the important loads and reactions. In cases involving combined torsion and bending, or for unsymmetrical bending, the method does not apply, and the maximum combined flange stress should be checked to see that it is within the safe stress limits of the material.

Since the compression flanges of structural beams may fail by localized buckling of the component parts, it is necessary to check the stress not only

for the stability of the flange as a whole, but also for local buckling. The critical stresses for flat plates forming parts of the compression flanges of a beam are determined in the same manner as given for flat plates in edge compression, using Eqs. 11-1 and 11-2 for the stress and Eqs. 11-3, 11-4, and 11-5 for the equivalent slenderness ratios.

The webs of beams and girders are subjected to a horizontal compressive stress induced by flexure, varying from zero at the neutral axis to a maximum at the compression flange. Comparatively thin webs may tend to buckle under the influence of these stresses, which exist at the compression edge of the clear height of the web; the critical stress is given by Eqs. 11-1 and 11-2, using the following value for the equivalent slenderness ratio.

$$L/k = 2\,h/3\,t \qquad (11\text{-}12)$$

where h is the clear height and t is the thickness of the web in inches.

When beam and girder webs are subjected to shearing forces, they are likely to buckle before the yield strength in shear of the material is exceeded. For girder webs supported by vertical stiffeners, as shown in Fig. 7-24, the critical stress is given by

$$S = \frac{51{,}000{,}000}{(b/t)^2}\left[1 + \frac{3}{4}\left(\frac{b}{d}\right)^2\right] \qquad (11\text{-}13)$$

where d is the long and b the short dimension or interior distance of the panel (bounded by the inner edges of the flanges and stiffeners), and t is the web thickness. If the girder has stiffeners spaced more closely than the clear height of the web, the distance d represents the clear height and the distance b the spacing between the interior edges of the stiffeners; if the stiffeners are widely spaced, b represents the clear height and d the stiffener spacing. If the girder has no stiffeners at all, distance d approaches infinity as a limit, and the factor $(b/d)^2$ in Eq. 11-13 becomes zero.

The critical stress values obtained from Eq. 11-13 should be compared with the yield strength in shear of the aluminum alloy used for the design. The smaller of the two values (either the critical stress or the shear yield stress) should control the design. For 17S-T alloy, for example, if the critical buckling shear stress is less than 22,000 psi., it will control the design; if the value of the critical buckling shear stress is greater than 22,000 psi., the latter value should be used.

The deflection of a beam or girder is often a limiting factor in design. Since aluminum alloys have a relatively low modulus of elasticity (10,300,000 psi.), and thereby tend to deflect almost three times as much as steel beams of similar section and stress application, such restrictions may require special consideration. The deflection of aluminum alloy members may be computed from the formulae given in Figs. 5-35 and 5-36, but in many instances, particularly in the preliminary stages of design, a high degree of precision is un-

warranted. For such cases, the following approximation for the deflection d at midspan of aluminum alloy beams may be used:

$$d = \frac{SL^2}{100,000,000\,c} \qquad (11\text{-}14)$$

where S is the maximum flexural stress, psi., at or near midspan, L is the beam span in in., and c is the distance from the neutral axis to the extreme fiber of the beam section.

In the foregoing, and in deflection equations obtained from Figs. 5-35 and 5-36, the shearing deformation has been disregarded. This practice is justified in solid-web beam and girder sections of comparatively large span-depth ratios, but is not correct for members with trussed or latticed webs. Such members should therefore be designed as trusses and not as beams, and the design analysis should include the computation of stresses in the web members and their connections.

Example 11-2. Select a suitable 17S-T alloy I beam section for a uniform load of 500 lbs. per ft. of span, if the beam span is 20 ft., and the deflection is limited to 1/360 of the span.

Solution. The preliminary selection of a suitable beam section is limited by the permissible deflection, the allowable flexural stress, and the maximum available size of 17S-T alloy beam sections. Since the deflection is of considerable importance, it may serve to initiate the solution. For a span of 20 ft. or 240 in., the permissible deflection is 240/360, or 0.67 in. If the maximum flexural stress of 15,000 psi. is assumed, the beam depth, $2c$, may be found from Eq. 11-14, as follows

$$c = \frac{SL^2}{100,000,000\,d} = \frac{15,000 \times 240^2}{100,000,000 \times 0.67} = 12.97 \text{ in.}$$

which would require a beam at least 26 in. deep. Since the maximum depth of beam available is 12 in., it is necessary to solve for the maximum permissible stress based upon the deflection of 0.67 in., and a value of c not exceeding 6 in., as follows

$$S = \frac{100,000,000\,cd}{L^2} = \frac{100,000,000 \times 6 \times 0.67}{240^2} = 6940 \text{ psi.}$$

which is therefore the maximum permissible flexural stress.

The end reaction is $500 \times 20/2$, or 5000 lbs., and the maximum moment, which occurs at the center of the span, is equal to $(5000 \times 10 \times 12) - (5000 \times 5 \times 12)$, or 300,000 in.-lbs. For a flexural stress of 6940 psi., the required section modulus Z, from Eq. 5-13, is equal to 300,000/6940, or 43.3 in.^{3} By reference to Table 7-2, a $12 \times 5\frac{1}{4}$-in. American Standard beam section, with an area of 11.84 sq. in., has a moment of inertia of 268.9 in.^{4}, with respect to the flexural axis xx, and the section modulus Z of this section is 268.9/6, or 44.8 in.^{3}, which is satisfactory.

The actual deflection is computed from Case 1, Fig. 5-35, as follows

$$d = \frac{5WL^3}{384EI} = \frac{5 \times 500 \times 30 \times 240^3}{384 \times 10.3 \times 10^6 \times 268.9} = 0.65 \text{ in.}$$

which will be satisfactory.

The compression flange of the beam should be investigated for lateral stability or sidewise buckling by using Eq. 11-1 or 11-2. The equivalent radius of gyration may be found by applying Eq. 11-10. The torsion factor J for a $12 \times 5\frac{1}{4}$-in. I beam with an area of

11.84 sq. in. is 1.78, from Table 11-6. The moment of inertia I_f may be taken equal to one half I_{yy}, which has a value of 13.8 in.^{4} from Table 7-2. Substituting

$$k = \sqrt{\frac{0.2}{44.8}\sqrt{13.8(1.78 \times 240^2) + 13.1 \times 6.9 \times 12^2}} = 2.38 \text{ in.}$$

The unsupported length of the flange is equal to the beam span or 240 in., and the L/k ratio is 240/2.38, or 101, necessitating the use of Eq. 11-12. The critical stress is

$$\frac{P}{A} = \frac{102{,}000{,}000}{101^2} = 10{,}000 \text{ psi.}$$

Since the actual stress in the compression flange, from the flexure formula, is very nearly 7000 psi., it is advisable to use some form of lateral support, such as a stiffener, at the center of the beam to halve the lateral unsupported length of the flange. For a condition in which the unsupported flange length is 120 in., the equivalent radius of gyration, from Eq. 11-10, is

$$k = \sqrt{\frac{0.2}{44.8}\sqrt{13.8(1.78 \times 120^2) + 13.1 \times 6.9 \times 12^2}} = 1.81 \text{ in.}$$

The L/k ratio is 120/1.81, or 66.3 in., necessitating the use of Eq. 11-1. The critical stress is

$$\frac{P}{A} = 43{,}800 - (350 \times 66.3) = 20{,}600 \text{ psi.}$$

which is approximately three times as great as the flexural stress and is thus satisfactory.

For localized buckling, Eq. 11-3 may be used to determine the equivalent slenderness ratio. The thickness t of the flange may be assumed to be at least equal to the web thickness, which is equal to 0.46 in., from Table 7-2. The overhang of the flange past the web may be taken as one half the flange width, or 0.5 × 5.25, or 2.63 in. Substituting in Eq. 11-3

$$L/k = \frac{1.5 \times 2.63}{0.46} = 8.5$$

Since this value for the equivalent slenderness ratio is less than the value used for investigating lateral stability, there is no danger of localized buckling.

The critical shear buckling stress is obtained from Eq. 11-13, by disregarding the stiffener at the center, and considering the entire height of the beam as equal to the interior distance of the panel. Substituting,

$$S = \frac{51{,}000{,}000}{(12/0.46)^2} = 16{,}800 \text{ psi.}$$

Since this value is greater than the yield strength in shear (22,000 psi.), it controls the design. The actual vertical shear on the beam is 5000 lbs. at either end, and the unit vertical shear, based upon the area of the web, is 5000/0.46 × 12, or 91 psi. Even if the horizontal or longitudinal shearing stress at the neutral axis is considered, the actual shear in the beam is far below the critical value, and there is no danger of buckling. It may be seen from the above that the investigation of the possibility of localized buckling of the flanges, or web buckling either in compression or shear, could have been neglected since the section is of standard proportions. Lateral stability, however, should always be considered, because of its dependence upon the span of the beam.

It may be of interest to compare the selection just made with a steel beam of the same flexural capacity. The required moment of inertia for a deflection of 0.67 is found by using the deflection formula from Case I, Fig. 5-35, with a value of 30 × 10^6 for the modulus of elasticity:

$$I = \frac{5WL^3}{384Ed} = \frac{5 \times 500 \times 20 \times 240^3}{384 \times 30 \times 10^6 \times 0.67} = 89.6$$

From Table 7-2, this value of I will require a $10 \times 4\frac{3}{4}$-in. American Standard beam, with an area of 7.38 sq. in. Checking the section, the flexural stress S is equal to $300{,}000 \times 5/122.1$, or 12,300 psi., which is satisfactory.

The weight of the steel beam is $3.4 \times 7.38 \times 20$, or 502 lbs. The unit weight of the 17S-T alloy beam is approximately 0.1 lb. per cu. in. The sectional area of the $12 \times 5\frac{1}{4}$-in. beam is 11.84 sq. in., and the toal weight is equal to $0.1 \times 11.84 \times 240$, or 284 lbs., slightly more than half the weight of a steel beam of equivalent capacity. Had the 17S-T alloy beam been selected on the basis of flexural stress alone, the reduction in weight would have been considerably greater, since the ratios of actual to allowable stresses for the steel and aluminum alloy beams are respectively 12,300/20,000 and 6940/15,000, or 0.615 and 0.463.

11-15. Miscellaneous Structural Details. Aluminum alloy tread plates, similar to steel plates with anti-skid surfaces, are used for flooring and deck and balcony surfacing. They may be obtained in 4S, 17S-O, 17S-T, and 53S alloys, in thicknesses from $\frac{1}{8}$ to $\frac{3}{8}$ in. by $\frac{1}{16}$-in. increments, in 60-in. widths (except for the $\frac{1}{8}$-in. size, which is 50 in. wide), and in lengths up to 24 ft. The weight of $\frac{1}{8}$-in. plate is about 2 lbs. per sq. ft. The design of flooring is handled by using the data of section 7-18 and Eqs. 7-23, 7-24, and 7-25.

Corrugated aluminum alloy sheet for roofing and wall surfacing is available in a variety of thicknesses and widths. Manufacturers' catalogs and the Structural Aluminum Handbook should be consulted for dimensional data and methods of application.

11-16. Fabrication of Structural Aluminum. Rivet or bolt holes in structural members should be drilled or sub-punched and reamed; all holes on material over $\frac{1}{2}$ in. thick should be drilled. Punches and drills used for steel fabrication can be used for aluminum alloy, although drilling speeds can be increased 50% over those used for steel. Aluminum alloy plates and shapes $\frac{1}{2}$ in. or less in thickness can be sheared with any of the types of equipment used for steel, but heavier material should be sawed. Flame cutting should not be attempted with aluminum alloys, since the excessive heat damages the metal, and the cut edge is very ragged.

Structural aluminum alloys do not rust, but in severe conditions of exposure it is desirable to protect structural members with paint, particularly where thin sections are employed. For ordinary conditions, such as trusses and interior structures, the finishing system used for steel construction can generally be employed with some changes in the surface preparation and the priming coat. For severe conditions, such as exposure to sea water, the manufacturer should be consulted.

PROBLEMS—CHAPTER 11

1. A $3 \times 3\frac{3}{8}$-in. 17 S-T aluminum angle serves as a tension member in a truss and is attached to a $6 \times 6 \times \frac{1}{2}$-in. chord angle by fillet welds. If the permissible length of weld on either edge of the angle is 5 in., determine the permissible load on the tension member.

2. For the member of Problem 1, find the number of rivets (maximum diameter) if the attachment to the chord member is accomplished by a $\frac{1}{2}$-in. gusset.

3. A pair of 17 S-T aluminum channels corresponding to 7 in., 1475 lb., structural steel members back to back and ½ in. apart serve as a building column 15 ft. long. Find the permissible axial load.

4. What is the maximum length and the corresponding load capacity of a 3 × 2½ × ½-in. 17 S-T aluminum angle when used as a main member column? A secondary member column?

5. The column of Problem 3 carries a bracket, similar to Fig. 7-18, which is used to support a crane runway. The applied load is 1800 lbs. located 9 in. from the centroid of the column. Determine the permissible axial load on the column.

6. Determine the stress in the rivets of the bracket of Problem 5 if the distance *m* (Fig. 7-18) is equal to 3 in., and if the rivet diameter is ⅝ in.

7. Design the rivet arrangement for the bracket of Problem 5 using six rivets of maximum size.

8. Redesign the vessel and substructure of Problem 1, Chapter 8, for 17 S-T aluminum construction. Limit the deflexion of the cradle beams to 1/360 of the span.

CHAPTER 12

CONCRETE CONSTRUCTION

12-1. Cement, Mortar, and Concrete. Portland cement is a powder made by heating rock of the proper mineral content through the calcining stage, and pulverizing the resulting sinter. The powder is an inorganic material containing oxides of calcium, silicon, and aluminum which, if mixed with water, forms jell-like compounds that subsequently stiffen into hard masses. Cement mortar is a mixture of cement, sand, and water; concrete is a mixture of cement, sand, water, and crushed rock or gravel (known as aggregate). Concrete is appreciably stronger than either cement or mortar.

Concrete structures, shapes, and bodies are made by filling wooden forms with the material in the plastic state, tamping or vibrating the plastic mass to insure complete filling, and then allowing it to set up or harden, after which the forms are removed. The proportions of sand, cement, aggregate and water have an effect on both the strength and the workability or consistency of the concrete. The strength of a concrete may be improved by proper grading of the materials, by adding cement, or by reducing the amount of water used. Workability can be improved by proper grading of the materials, by the use of rounded aggregate, or by adding water or cement and water. Representative proportions for concrete mixtures of different strengths are shown in Table 12-1.

Since the tensile strength of concrete is usually less than one thirtieth of its compressive strength, concrete bodies or structures subjected to any degree of tension are reinforced or strengthened by steel rods embedded in the mixture at the time it is poured. The resulting product is known as reinforced concrete. Concrete walls and slabs subjected primarily to compressive stresses are often reinforced with wire mesh to prevent expansion or temperature cracks. Workability is of considerable importance in reinforced concrete, since the plastic mixture should flow around the reinforcement with a minimum of prodding or vibration. Workability is measured by a slump test, which relates consistency to the loss in height of a truncated cone of freshly mixed concrete, 12 in. high, when released from a standard bucket form. A slump range of from 3 to 6 in., based upon the original height of 12 in., is permissible in building construction; the data in Table 12-1 is based upon a 4-in. slump, which is considered an average value.

Incorrect or careless proportioning or working of concrete may result in early deterioration and lack of strength and will be especially noticeable in varying atmospheric conditions, or in cases where the concrete is subjected to intermittent wetting. Proper specification, inspection, and guidance should be

TABLE 12-1.—JOINT COMMITTEE GUIDE FOR PROPORTIONING CONCRETE MIXES

Estimated 28-Day Compressive Strength psi.	Maximum Size Coarse Aggregate, Inches	Cement Factor—Sacks of Cement per Cu. Yd. of Freshly Mixed Concrete	Maximum Water per Sack of Cement, Gallons	Fine Aggregate Percentage of Total Aggregate *	Approximate Weights in Lbs. Saturated Surface-Dry Aggregate per Sack of Cement **		
					Total Agg.	Fine Agg.	Coarse Agg.
2250	1	4.9	8	40-46	660	280	380
2250	2	4.5	8	37-43	740	300	440
2250	3	4.1	8	34-40	840	310	530
3300	1	6.5	6	37-43	470	190	280
3300	2	6.0	6	35-41	530	200	330
3300	3	5.5	6	33-39	600	220	380
4250	1	8.0	5	35-41	370	140	230
4250	2	7.4	5	33-39	420	150	270
4250	3	6.8	5	31-37	470	160	310

These proportions are valid for a 4-in. slump. For each 1 in. difference in slump, the cement should be changed by ⅛ sack per cubic yard. The cement proportion should be increased for slumps greater than 4 in. and decreased for slumps less than 4 in.

* The limits shown are approximate and percentages falling outside of the limits shown may frequently be found necessary to produce concrete of the desired workability. When expressed in terms of absolute volume, they are applicable to aggregates of different specific gravities; when expressed in terms of weight, differences in specific gravity should be taken into account.

** These approximate weights are based on a bulk specific gravity of 2.65 in a saturated surface-dry condition.

given to even the smallest job. In many organizations, particularly in small or medium sized plants, concrete work is handled by the maintenance division, or by common labor. In such cases, it is advisable for the processing engineer to check the proportions of sand, aggregate and cement as they are delivered to the mixing board, to see that the plastic mass is properly mixed, and to supervise the actual pouring. In many urban areas it is possible to purchase ready-mixed concrete in any desired quantity, delivered to the job. Such material should only be obtained from a reputable supplier, since it is difficult or impossible to determine the respective proportions of sand, cement, aggregate and water after the concrete has been mixed.

Curing or hardening of the concrete structure is as important as proper working and proportioning, since concrete attains its maximum strength about a month after pouring. Too rapid curing usually results in an appreciable loss of strength and hardness, although quick-setting cements for high early strength concretes are commercially available; they are used where the necessity for high strength in a short period of time overshadows the added cost. Cements with good resistance to acids are used for concrete structures exposed to such conditions. Lime and asphaltic base cements can also be obtained. Common salt or calcium chloride in small proportions added to the mix will improve the strength of concretes cured at freezing temperatures. In reinforced concrete, however, salt is likely to cause corrosion of the steel. Waterproofing compounds are sometimes added in powdered or liquid form to obtain a more impervious concrete, with more or less successful results.

12-2. Expansion and Shrinkage. Temperature and moisture cause significant volume changes in concrete. The coefficient of expansion is $6/10^6$ in. per in. per ° F.; the coefficient of shrinkage of lean dry mixes is about $3/10^4$ in. per in. A shrinkage of 0.0003 in. per in. of length, due to a dry mix, is thus comparable to a contraction caused by a temperature decrease of 50° F. Shrinkage cracks are discernible in most concrete structures; for this reason, and because of expansion, concrete bodies having large surface areas are usually made with open joints. Unheated, uninsulated buildings, for example, require expansion joints at 200-ft. intervals across the entire building, while subjoints at 100-ft. intervals are required in the roof. The volume of concrete is affected to some extent by its moisture content, but this change has little effect upon its weight per unit volume, which is usually taken as 150 lbs. per cu. ft.

12-3. Strength and Other Properties of Concrete. The design and construction of concrete bodies and structures are controlled by specifications prepared by a Joint Committee organized by the following groups: The American Concrete Institute, The American Institute of Architects, The American Railway Engineering Association, The American Society of Civil Engineers, The American Society for Testing Materials, and The Portland Cement Association. In this text, these specifications will be referred to as the JC Code.

TABLE 12-1.—JOINT COMMITTEE GUIDE FOR PROPORTIONING CONCRETE MIXES

Estimated 28-Day Compressive Strength psi.	Maximum Size Coarse Aggregate, Inches	Cement Factor—Sacks of Cement per Cu. Yd. of Freshly Mixed Concrete	Maximum Water per Sack of Cement, Gallons	Fine Aggregate Percentage of Total Aggregate *	Approximate Weights in Lbs. Saturated Surface-Dry Aggregate per Sack of Cement **		
					Total Agg.	Fine Agg.	Coarse Agg.
2250	1	4.9	8	40-46	660	280	380
2250	2	4.5	8	37-43	740	300	440
2250	3	4.1	8	34-40	840	310	530
3300	1	6.5	6	37-43	470	190	280
3300	2	6.0	6	35-41	530	200	330
3300	3	5.5	6	33-39	600	220	380
4250	1	8.0	5	35-41	370	140	230
4250	2	7.4	5	33-39	420	150	270
4250	3	6.8	5	31-37	470	160	310

These proportions are valid for a 4-in. slump. For each 1 in. difference in slump, the cement should be changed by 1/8 sack per cubic yard. The cement proportion should be increased for slumps greater than 4 in. and decreased for slumps less than 4 in.

* The limits shown are approximate and percentages falling outside of the limits shown may frequently be found necessary to produce concrete of the desired workability. When expressed in terms of absolute volume, they are applicable to aggregates of different specific gravities; when expressed in terms of weight, differences in specific gravity should be taken into account.

** These approximate weights are based on a bulk specific gravity of 2.65 in a saturated surface-dry condition.

given to even the smallest job. In many organizations, particularly in small or medium sized plants, concrete work is handled by the maintenance division, or by common labor. In such cases, it is advisable for the processing engineer to check the proportions of sand, aggregate and cement as they are delivered to the mixing board, to see that the plastic mass is properly mixed, and to supervise the actual pouring. In many urban areas it is possible to purchase ready-mixed concrete in any desired quantity, delivered to the job. Such material should only be obtained from a reputable supplier, since it is difficult or impossible to determine the respective proportions of sand, cement, aggregate and water after the concrete has been mixed.

Curing or hardening of the concrete structure is as important as proper working and proportioning, since concrete attains its maximum strength about a month after pouring. Too rapid curing usually results in an appreciable loss of strength and hardness, although quick-setting cements for high early strength concretes are commercially available; they are used where the necessity for high strength in a short period of time overshadows the added cost. Cements with good resistance to acids are used for concrete structures exposed to such conditions. Lime and asphaltic base cements can also be obtained. Common salt or calcium chloride in small proportions added to the mix will improve the strength of concretes cured at freezing temperatures. In reinforced concrete, however, salt is likely to cause corrosion of the steel. Waterproofing compounds are sometimes added in powdered or liquid form to obtain a more impervious concrete, with more or less successful results.

12-2. Expansion and Shrinkage. Temperature and moisture cause significant volume changes in concrete. The coefficient of expansion is $6/10^6$ in. per in. per ° F.; the coefficient of shrinkage of lean dry mixes is about $3/10^4$ in. per in. A shrinkage of 0.0003 in. per in. of length, due to a dry mix, is thus comparable to a contraction caused by a temperature decrease of 50° F. Shrinkage cracks are discernible in most concrete structures; for this reason, and because of expansion, concrete bodies having large surface areas are usually made with open joints. Unheated, uninsulated buildings, for example, require expansion joints at 200-ft. intervals across the entire building, while subjoints at 100-ft. intervals are required in the roof. The volume of concrete is affected to some extent by its moisture content, but this change has little effect upon its weight per unit volume, which is usually taken as 150 lbs. per cu. ft.

12-3. Strength and Other Properties of Concrete. The design and construction of concrete bodies and structures are controlled by specifications prepared by a Joint Committee organized by the following groups: The American Concrete Institute, The American Institute of Architects, The American Railway Engineering Association, The American Society of Civil Engineers, The American Society for Testing Materials, and The Portland Cement Association. In this text, these specifications will be referred to as the JC Code.

The ultimate compressive strength of concrete is evaluated by crushing a standard cylinder 6 in. in diameter and 12 in. high, cured under moist conditions, and tested at the end of 28 days. Estimated ultimate compressive strengths for mixes of different proportions are given in Table 12-1. Fig. 12-1 shows stress-strain data as obtained from standard tests for concretes of varying proportions. From these data, it may be seen that the modulus of elasticity of concrete is constant only over short ranges and at low stresses. In the upper stress ranges, the concrete is not elastic and permanent changes of shape take place before actual failure occurs. The modulus of elasticity E_c is approximated by

$$E_c = 1000\, S_c' \qquad (12\text{-}1)$$

where S_c' is the estimated 28-day compressive strength of the material.

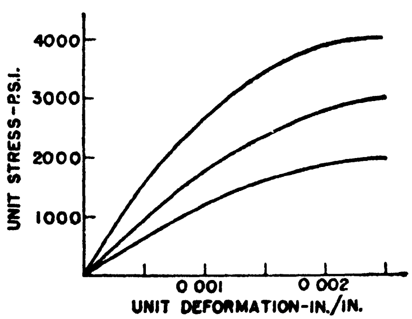


FIG. 12-1. Stress-strain Diagram for Concrete.

For fully loaded areas the JC Code specifies that the allowable bearing stress in concrete bodies, such as foundations, beam supports, etc., or the axial compression in pedestals, should not exceed 25% of the ultimate compressive stress. The allowable bearing stress should not exceed 37.5% of the ultimate strength if the load is carried by a part of the area of the concrete. This differentiation in allowable stress is introduced to eliminate or reduce the possibility of spalling or cracking the edges of the concrete body. For flexure, the maximum permissible fiber stress in compression is 45% of the ultimate compressive strength.

On account of the non-homogeneity of concrete, it is difficult to produce a true tensile failure. The tensile strength is usually measured as a function of the modulus of rupture, but even this evaluation is subject to considerable variation. The usual practice is to specify the allowable flexural tensile strength as one-fifteenth the allowable compressive strength, or 3% of the ultimate compressive strength, and to limit the application of tensile stresses to plain concrete footings where temperature stresses are practically non-existent.

Shearing stresses in concrete bodies subjected to flexural stresses are always accompanied by diagonal tension and compression. The unit diagonal tension is equivalent to the unit shear, and the shearing resistance of concrete is thus limited by the permissible tensile strength of the material. The average unit shearing stress, by the JC Code, is taken as 2% of the ultimate compressive strength, but since the maximum permissible shear for a rectangular section is 50% greater than the average, the allowable maximum stresses for tension and shear in beams and slabs are identical.

12-4. Bearing Power of Soils. Heavy foundations and footings exert considerable pressure on soils, and it is imperative that data be available as to the ability of the soil to handle such loads. Practically all soils will flow under pressure; the science of soil mechanics is concerned with the study of this and related phenomena. The permissible bearing pressure of soils depends upon the shape and size of the foundation as well as the depth of excavation and the moisture content and soil composition. To date, however, there are few available data for predicting a soil's bearing capacity on a purely analytical basis, and it is still necessary to make bearing tests and soil analyses whenever major foundation design is being considered. If accurate test data on the soil in question are not available, the safe bearing pressures given in Table 12-2 may be used, since they represent average values based upon experience and analysis of existing structures. In major design, however, bearing tests should be made.

TABLE 12-2.—SAFE BEARING POWER OF SOILS

Material	Minimum Tons per Sq. Ft.	Maximum Tons per Sq. Ft.
Alluvial soil	½	1
Clays	1	4
Sand (confined)	1	4
Gravel	2	4
Cemented sand and gravel	5	10
Rock	5	

As a rule, foundations and footings should extend above the ground level, and should be deep enough so that the base lies below the frost line. If foundations are too shallow, heaving or lifting may take place, which destroys the alignment of the footing or the equipment supported by it. The depth of the frost line varies considerably, depending upon climatic conditions, but a value of about 2 ft. can be assumed with reasonable safety for ordinary construction. In some instances, footings are provided with frost batter (i.e., poured in the shape of the frustum of a pyramid) so that the resistance of the soil above the inclined faces of the foundation prevents heaving.

Foundations for heavy machinery and footings for columns and walls are subject to the effects of earth settling. Unequal settlement will usually cause cracks in the structure, and may result in misalignment of machinery or other equipment. Settlement of foundations and footings is not objectionable, however, if it is reasonably uniform and does not result in instability. Since the dead load is continuously applied, it is of greatest significance in foundation design. Live loads are usually considered of temporary character, and will

produce little settlement. In designing foundations, particularly where two or more footings are used to support the same piece of equipment, the bearing areas of the separate footings should be balanced so that each carries approximately the same unit load. In proportioning footing areas, it is necessary to provide for the dead load, which should include the weight of the foundation itself and the live load; in balancing foundation areas for anticipated settlement, however, it is common practice to provide only for the sum of the dead load and one third the live load.

12-5. Eccentric Loads. When a foundation or footing is subjected to a vertical load applied at a point or region other than its centroid, the soil bearing condition is analogous to the stress condition of the eccentrically loaded prism described in section 5-18. If a footing of width m and of unit length

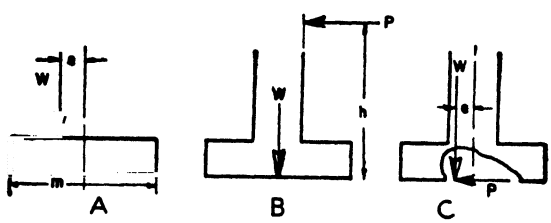


FIG. 12-2. Eccentric Load Analysis for Footings.

is subjected to a load W with an eccentricity e, as shown in Fig. 12-2A, the unit soil pressure at either the right or the left edge, from Eq. 5-16, is

$$S = \frac{W}{m} \pm \frac{6We}{m^2}$$

(in the substitution in Eq. 5-16, m represents a unit of area, and $m^2/6$ the section modulus Z of a unit length). Factoring, S is equal to $(m \pm 6e)W/m^2$. From this expression, it may be seen that the soil pressure at the right edge will be zero if the load eccentricity e is equal to one sixth the width m of the foundation. If e is less than one sixth m, some bearing load will exist at the right edge; if e is greater than one sixth m, the position of zero bearing load will lie at some distance from the right edge.

When a wall is subjected to a horizontal force, such as wind pressure, the moment of the horizontal form tends to overturn the wall; this moment increases the bearing load at one edge of the footing, and decreases it at the other. Fig. 12-2B shows a wall subjected to a horizontal pressure P at a distance h from the ground; W is the dead weight or vertical load on the soil. The force W at the centroid of the footing can be replaced by an equal force W located a distance e from the centroid; if the magnitude of e is taken as Ph/W, and the force P translated to the base of the footing, as shown in Fig 12-2C, this force system is equivalent to that of Fig. 12-2A (as far as vertical forces and

moments are concerned). If the magnitudes involved are such that *e* is equal to or less than one sixth the footing width *m*, compression will exist over the entire surface of the footing.

From the three equations of static equilibrium, the foundation will be stable and will not overturn as long as the virtual load *W*, Fig. 12-2C, lies within the base area of the footing. If the eccentricity is so great, however, that compression exists over only a portion of the footing, the soil pressure at the left edge may be so high that instability and possible overturning will result by virtue of the failure of the soil at that edge. If the entire bearing area underneath the foundation is subjected to compression, the maximum compression at the left edge will not exceed twice the average compression. For these reasons, it has been usual practice to limit the virtual eccentricity *e* to one sixth the base width *m* or, in other words, to require the virtual load *W* to fall within the middle third of the base. In modern design, however, there is often some departure from this practice. For example, the highway department of an eastern state allows the virtual load to fall within the middle half of the footing, rather than the middle third. In masonry tower and chimney design, it has been customary in the past to limit the eccentricity of the virtual load so that no tension exists in the structure. In modern practice, however, if the virtual load *W* falls within the base ring of the chimney, and if the safe unit compressive stresses within the masonry are not exceeded, a partial crack on the windward side of the chimney does not necessarily imply failure, since there is no loss in stability normal to the wind direction, provided the crack does not occur over more than one half the area of the base ring. A detailed discussion of this mode of analysis is given in "Power Plant Engineering and Design." [42]

12-6. Footing Analysis and Design. When equipment is supported by two or more walls and footings, as shown in Fig. 12-3, two methods of computing the effect of an overturning moment on the footings may be used. In the first method, each footing is considered individually, and is presumed to afford resistance to overturning in direct proportion to the section modulus of its base area. In the second method, all the footings are assumed to act as a unit, and the stress on each varies over its base area and is dependent upon the distance of the area from the probable axis about which overturning may take place. Since the footings are not mechanically connected at their bases, the first method of analysis might be assumed to give results more in accord with theory than the second. These analyses, however, do not take into account the rigidity and unity afforded by the vessel at the top of the walls and the effect of the earth friction at the base. It is probable that the maximum soil pressure actually existing under the footings will lie somewhere between the values obtained from the two methods.

Example 12-1. Design plain concrete walls and footings for the oil storage tank shown in Fig. 12-3. The contents of the tank weigh 9600 lbs., the tank shell 1200 lbs., and

the heads 425 lbs. each. The footings are to be at least 2½ ft. thick, the width w is 5 ft., and the height H about 4 ft.

Solution. The tank and its contents are analogous to a uniformly loaded continuous beam of two equal spans. From Fig. 5-37 and Table 5-1, the center reaction is ⅝ of the total load; the end reactions are each 3/16 of the total load. The combined weight of the shell and the contents of the tank are 1200 + 9600, or 10,800 lbs. The center reaction is 0.625 × 10,800, or 6750 lbs.; the end reaction is 0.188 × 10,800, or 2025 lbs. Since the heads are carried only by the end supports, their weight will increase each end reaction to 2450 lbs., and are not included in the center reaction.

The minimum wall thickness t or t', from the standpoint of pouring and working the concrete, is 6 in. If the area bearing on the shell is assumed equal to the projected area of the vessel, the unit bearing stress on the tank at the center support is 6750/6 × 60, or about 19 psi., a very low value. The approximate average height of each wall is 2 ft., and the weight of each wall, based upon the density of concrete as 150 lbs. per cu. ft., will be

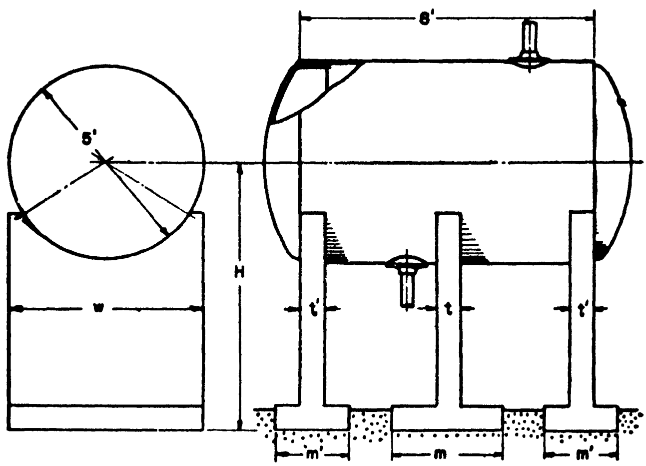


Fig. 12-3. Horizontal Tank Supported by Concrete Piers.

5 × 2 × 0.5 × 150, or 750 lbs., increasing the magnitudes of the center and end reaction loads to 7500 and 3200 lbs., respectively. From Table 12-2, the permissible bearing pressure of alluvial soils is taken as one half ton, or 1000 lbs., per ft. Since the footing has a minimum thickness of 2 ft., 6 in., its weight must be considered in computing the widths m and m', Fig. 12-3. The unit weight of the tank and the walls is $7500/5m$ and $3200/5m'$ for the center and end footings, respectively. The width of the footings is then given by:

$$\frac{7500}{5m} + 375 = 1000, \text{ or } m = 2.4 \text{ ft., center footing}$$

and

$$\frac{3200}{5m'} + 375 = 1000, \text{ or } m' = 1.02 \text{ ft., end footings}$$

The center footing has a width of 28.8 in.; the end footings 12.3 in. These can be reduced to 28 and 12 in., respectively, to give integral figures and to maintain the same bearing load ratio.

Since the footing has an actual height of 30 in., and the maximum projection past the wall of the center footing is only 11 in., the shearing and flexural stresses will be very

small. For a soil pressure of one-half ton per foot, the unit pressure per inch of length is 1000/144, or 6.95, say 7 psi. The moment induced by the soil pressure at the face of the wall, per inch of length, is $7 \times 11^2/2$, or 424 in.-lbs. The effective depth of the footing is equal to the actual depth minus 2 in., to allow for poor concrete in contact with the soil; the section modulus Z, from Chap. 5, is

$$Z = \frac{bd^2}{6} = \frac{1 \times 28^2}{6} = 131 \text{ in.}^3$$

and the unit stress at the face of the wall is

$$S = \frac{M}{Z} = \frac{424}{131} = 3.2 \text{ psi.}$$

The permissible unit stress in tension should not exceed 3% of the estimated 28-day strength of the concrete; if 2250-lb. concrete is assumed, the permissible unit tensile stress is 0.03×2250, or 67.5 psi., which is considerably greater than the induced stress.

The vertical shear at a distance 2 in. from the face of the wall, for a unit length, is 7×9, or 63 lbs. The unit shearing stress is 63/28, or 3.3 psi., which is also negligible.

Example 12-2. Check the soil bearing load and the stability of the foundation of Example 12-1, if a wind pressure of 30 lbs. per sq. ft. of vertical projection is assumed.

Solution. The wind pressure on a cylindrical surface is based upon the projected area of the cylinder, and is usually taken as two-thirds the unit pressure on a flat surface. From Fig. 12-3, the length and diameter of the cylindrical portion of the tank are 8 and 5 ft., respectively; from manufacturers' data, the projection of the dished head at each end is about 10 in. If a virtual or equivalent length of 9 ft. is assumed, to compensate for the projection of the heads, the projected area of the tank is 9×5, or 45 sq. ft. For pressure of $30 \times 2/3$ lbs. per sq. ft. (psf.), the total wind pressures is 45×20, or 900 lbs. For purposes of moment computation, this force may be assumed concentrated in the horizontal plane through the axis of the vessel, resulting in a moment of $900 \times 4 \times 12$, or 43,200 in.-lbs.

From Eq. 5-16 the unit bearing pressure at the extreme edge of the footing, induced by an overturning moment M, is equal to M/Z, where Z is the section modulus of the footing area. For wind pressure on the side of the tank, all three footings resist overturning. The section modulus of the end footing is $12 \times 60^2/6$, or 7200 in.^{3}; that of the outer footing $2 \times 7200 + 16{,}800$, or 31,200 in.^{3} The maximum stress, due to the overturning tendency, is

$$S = \frac{43{,}200}{31{,}200} = 1.37 \text{ psi., or 199 psf.}$$

The actual stress at one edge is equal to the sum of the flexural and vertical stresses, and is equal to $199 + 1000$, or about 1200 psf. Although this value is greater than the minimum of 1000 psf. from Table 12-2, it is considerably less than the maximum of 2000 psf.

It may be noted that the effect of the wind pressure on the edges of the walls has been disregarded; the total area is $3 \times 1.5 \times 0.5$, the total wind pressure is 30×2.25, and the overturning moment is $6.75 \times 0.75 \times 12$, or about 61 in.-lbs. This value is so small that it does not warrant the expenditure of time used in computing it.

In calculating the effect of the wind pressure acting on the circular end of the vessel, the two methods of analysis previously described may be used. The area subjected to wind pressure may be assumed, without serious error, to be a rectangle 6 ft. high and 5 ft. wide. The total wind pressure will be 30×30, or 900 lbs., considered concentrated at a point 3 ft. from the base of the footing, which results in a moment of $900 \times 3 \times 12$, or 32,400 in.-lbs. The section modulus of the three footings is $2 \times 1440 + 7840$, or 10,720 in.^{3} Proportioning the moment to correspond to the section moduli, the maximum stress S at the edge of the footing, due to the overturning tendency, is 32,400/10,720, which is equal to 3.02 psi. or 435 psf. The actual unit pressure at the edge is equal to the sum of the flexural and vertical stresses, and is $435 + 1000$, or 1435 psf.

For the second method of analysis, the force distribution under the footings is shown in Fig. 12-4B, where the wind pressure is considered to act from the right. If F represents the pressure on a footing section 60 in. long and 1 in. wide at a unit distance from the axis of rotation Q, then the pressure under each footing varies as its distance from the axis. To illustrate, the pressure under the left footing varies from a maximum of $102F$ to $90F$, that under the center footing from $65F$ to $37F$, and that under the right footing from $12F$ to 0. The total pressure on each footing may be represented by the area of a trapezoid whose depth is the average of the edge pressures, and whose length is equal to the width of the footing. The resisting moment of these pressures is equal to the product of their areas and centroidal distances, and is equal to the overturning moment. The equation of equilibrium, obtained by dividing the trapezoidal areas into rectangles and

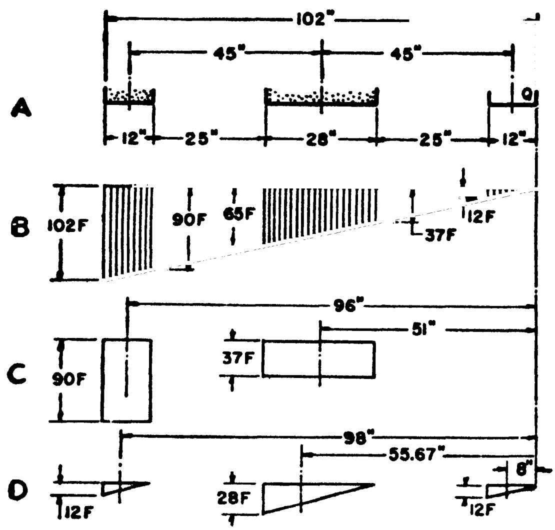


FIG. 12-4. Distribution of Stresses under Footings.

triangles as indicated in Fig. 12-4C and D, and using the distances from the axis Q to their centroids, is

$$(90F \times 12 \times 96) + (37F \times 28 \times 51) + \left(\frac{1}{2} \times 12F \times 12 \times 98\right) + \left(\frac{1}{2} \times 28F \times 28 \times 55.67\right)$$

$$+ \left(\frac{1}{2} \times 12F \times 12 \times 8\right) = 32{,}400$$

or

$$F = \frac{32{,}400}{131{,}836} = 0.246$$

The pressure at the left edge of the left footing is $102F$ or 25.1 lbs. for the 60-in. width, which is equivalent to $25.1 \times 144/60$, or 60 psf. The unit pressure based upon the sum of the vertical and flexural stresses is $1000 + 60$, or 1060 psf.

From the preceding methods of computation, the actual pressure at the edge of the footing may be estimated as somewhere between 1435 and 1060 psf. Although these values are greater than the minimum soil pressures of 1000 psf., they are less than the maximum permissible pressure of 2000 psf.

It may be of interest to ascertain whether the virtual load W falls within the middle third of the end footing. The magnitude of the effective moment, proportioned upon the

section modulus of the base, is 32,400 × 1440/10,720, or 4350 in.-lbs. The total downward pressure W on the end footing, based upon an *empty* tank, is 225 + 400 + 750 + 1875, or 3250 lbs. The virtual eccentricity e is 4350/3250, or 1.34 in. Since the middle third of the footing extends 2 in. on either side of the centerline of the footing, the point of application of the virtual load falls within the middle third.

From the preceding computations, it is evident that an analysis of the stresses induced by the horizontal forces can be omitted with safety if the height of the structure is not too great. In doubtful cases, however, where the designer of a foundation has had no previous experience, or where a record of successful design is not at hand, an analysis that includes the effect of horizontal forces is usually advisable.

Example 12-3. Design footings for the tank of Fig. 12-3 and Example 12-1 if the height H is 12 ft., 6 in. instead of 4 ft.

Solution. The average height of the wall may be taken as 10 ft., 6 in., giving a wall height of 5 × 10.5 × 0.5 × 150, or 3940 lbs. The magnitudes of the reaction, induced by the weight of the tank and contents at the end and at the center walls are 2450 and 6750 lbs. The total end and center reactions, exclusive of the footing weights, are 3940 + 2450, or 6390 lbs., and 3940 + 6750, or 10,690 lbs., respectively. The footing widths are obtained by

$$\frac{10{,}690}{5m} + 375 = 1000, \text{ or } m = 3.42 \text{ ft., center footing}$$

$$\frac{6390}{5m'} + 375 = 1000, \text{ or } m' = 2.04 \text{ ft., end footing}$$

These values give footing widths of 41 in. and 24.5 in., respectively, for which widths of 40 and 24 in. may be used. The weight of these footings are (40/12)(2.5 × 5 × 150), or 6250 lbs. for the center footing, (24/12)(2.5 × 5 × 150), or 3750 lbs. for the end footing. The unit soil pressure, based upon vertical load only, is (6390 + 3750)/(2 × 5), or 1014 psf. for the end footing, and (10,690 + 6250)/(3.3 × 5), or 1020 psf. for the center footing.

The area subjected to pressure may be assumed as a rectangle 14.5 ft. high and 5 ft. wide. The total wind pressure will be 30 × 14.5 × 5, or 2180 lbs., concentrated at a point 7 ft. 3 in. from the base, thus giving an overturning moment of 2180 × 7.25 × 12, or 189,000 in.-lbs. The section modulii of the end and center footings are $60 \times 24^2/6$, or 5760 in.^{3}, and $60 \times 40^2/6$, or 16,000 in.^{3} The pressure at the edge of the footing will be 189,000/(5760 + 5760 + 16,000) which is equal to 6.88 psi. or 990 psf. The combined pressure based upon vertical and flexural load is 990 + 1020, or 2010 psf., which is just over the maximum permissible pressure.

Using the second method of analysis, in which the footings are considered as integral units, and following the procedure outlined in Example 12-2, the equation of equilibrium is

$$(90F \times 24 \times 102) + (37F \times 40 \times 57) + \left(\frac{1}{2} \times 24F \times 24 \times 106\right) + \left(\frac{1}{2} \times 40F \times 40 \times 63.7\right)$$
$$+ \left(\frac{1}{2} \times 24F \times 24 \times 16\right) = 189{,}000$$

or

$$F = \frac{189{,}000}{391{,}360} = 0.483$$

The pressure at the left edge of the left footing is $114F$, or 114 × 0.483, which is equal to 55 lbs. for the 60-in. length, or 55 × 144/60, or 132 psf. This gives an actual soil pressure of 1020 + 132, or 1152 psf.

The total downward pressure on the end footing, based upon an empty tank, is 225 + 400 + 3940 + 3750, or 8315 lbs. The effective moment on this footing, proportioned upon the section modulus of the base, is 189,000 × 5760/27,520, or 39,600 in.-lbs. The virtual eccentricity e is 39,600/8315, or 4.77 in. The middle third extends 4 in. and the middle half 6 in. on either side of the centerline of the end footing, and it is seen that although the

point of application of the virtual load falls slightly outside the middle third, it is still within the middle half of the footing width.

Computations for load distribution based upon wind pressure normal to the axis of the tank, and determination of the flexural stress in the footing itself, have been omitted, since they are analogous to the analysis made in Examples 12-1 and 12-2.

12-7. Machinery Foundations. The design of plain or unreinforced concrete bases or footings for machinery often necessitates a considerably larger area at the foot of the foundation than is required at the top. A concrete base for a motor, for example, may require a comparatively small upper area to support the motor, but a larger footing is required to keep the base within the allowable bearing pressure of the soil. In such cases, the depth of the base will have to be sufficiently great to eliminate the possibility of tension failure in the stepped or sloping sides of the base. In Fig. 12-5, if P represents the permissible unit soil bearing pressure, psf., and S_t is the permissible extreme fiber stress in tension for plain concrete, which is equal to $0.03S_c$, the bending moment at point X, for a unit length of 12 in., will be

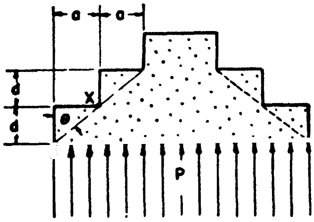


FIG. 12-5. Stepped Footing for Machine Foundation.

$$M = SZ = \frac{12d^2S_t}{6} \text{ in.-lbs.}$$

where d is given in inches.

If the projection of the footing to the left of X is considered a uniformly loaded cantilever, the moment at X is

$$M = 12Pa^2/(144 \times 2) = Pa^2/24 \text{ in.-lbs.}$$

and combining

$$Pa^2/24 = 12d^2S_t/6$$

or

$$d = a\sqrt{\frac{P}{48S_t}} \qquad (12\text{-}2)$$

For a concrete having an allowable compressive strength of 2250 psi., the expression becomes:

$$d = \frac{a}{57}\sqrt{P} \qquad (12\text{-}3)$$

For a 3300-psi. concrete,

$$d = \frac{a}{69}\sqrt{P} \qquad (12\text{-}4)$$

For a 4250-psi. concrete,

$$d = \frac{a}{78}\sqrt{P} \qquad (12\text{-}5)$$

where a and d are in inches, and P is in psf.

If a pedestal with sloping sides instead of stepped sides is used, the angle θ with the vertical is found from

$$\tan\theta = \frac{d}{a} \tag{12-6}$$

In machine or motor foundations and bases, particularly those in which the machine is subjected to horizontal forces, sufficient weight to eliminate movement or possible overturning may be an important factor in the selection of the base dimensions. If the coefficient of friction between concrete and soil be assumed as about 0.30, which is a very conservative value, the total weight of the machinery and base need only be three times the horizontal force. For resistance to overturning, the moment of the foundation and machine weight about its edge must be greater than the moment of the horizontal force.

In addition to the function of distributing the weight of equipment over an area of sufficient size, foundations for machinery must be sufficiently rigid to prevent excessive deflection of the component parts supported by the base. Foundations should also provide sufficient mass to absorb machine vibration, and to compensate for any unbalanced vertical kinetic forces. The required minimum weight of a foundation to absorb vibration is not easily calculated; from past experience, it has been found that for single-cylinder units per brake horsepower, the following will give good results: foundation weights of 2500 lbs. for gas engines, 2000 lbs. for Diesel engines, and 700 lbs. for steam engines. Multi-cylinder engines require foundation masses of about three fifths these values.

Example 12-4. A 50-HP gear motor drives a conveyor through the medium of a roller chain, using a sprocket approximately 17 in. in diameter. The output shaft of the motor rotates at 220 RPM, and the weight of the motor and rails is approximately 2700 lbs.[48] Design a suitable base made of 2000-lb. concrete, for mounting on alluvial soils.

Solution. A cross-section layout is shown in Fig. 12-6. The horizontal pull exerted by the chain is obtained from Eq. 14-4, and is

$$E = \frac{63{,}025 \times 50}{220 \times 17/2} = 1610 \text{ lbs.}$$

To eliminate any possibility of the foundation slipping, the weight of the motor and base must be at least three times this value, or 4830 lbs. If the weight of the motor, 2700 lbs., is deducted, the weight of the base itself must be 2130 lbs.

The overall length of the motor feet, from manufacturers' catalogs, is 30 in.; the rail width, from Fig. 12-6, is approximately 24 in.; the top of the base must therefore have a minimum of 24 in. $\times$ 30 in., or 5 sq. ft. The minimum bearing pressure for an alluvial soil, from Table 12-2, is one half ton per sq. ft. For a total load of 4830 lbs., the required base area will be 4830/1000, or approximately 5 sq. ft. From the standpoint of soil pressure, the upper and lower base areas can be equal. If the weight of a cubic foot of plain concrete be assumed as 140 lbs., the required height of the base, to provide sufficient weight to prevent slippage, will be $(4830 - 2700)/(5 \times 140)$, or 3.04 ft. This height is obviously out of proportion to the length and width of the base; it is consequently advisable to assume a base with an upper surface 3½ ft. wide and 3 ft. long and a lower surface 4½ ft. wide and 4 ft. long. The average horizontal area of this design will contain 4×3.5, or 14 sq. ft., and the required thickness of the base will be $(4830 - 2700)/(14 \times 140)$, or 1.085 ft., equal to 13 in.

The unit soil pressure is 4830/(4.5 × 4), or 268 psf. For 2000-lb. concrete, the maximum permissible slope of the lateral sides, from Eqs. 12-3 and 12-6, is

$$d = \frac{a}{53}\sqrt{268} = 0.31a$$

and

$$\tan\theta = \frac{a}{d} = \frac{a}{0.31a} = 3.2$$

From the standpoint of strength, the lower 2 in. of concrete are usually disregarded so the net height of the base may be taken as 11 in. The actual slope is

$$\tan\theta = \frac{(4.5 - 3.5)12}{11 \times 2} = 0.55$$

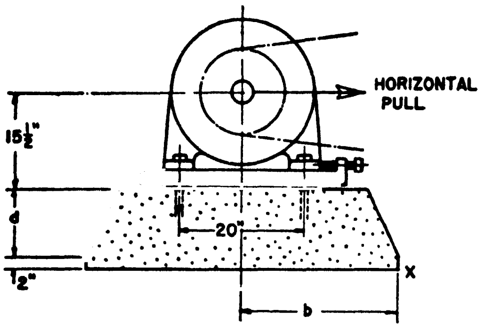


Fig. 12-6. Motor Foundation.

Since the actual slope is considerably less than the permissible, the footing is safe as regards tensile failure.

The foundation should be checked against possible lifting or overturning. The moment arm of the horizontal force, with respect of point X, is $15.5 + d + 2$, or 28.5 in., and the moment is 1610 × 28.5, or 46,000 in.-lbs. The moment that resists turning about point X is equal to the product of the total weight and the distance b, or 4830 × 2.25 × 15, or 130,600 in.-lbs., which is ample.

12-8. Design of Plain Floors and Slabs. Plain floors for shops and buildings may be made of unreinforced concrete and placed directly on a soil foundation or, in some instances, on a cinder sub-foundation. Plain concrete floors are usually from 4 to 6 in. thick; if thicknesses greater than 6 in. are required, because of moving loads or because the flooring is used as a support for machinery, it is usually advisable to localize the regions of greater depth and make the balance of the floor area of a uniform minimum thickness. On account of their heat conductivity and comparative hardness, concrete floors are unpleasant to stand on, especially when it is necessary to remain continuously in one spot, as in immobile operation, and wooden platforms or bases should be provided at such points.

The adhesion of new concrete to work previously placed is of importance where additions to construction must be made. Tests indicate that thorough cleaning and roughening of the old surface, and treatment with hydrochloric acid or the application of a cement grout, will give joint efficiencies up to 80%. Merely cleaning and wetting the surface will give a joint efficiency in the neighborhood of 40%.

Reinforced Concrete

12-9. Since the tensile strength of concrete is very low, beams or other members subjected to flexural stresses must be of excessive or uneconomical

size if made of plain concrete. For this reason steel reinforcing members are usually imbedded in the concrete at or near the regions subjected to tensile stresses, as shown in Fig. 12-7. Reinforcing bars are used so that the concrete will develop numerous small cracks in preference to a few open cracks when the concrete is stretched beyond its deformable range. Reinforcement used in this manner is known as "temperature steel." Reinforcing bars are also employed to act with the concrete in resisting compression in order that the overall dimensions of the members may be reduced.

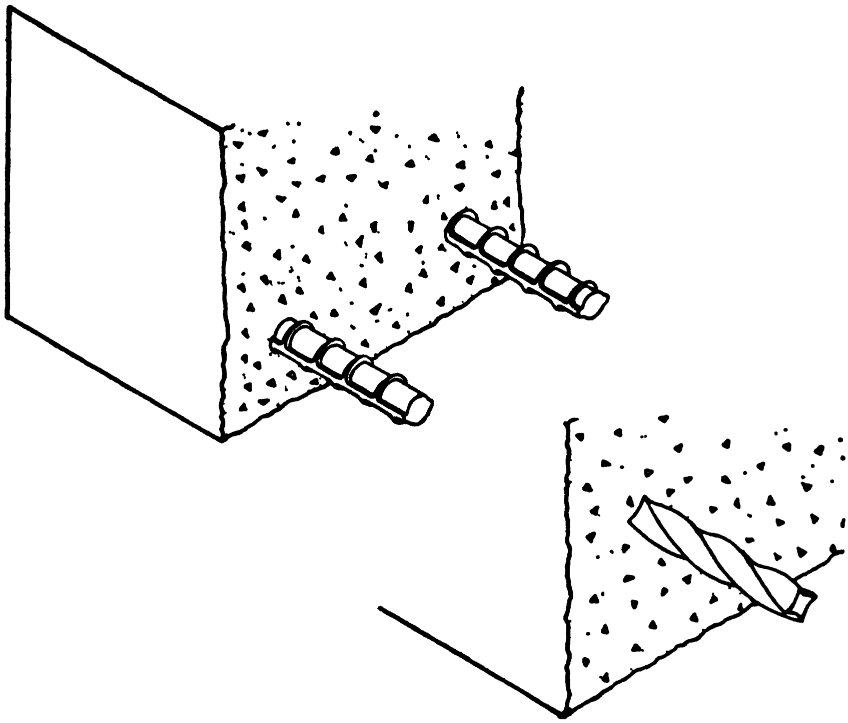


FIG. 12-7. Reinforced Concrete Beams.

Reinforcing bars must be supported inside the forms by wire ties or other supporting media, so that displacement will not occur when the concrete is poured and tamped. In order that the aggregate may pass between the bars, the open clearance must be at least 50% greater than the maximum size of the stone, but should not be less than 1 in. for beams and flat slabs, or 1½ in. for columns and walls. In addition, the minimum spacing for parallel bars in 2.50*D* for round, and 3*D* for square bars, where *D* is the diameter of the round bar, or the size of the square bar.

Out-of-door reinforced concrete structures should have a concrete "cover" of at least 2 in., and preferably 3 in., over the steel bars to prevent atmospheric

corrosion. In interior fabrication, the thickness of the concrete outside the bars, or cover, is properly considered a means of fireproofing. Structures are rated by the JC Code in terms of the hours they will withstand a standard fire test without structural failure. For beams, girders, and floor joists, a 2-in. cover is presumed to give four hours of fire protection, a 1½-in. cover three hours, and a 1-in cover one hour. Solid slabs and walls protected by at least ¾ in. of plaster are from 50 to 100% more effective in resisting fire than plain concrete surfaces.

The permissible tensile stress in structural grade steel bars and shapes, from the JC Code, is 18,000 psi. Bars made of billet, rail, or axle steel may be stressed to 20,000 psi. The permissible tension in web reinforcement, for all grades of steel, is 16,000 psi. Wire mesh or bars not over ½-in. diameter, when used in one-way slabs, can be stressed to 50% of the yield point of the material, but the stress cannot exceed 25,000 psi. in any case.

12-10. Reinforced Concrete Beams and Slabs. Wooden and steel members subjected to flexure must be investigated for possible failure caused by flexure or by shear, and should be proportioned to resist the maximum stresses developed in either condition. In reinforced concrete construction, failure may occur either by flexure, by shear, or by diagonal tension induced by horizontal flexural tension and tension due to shear. Each of these will be considered separately.

In the analysis of flexural members made of a homogeneous material, it has been assumed that a cross section plane before bending remains a plane after bending, that the strain is proportional to the distance from the axis of rotation of the section, and that the stress is proportional to the strain. From a consideration of the stress-strain diagram, Fig. 12-1, it is apparent that such assumptions do not hold rigorously for concrete. In addition such factors as tensile strength, temperature and shrinkage stresses, and plastic flow require special consideration in reinforced concrete beam design. Since the tensile strength of concrete is very low, it is customary to disregard it entirely, and consider the reinforcement to carry all of the tensile stress. Temperature and shrinkage stresses and plastic flow are important, but are disregarded because they are indeterminate. Plastic deformations are not high at working stresses and tend to retard the development of high stress concentrations; neglecting plastic flow is therefore ordinarily on the side of safety. The allowable stresses specified in the JC Code are sufficiently low to compensate for temperature and shrinkage stresses, and for plastic flow. The process engineer or the occasional designer of equipment should bear in mind that major design projects and analyses, in which special conditions of stress, load, or temperature control the construction, should be handled in consultation with an experienced reinforced-concrete design engineer.

12-11. Beam Flexure. The conventional theory of beam flexure postulates a beam of homogeneous material, in which the neutral axis of the cross section passes through its centroid. Since this relationship does not hold for a

beam composed of both concrete and steel, it is convenient to employ a substitute homogeneous cross section composed entirely of concrete, having the same characteristics as the actual steel-concrete section. Such a substitution is termed a transformed section, shown in Fig. 12-8, and consists of two parts: the existing area A_c of concrete above the neutral axis; and a fictitious area A_t of tensile concrete below the neutral axis, replacing the actual area A_s of the reinforcing steel. The ratio n of the modulii of the elasticity of steel and concrete is given by

$$n = \frac{30 \times 10^6}{E_c} \tag{12-7}$$

where E_c is obtained from Eq. 12-1, and 30×10^6 represents the modulus of elasticity for steel. Since the actual elongation is n times as great for steel as for concrete, the area A_t must be equal to nA_s, so that the unit stress in the fictitious tensile concrete will be only $1/n$ times the actual steel stress. In the transformed section, the fictitious concrete area is located at the same distance from the neutral axis as the reinforcing steel; the change in area is obtained by increasing the width, leaving the vertical dimensions equivalent to that of the steel.

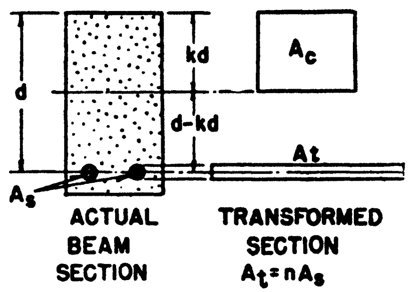


Fig. 12-8. Transformed Section of Reinforced Concrete Beam.

In Fig. 12-8, the distance from the upper edge of the beam section to the neutral axis is kd, where d is the effective depth, or distance, from the upper edge to the centroid of the reinforcement. and k is a factor less than unity. The position of the neutral axis is found by equating the static moment of the areas A_c and A_t as follows:

$$b(kd)(kd/2) = A_t(d - kd)$$

The ratio p between the steel and concrete areas is given by

$$p = \frac{A_s}{bd} \tag{12-8}$$

Since A_t is equal to nA_s, the expression for the static moments of the areas is

$$\frac{bd^2k^2}{2} = pbdn(d - kd)$$

Rearranging and completing the square

$$k^2 + 2kpn - 2pn = 0$$

or

$$k = \sqrt{2pn + (pn)^2} - pn \tag{12-9}$$

Two methods of analysis are commonly used for determining the stresses in the transformed section. In the first, the "beam flexure" method, the moment of inertia about the centroid of the transformed section is computed, and the stresses in the actual compressive and fictitious tensile concrete areas are obtained from the flexure formula, Eq. 5-13. The actual stress in the steel is then obtained by finding the product of the tensile stress in the concrete and the factor n. In the second, called the "couple" method, the tensile and compressive resisting forces T and C are considered concentrated at the level of the tensile stress and at the centroid of the compressive resistance, respectively, as shown in Fig. 12-9. Since the horizontal summation must be equal to zero, T and C are equal, and comprise a couple whose resisting moment is equal to the external flexural moment. The centroid of the compressive resistance is located at a distance $kd/3$ from the upper edge of the section, and the distance between T and C, or the arm of the couple, is equal to $d - kd/3$, or jd, where j is less than unity. From the preceding expression, j is equal to $1 - k/3$.

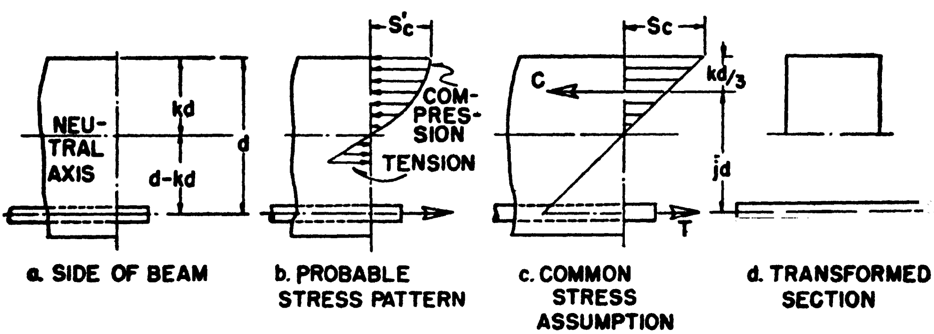


FIG. 12-9. Stress Assumptions in Reinforced Concrete Beams.

If S_s represents the unit stress in the steel, the tensile resisting force T is equal to S_sA_s. Since the moment Tjd of the couple is equal to the external moment M, the unit stress in the steel is given by

$$S_s = \frac{M}{A_sjd} \tag{12-10}$$

If S_c represents the unit stress in the concrete, at the extreme fiber, the compressive resistance C is equal to $S_cA_c/2$. Since the compressive area A_c is equal to the product of the breadth b and the depth kd, and the moment Cjd of the couple is equal to M, the unit stress at the extreme fiber of the concrete is given by

$$S_c = \frac{2M}{A_cjd} = \frac{2M}{jkbd^2} \tag{12-11}$$

Example 12-5. A reinforced concrete beam has a span of 20 ft., and carries a load of 350 lbs. per ft. of length in addition to its own weight. The beam section is shown

in Fig. 12-8; the beam width is 8 in., the effective depth is 16 in., and the total depth is 18 in. The two reinforcing rods are 1 in. in diameter. The beam was originally designed for 3300-lb. concrete. Determine the stresses in the concrete and reinforcing steel.

Solution a. (Beam Flexure Method) The gross area of the beam section is $8 \times 18/144$, or 1 sq. ft. The weight of the beam per foot of length is 150 lbs., making the total uniform load $350 + 150$, or 500 lbs. The maximum flexural moment exists at the center of the span, and is equal to $wL^2/8$; by substitution, the flexural moment is equal to $500 \times 20^2/8$, or 25,000 ft.-lbs., or 300,000 in.-lbs. The actual area A_s of the steel is $2\pi D^2/4$, or 1.571 sq. in. The modulus of elasticity E_c of the concrete, from Eq. 12-1, is

$$E_c = 1000 \times 3300 = 3{,}300{,}000 \text{ psi.}$$

The ratio n, from Eq. 12-7, is

$$n = \frac{30 \times 10^6}{3{,}300{,}000} = 9.1$$

The fictitious area A_t is equal to 9.1×1.571, or 14.3 sq. in., giving a width of 14.3 in. for a depth of 1 in.

From Eq. 12-8, the ratio p is

$$p = \frac{1.571}{8 \times 16} = 0.0123$$

From Eq. 12-9, the value of k is

$$k = \sqrt{2 \times 0.0123 \times 9.1 + (0.0123 \times 9.1)^2} - (0.0123 \times 9.1) = 0.373$$

and $$kd = 0.373 \times 16 = 5.96 \text{ in.}$$

The moment of inertia of the concrete area A_c, from Fig. 5-12, is $8 \times 5.96^3/3$, or 565 in.[4] The moment of inertia of the fictitious area A_t, from Eq. 5-6, is $I_{gg} + A_t(d - kd)^2$. Substituting, $[14.3 \times 1^3/12 + 14.3(16 - 596)^2]$ which is equal to 1447.2 in.[4] The unit compressive stress, from the flexure formula, is

$$S_c = \frac{Mkd}{565 + 1447.2} = \frac{300{,}000 \times 5.96}{2012.2} = 890 \text{ psi.}$$

The unit tensile stress in the transformed steel area is

$$S_t = \frac{M(d - kd + D/2)}{565 + 1447.2} = \frac{300{,}000(16 - 5.96 + 0.5)}{2012.2} = 1570 \text{ psi.}$$

The actual stress in the steel is $1570n$, or 14,300 psi.

Solution b. (Couple Method) The location of the centroid of the section, and the determination of the factor k, are the same as in Solution *a*. From Fig. 12-9, the factor j is equal to $1 - k/3$, or $1 - 0.373/3$, or 0.876. From Eq. 12-10, the tensile stress in the steel is

$$S_s = \frac{2 \times 300{,}000}{1.571 \times 0.876 \times 16} = 13{,}530 \text{ psi.}$$

The maximum compressive stress in the concrete, from Eq. 12-11, is

$$S_c = \frac{2 \times 300{,}000}{0.876 \times 0.373 \times 8 \times 16^2} = 890 \text{ psi.}$$

The difference between the values of S_s obtained in the beam flexure and couple methods of solution is caused by taking the extreme or lowest fiber of the steel in the first method, and the fiber at the centroid in the second. The beam flexure method is more exact, but the error may be neglected for the usual proportions employed in building construction.

12-12. T Beams. Building floor and roof slabs are often poured integrally with the supporting beams, forming T-beam sections, as shown in Fig. 12-10. In such cases, a portion of the slab is assumed to form the flange of the beam and to aid in resisting the flexural moment. The proportions of the slab that are so considered are shown in Fig. 12-10 for both T and L (one-sided flange) beams. In addition to these data, the effective flange width cannot exceed the spacing from center to center of adjacent stems, or one-fourth the span length of the beam, for T beams. For L beams, the effective flange width must not be in excess of the width of the stems plus one-half the clear distance to the next beam, or the width of the stem plus one-twelfth the span of the beam.

The moment resistance of the concrete below the neutral axis of a T beam is completely neglected. There are two possible conditions of analysis, dependent upon the position of the neutral axis, Fig. 12-11. Case A is identical with the analysis of a rectangular beam having the same effective depth *d*, and a width *b*

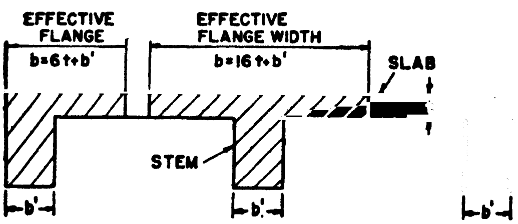


Fig. 12-10. Integral Slab and Beams.

equal to the effective flange width of the T beam. In Case B, the neutral axis lies below the flange, and the transformed area consists of a shallow T section above the neutral axis, and a narrow transformed tensile area at the level of the steel. This analysis may be simplified by disregarding that portion of the stem that is effective in resisting compression, and considering the compressive area as a simple rectangle of width *b* and depth *t*. For analysis in which the neutral axis lies in the flange, Case A, the couple method may be employed; for Case B, the beam flexure method of analysis is usually easier to apply.

12-13. In beams with built-in ends, or in continuous multi-span beams, conditions of negative moment exist at the supports, inducing tensile stresses in the upper portion of the beam section. Tensile reinforcement must be provided at such regions. In slabs and shallow beams, it is usual practice to bend the reinforcing steel provided for the lower portion of the beam section, as illustrated in Fig. 12-12A. For beams of considerable depth, however, the cost of fabrication is excessive, and the use of short sections of tensile steel at the regions of support, shown in Fig. 12-12B, is common practice. The analysis and design of such tensile steel in the upper portion of the beam section are essentially the same as that described in section 12-11.

In reinforced concrete beams of long span, the beam depth is comparatively great, and in building floor construction will result in a decrease of available headroom. Beam depth can be minimized at the expense of a disproportionate increase in beam width, but economy and modern practice usually dictate the use of compressive steel reinforcement, illustrated in Fig. 12-13. The amount of compressive steel is usually somewhat less than the tensile steel, since the concrete above the neutral axis carries its proportionate share of the flexural compressive stress.

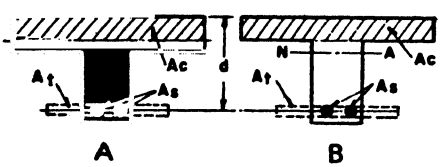


FIG. 12-11. Tee-beam Analysis.

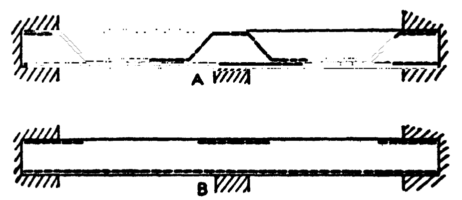


FIG. 12-12. Tensile Reinforcement in Multispan Beams.

The permissible stress in compressive reinforcing steel is equal to 2 $(n-1)$ times the compressive stress in the concrete at the same level, since plastic flow or time yield has been demonstrated to produce greater steel stresses in compressive reinforcement than is indicated by the theory of straight-line stress distribution. The stress, however, must not exceed 16,000 psi. The compressive reinforcement must be secured against buckling by ties or stirrups adequately anchored in the concrete and spaced not more than sixteen bar diameters apart.

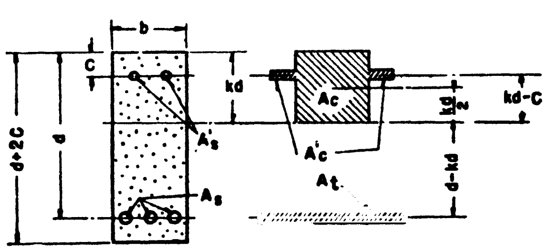


FIG. 12-13. Concrete Beam Action with Double Reinforcement.

The transformed section method can be used to compute the efficiency of beams with double reinforcement by adding a fictitious area A_c' to the compressive area A_c of the concrete. The size of this fictitious area is obtained from

$$A_c' = 2(n-1)A_s' \tag{12-12}$$

where A_s' represents the area of the compressive reinforcing steel.

Example 12-6. The doubly reinforced beam section shown in Fig. 12-13 has a depth d of 12 in., a width b of 6 in., with two ⅜-in. round bars for compressive reinforcement, and three ⅝-in. round bars for tensile reinforcement. The centers of the bars are

2 in. from the upper and lower faces of the section. Determine the stresses in the concrete and steel for a flexural moment of 160,000 in.-lbs., if the original design was based on 3000-lb. concrete.

Solution. The neutral axis is located by equating the statical moment of the tensile and compressive areas. The modulus of elasticity of the concrete, from Eq. 12-1, is

$$E_c = 1000 \times 3000 = 3{,}000{,}000 \text{ psi.}$$

The ratio n, from Eq. 12-7, is

$$n = \frac{30 \times 10^6}{3{,}000{,}000} = 10$$

The area of a ⅝-in. round bar is 0.31 sq. in., and the fictitious tensile area A_t is equal to $10 \times 3 \times 0.31$, or 9.3 sq. in. The moment of this area is $9.3(10 - kd)$. The area of the concrete is $6kd$, and its moment is $3(kd)^2$. The area of a ⅜-in. round bar is 0.11 sq. in., and the fictitious tensile area A_c', from Eq. 12-12, is $2(10 - 1)2 \times 0.11$, or 3.96 sq. in. The moment of this area is $3.96(kd - 2)$. Equating the statical moments of the tensile and compressive areas,

$$93 - 9.3kd = 3(kd)^2 + 3.96kd - 7.92$$

Collecting and solving,

$$(kd)^2 + 4.42kd = 33.64$$

$$(k + 2.21)^2 = 38.52$$

or

$$kd = 4.00 \text{ in.}$$

The moment of inertia of the entire section is equal to the sum of the moments of inertia of its component parts, or

$$I \text{ (tensile steel)} = 9.3(10 - 4.00)^2 = 334.0$$

$$I \text{ (compressive steel)} = 3.96(3.92 - 2)^2 = 15.8$$

$$I \text{ (concrete)} = 6(3.92)^3/3 = 128.0$$

$$= 477.8$$

The fiber stresses, from the flexure formula, are

$$S_s = \frac{160{,}000 \times 6.08 \times 10}{477.8} = 20{,}300 \text{ psi.}$$

$$S_s' = \frac{160{,}000 \times 1.93 \times 10 \times 2}{477.8} = 12{,}900 \text{ psi.}$$

$$S_c = \frac{160{,}000 \times 3.93}{477.8} = 1315 \text{ psi.}$$

The actual stress S_s in the tensile steel is slightly higher than the permissible value of 20,000 psi. given for billet or rail steel, but will probably be satisfactory. The stress S_s' in the compressive steel has been computed on the basis that it is equal to twice the stress in the concrete at the same level; since the resultant stress is less than 16,000 psi., it is satisfactory. The allowable stress in 3000-lb. concrete (section 12-3) is 0.45×3000, or 1350 psi., and the resultant value of S_c is therefore within safe limits.

12-14. Shear and Diagonal Tension. The criterion of resistance to shearing forces is fully as important in concrete structures as the tensile or compressive resistance of the material, particularly so when the comparatively low shearing strength of the material is kept in mind. The average unit vertical shearing stress in a reinforced concrete beam of rectangular section is equal to the total shear V, divided by the effective area bd of the section, and should not

exceed 2% (section 12-2) of the ultimate compressive strength of the material. The unit horizontal shear v in a concrete beam of rectangular section is given by

$$v = \frac{V}{bjd} \tag{12-13}$$

(The derivation of this expression may be found in any standard text on reinforced-concrete design.)

Permissible working stresses specified by the JC Code for shear allow 2% of the ultimate strength of the concrete for beams without web reinforcement or end anchorage of the longitudinal steel, and 3% for beams without web reinforcement but in which end anchorage is provided. Beams with web reinforcement without end anchorage may have shear stresses up to 6% of the ultimate compressive strength. If web reinforcement to carry all the vertical shear and suitable end anchorage are provided, the allowable shear may be as great as 12%.

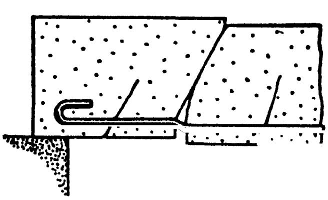


FIG. 12-14. Shear Failure in Concrete Beams.

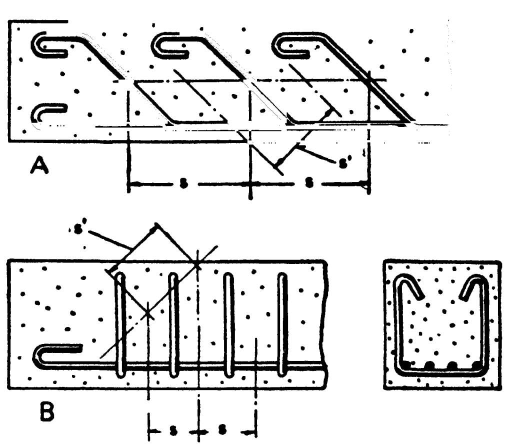


FIG. 12-15. Reinforcement for Shear and Diagonal Tension.

The combination of vertical and horizontal shearing stresses induce a diagonal tension whose intensity will be equal to the horizontal shear but which acts at an angle of 45° to the horizontal. Since the concrete below the neutral axis is not considered to carry any flexural stress, the diagonal tension remains constant over the lower part of the beam section. That portion of the beam section above the neutral axis is subjected to a compressive stress (caused by flexure) in addition to the horizontal and vertical shearing stresses, which affects both the direction and magnitude of the diagonal tension. Such variation, however, is disregarded and it is assumed that the diagonal tension existing at the neutral axis is uniform over the entire section of the beam. Diagonal tension in unreinforced beams may result in shear failure as indicated in Fig. 12-14. To guard against such failure, one of the two methods of reinforcement shown in Fig. 12-15 are used. The detail at A is the more effective method of reinforcement since the bars are perpendicular to the cracks which might open in the beam

Since reinforced concrete beams have horizontal bars near the lower surface, the web reinforcements can also be cared for by vertical stirrups, as shown in detail B. The stirrups will resist the vertical component of the diagonal tension, leaving the horizontal component to be resisted by the tensile steel of the beam itself.

For sloping bars, Fig. 12-15A, the unit stress f_s in the web reinforcement is given by

$$S_s = \frac{(v - v')sb}{(\cos\theta + \sin\theta)A_v} \qquad (12\text{-}14)$$

where v is the unit shearing stress, psi., from Eq. 12-13, v' that portion of the unit shearing stress that is carried by the concrete, b the breadth of the beam section, s the bar or stirrup spacing or horizontal distance, and A_v the cross-sectional area of the bent bars or stirrups in any plane. For bars bent at an angle of 45° to the horizontal, Eq. 12-14 reduces to

$$S_s = \frac{(v - v')sb}{\sqrt{2}A_v} \qquad (12\text{-}15)$$

For vertical stirrups, shown in Fig. 12-15B,

$$S_s = \frac{(v - v')sb}{A_v} \qquad (12\text{-}16)$$

The permissible stress in web reinforcement is 16,000 psi., regardless of the grade of steel used. The horizontal length of the web reinforced by a set of bent bars in one plane should not exceed three quarters of the effective depth d of the beam. The web reinforcement should be spaced so that any 45° crack below the mid point of the beam section will be intersected by at least one bar, and at least two bars in different planes should intersect each possible 45° crack when the unit shear exceeds 6% of the ultimate compressive strength of the concrete. If the moment diagram for the entire beam is drawn, it is possible to determine the point at which the bars may be bent up to provide resistance to diagonal tension without materially affecting the flexural resistance of the beam. The moment resistance of the cross section with the various reduced areas of tensile steel is computed and the bars are bent at a point approximately 1 ft. beyond which the moment resistance of the remainder of the section is equal to or in excess of the flexural moment. The bars are usually bent up in pairs.

Modern practice avoids the use of bent bars for diagonal tension because of the difficulty and expense of proper fabrication. For this reason, vertical stirrups are usually employed, except in flat slabs where the effective depth of the section is so small that a comparatively short bend can be made in the bar.

Reinforcing steel, whether for tensile or for shearing reinforcement, should be carefully placed and securely held in the form so that movement of the steel does not occur when the concrete is poured. Many forms of ties and supports

are commercially available for this purpose, several of which are shown in Fig. 12-16. The cost of such positioning steel is only a few dollars per ton and its use insures construction and fabrication in accordance with specifications. For exposed ceilings where no plaster will be used the legs of the supports or chairs that come into contact with the forms, and will consequently be exposed in the final construction, should be hot galvanized to prevent rusting and staining of the surface.



Fig. 12-16. Supports for Holding Reinforcing Steel in Place.

12-15. Bond Resistance. Since a reinforced concrete member is considered an integral unit, its capacity depends upon the bond resistance between the concrete and the steel. Bond resistance varies directly with the crushing strength of the concrete, and is usually expressed as a percentage of the ultimate strength of the concrete. For beams, slabs, and one-way footings, the bond resistance per square inch of bar surface may be taken as 4% of the ultimate strength of the concrete for plain bars, and as 5% of the ultimate strength for deformed bars. For multiple-way footings, in which bars run in perpendicular directions, 3% and 3.75% of the ultimate strength are permissible for plain and deformed bars, respectively. In no case, however, may the bond resistance values exceed 160 psi. for plain bars or 200 psi. for deformed bars. When the bars are provided with semi-circular hooks or loops at the ends, to give end anchorage, permissible bond resistance stresses 50% greater than those given may be used, provided that maximum values of 200 psi. for plain bars, and 250 psi. for deformed bars, are not exceeded. Hooks and loops must be of proper size, in accordance with the specifications of the JC Code.

The vertical shear is a measure of the change in longitudinal stress per unit length along a beam, as follows

$$T_1 - T_2 = \frac{V}{jd}$$

The force tending to induce slip of the embedded bars is the difference in the tensile forces on either side of the section under consideration; if the perimeter of all the bars is designated by P, the unit bond stress u is

$$u = \frac{V}{Pjd} \qquad (12\text{-}17)$$

The efficiency of the bond is materially affected by the presence of cracks in the concrete, and the above expression should only be considered as a measure of the bond stress relationship between design and test practice. For design purposes, however, the values of the permissible bond stresses given in the preceding paragraphs are sufficiently conservative to insure safety.

Rather simple relationships between the bar diameter and the length of embedment may be obtained from the preceding data. The bond resistance of

a plain bar of diameter D and length L is $\pi DL \times 0.04S_c$. The tensile strength of the bar is $\pi D^2 S_t/4$. Equating,

$$L = \frac{DS_t}{0.16S_c'} \qquad (12\text{-}18)$$

By substituting suitable values for S_t and S_c', the necessary length of embedment L, for plain structural steel bars, is

$$2250\text{-lb. concrete, } L = 50D \qquad (12\text{-}19)$$

$$3300\text{-lb. concrete, } L = 34D \qquad (12\text{-}20)$$

$$4250\text{-lb. concrete, } L = 26.5D \qquad (12\text{-}21)$$

For deformed bars, the values for L are 80% of those given for plain bars. For bars with end anchorage, the values of L are two thirds of those given. These expressions provide that pull-out will not occur until the bar fails in tension.

Stirrups and other forms of vertical web reinforcement should have lengths and diameters such that embedment will be carried by the upper four tenths of the beam section. In a beam with an effective depth of 20 in., for example, made of 2250-lb. concrete, the length of embedment will be 0.4×20, or 8 in., and the diameter of the bar will be limited to $2L/50$, or $2 \times 8/50$, or 0.32 in. A stirrup with a diameter greater than $\frac{5}{16}$ in. will be useless, since the length of embedment rather than the bar strength is the limiting factor in this case.

Foundation bolts, similar to those shown in Fig. 7-25, are provided either with bent ends, or with a head and a large washer to prevent pulling out. Since the tensile strength of foundation bolts is based on the area at the root of the threads, such bolts may be considered safe against pulling out if the length of embedment is equal to thirty times the nominal diameter.

Example 12-7. The beam shown in Fig. 12-17 has a span of 24 ft., and carries a uniform load, inclusive of its own weight, of 1800 lbs. per ft. of length. The beam was originally designed for 3300-lb. concrete, and 20,000-lb. steel. Determine the possible modes of failure and check the stresses.

Solution. The shear and moment diagrams for the beam are shown directly below the front elevation; the moment diagram is of parabolic form, with a maximum moment of 1,296,000 in.-lbs. at the center of the span. In determining the flexural resistance, consideration must be given to the fact that six bars serve to resist tension from the mid-span of the beam to section AA, but five bars are available for this purpose between sections AA and BB, and only the three lower bars between section BB and the end support. The flexural resistance must therefore be determined for these three conditions, which will be referred to as conditions a, b, and c.

Condition a. Flexure between mid-span and section AA. The area A_s of a single bar is $\pi(0.875^2)/4$, or 0.60 sq. in. From Eq. 12-1, the ratio n is equal to $(30 \times 10^6)/(1000 \times 3300)$, or 9.1. The fictitious area A_t of each set of three bars is $3 \times 0.60 \times 9.1$, or 16.38, sq. in. From detail F, the position of the neutral axis is found by equating the static moments of the compressive concrete area A_c and the fictitious steel area A_t, as follows:

$$12(kd)^2/2 = (22 - kd)16.38 + (25 - kd)16.38$$

$$(kd)^2 = 128.31 - 5.46\,kd$$

Completing the square and solving,

$$(kd + 2.73)^2 = 128.31 + 7.45$$

$$kd = 8.9 \text{ in.}$$

Using the beam flexure method of solution, the moment of inertia of the concrete area A_c is $12 \times 8.9^3/3$, or 2820 in.^{4} The width of the fictitious area of the upper row of bars is

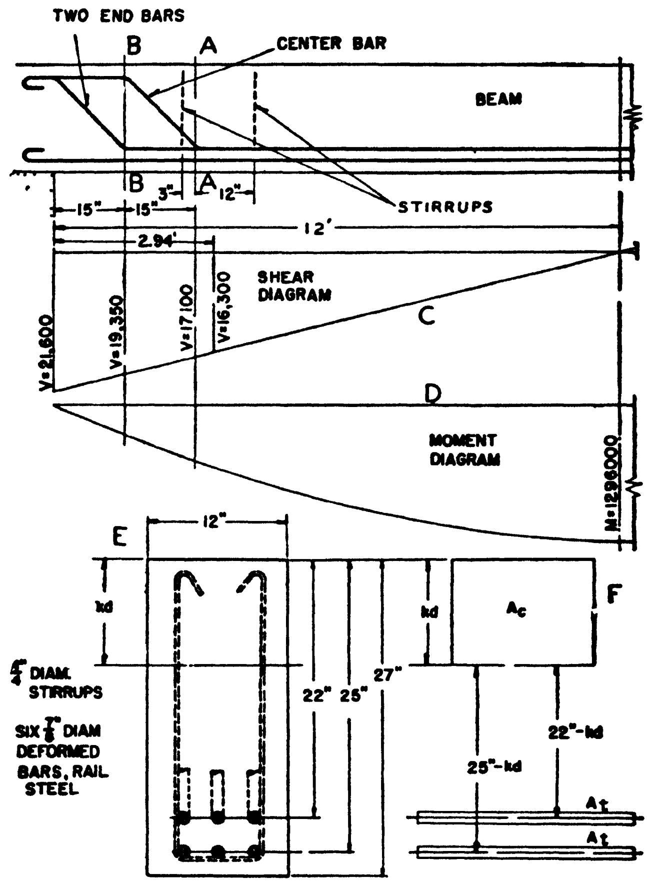


FIG 12-17 Reinforced Concrete Beam Analysis.

16.38/0.875, or 18.7 in. The moment of inertia of this area, with respect to the neutral axis, is $I_{gg} + A_t(d - kd)^2$ Substituting, $[18.7 \times 0.875^3/12 + 16.38(22 - 8.9)^2]$ which is equal to 2810 in.^{4} Similarly, the moment of inertia of the fictitious area of the lower set of bars is $[18.7 \times 0.875^3/12 + 16.38(25 - 8.9)^2]$, or 4250 in.^{4} The unit compressive stress in the concrete, from the flexure formula, is

$$S_c = \frac{Mkd}{2820 + 2810 + 4250} = \frac{1{,}296{,}000 \times 8.9}{9880} = 1170 \text{ psi.}$$

The actual stress in the extreme fiber of the lower bars is

$$S_s = \frac{nM(d - kd + D/2)}{9880} = \frac{9.1 \times 1{,}296{,}000(25 - 8.9 + 0.875/2)}{9880} = 19{,}800 \text{ psi.}$$

Since the tensile stress is less than 20,000 psi., and the compressive stress is less than 45% of 3300, or 1485 psi., the flexural stresses are within safe limits.

Condition b. Flexure between sections AA and BB. The fictitious area A_t of the three lower bars is 16.38 sq. in.; that of the two upper bars is 10.92 sq. in. Equating the static moments of the areas,

$$12(kd)^2/2 = (22 - kd)10.92 + (25 - kd)16.38$$

Solving,

$$kd = 8.32 \text{ in.}$$

The moment of inertia of the concrete is $12 \times 8.32^3/3$, or 2300 in.^{4} The width of the fictitious area, for the upper two bars, is 10.92/0.875, or 12.5 in., and the moment of inertia of this area is $[12.5 \times 0.875^3/12 + 12.5(22 - 8.32)^2]$, or 2340 in.^{4} The moment of inertia of the lower row of bars is $18.7 \times 0.875^3/12 + 16.38(25 - 8.32)^2$, or 4550 in.^{4} For a permissible concrete stress of 1485 psi., the maximum moment will be

$$M = \frac{(2300 + 2340 + 4550)20{,}000}{9.1(25 - 8.32 + 0.875/2)} = 1{,}180{,}000 \text{ in.-lbs.}$$

The actual flexural moment at section AA is 580,000 in.-lbs. The middle bar of the upper row is actually unnecessary at a point about 2 ft. from the center of the span.

Condition c. Flexure between section BB and the end support. This analysis is essentially the same as in the two preceding cases, except that the lower set of bars only is involved in the analysis. Investigation will show that the three lower bars have sufficient tensile strength to take care of the flexure in this region.

It is worth noting that a simplified analysis, based upon the combined area A_t of the six bars, and located at a distance $23.5 - kd$ from the neutral axis, may be used instead of the preceding detailed analysis. In this analysis, the actual area A_s of the steel is $6\pi \times 0.875^2/4$, or 3.6 sq. in., and the ratio p between the steel and concrete areas, from Eq. 12-13, is $3.6/(12 \times 23.5)$, or 0.0128 in., giving a value of 0.0128×9.1, or 0.1165 for pn. The ratio k, from Eq. 12-9, is

$$k = \sqrt{2 \times 0.1165 + 0.1165^2} - 0.1165 = 0.379$$

The value of j is

$$j = 1 - 0.379/3 = 0.874$$

The unit stress in the steel, from Eq. 12-10, at the centroid of the area, is

$$S_s = \frac{1{,}296{,}000}{3.6 \times 0.874 \times 23.5} = 17{,}500 \text{ psi.}$$

The maximum unit stress in the concrete, from Eq. 12-11, is

$$S_c = \frac{2 \times 1{,}296{,}000}{0.874 \times 0.379 \times 12 \times 23.5^2} = 1180 \text{ psi.}$$

By comparison with the results obtained in the computation for condition a, it is seen that the compressive stresses in the concrete are practically identical, but that an appreciable difference is found for the steel stresses. The simplified analysis may be incorrect by as much as 10% to 20% and, since the error is not on the side of safety, due allowance should be made in the permissible stress values for the steel if the simplified method of solution is used.

The construction shown in Fig. 12-17 shows the three upper bars bent up to take care of the diagonal tension and end shear. This leaves the three lower bars for carrying the

bond stresses caused by tension. The perimeter of one bar is $\pi \times 0.875$, or 2.75 in. The end shear is 21,600 lbs., and the unit bond stress, from Eq. 12-17, is

$$u = \frac{21{,}600}{3 \times 2.75 \times 0.874 \times 23.5} = 127 \text{ psi.}$$

The permissible bond stress, for deformed bars without special anchorage, is 5% of the ultimate strength of the concrete, which is equal to 0.05×3300, or 165 psi.

The maximum shearing stress, from Eq. 12-13, is

$$v = \frac{21{,}600}{12 \times 0.874 \times 23.5} = 87.6 \text{ psi.}$$

The permissible shearing stress on the concrete itself, from section 12-3, is 2% of the ultimate compressive strength of the concrete, which is equal to 0.02×3300, or 66 psi. The total vertical shear V, corresponding to this permissible stress, from Eq. 12-13, is

$$V = bjdv = 12 \times 0.874 \times 23.5 \times 66 = 16{,}300 \text{ lbs.}$$

Since the vertical shear in Fig. 12-17C has a straight-line variation, this value is located at a distance $(16{,}300 \times 12)/21{,}600$, or 9.06 ft., from the center of the beam.

The spacing for both the bent bars and the stirrups is 15 in., which is less than 0.75×23.5, or 17.6 in., as required by the JC Code. The shearing stress between the end reaction and section BB is taken by the two end bent bars, and the diagonal tension on the bars, from Eq. 12-15, is equal to

$$S_s = \frac{(87.6 - 66)(15 \times 12)}{\sqrt{2} \times 2 \times 0.60} = 2280 \text{ psi.}$$

The shearing stress between sections AA and BB is taken by the central bent bar. The shear is a maximum at section BB, and is equal to $(21{,}600 \times 129)/144$, or 19,350 lbs.

The unit shearing stress, from Eq. 12-13, is

$$v = \frac{19{,}350}{12 \times 0.874 \times 23.5} = 78.5 \text{ psi.}$$

The stress on the bar is

$$S_s = \frac{(78.5 - 66)(15 \times 12)}{\sqrt{2} \times 0.60} = 2660 \text{ psi.}$$

The shearing stress at section AA is $(21{,}600 \times 114)/144$, or 17,100 lbs. The unit shearing stress is

$$v = \frac{17{,}100}{12 \times 0.874 \times 23.5} = 69.2 \text{ psi.}$$

The cross-sectional area of the stirrup is $2\pi \times 0.25^2/4$, or 0.10 sq. in. The stress in the stirrup, from Eq. 12-16, is

$$S_s = \frac{(69.2 - 66)(15 \times 12)}{0.10} = 5620 \text{ psi.}$$

The stresses in the bent bars and in the stirrups are appreciably less than the maximum permissible stress of 16,000 psi. specified by the JC Code.

From Eq. 12-20, the necessary length of embedment for the bent bars is

$$L = 34 \times 0.875 = 29.8 \text{ in.}$$

These values apply, however, to bars stressed to their full capacity. Since the bent bars are stressed to less than one-fifth the maximum, the actual length of embedment required is approximately 6 in. The distance from the neutral axis to the upper surface of the beam

is kd or 0.379 × 23.5, or 8.9 in. Allowing 2 in. for cover, the effective vertical height is 6.9 in. The actual embedment of the diagonal position of the bar alone is $6.9\sqrt{2}$, or 9.8 in.

The necessary length of embedment for a stirrup bar of full strength, from Eq. 12-20, is

$$L = 34 \times 0.25 = 8.5 \text{ in.}$$

Since the actual stress in the bar is about one third the maximum, a length of 3 in. is sufficient. The actual embedment is 0.4 × 23.5, or 9.4 in.

12-16. Reinforced Slabs and Floors. Flat slabs and floors supported at two opposite parallel edges are essentially simply-supported beams, and the analysis employed in section 12-11 can be used to determine the stresses and the load capacity. Many reinforced concrete slabs, however, are supported at all four edges, or are built into the surrounding structure in such a manner that they more nearly resemble built-in plates. Such slabs are usually poured integrally with their supports, and are often continuous across several panels. For such conditions, which are beyond the scope of this text, reference should be made to texts and reference works on reinforced concrete design.

Slabs supported at all four edges are often provided with two mutually perpendicular sets of reinforcing steel. If each of the sets of perpendicular elements is assumed to carry its equivalent share of the total load on the slab, it is obvious that each set must have the same deflection at the center. From Fig. 5-44, the deflection of a uniformly loaded simply-supported beam is $5wL^4/384EI$, where w is the unit load. For a uniformly loaded slab, therefore,

$$\frac{w_1}{w_2} = \frac{L_2^4}{L_1^4} \tag{12-22}$$

where L_1 and L_2 are the mutually perpendicular long and short spans, and w_1 and w_2 that proportion of the unit load per square foot of area of the slab that each span is presumed to carry. To illustrate, if a slab 25 ft. long and 20 ft. wide is subjected to a load of 250 lbs. per sq. ft. of area, the load distribution is found by

$$\frac{w_1}{w_2} = \frac{w_1}{(250 - w_1)} = \left(\frac{20}{25}\right)^4$$

w_1 is equal to 73 and w_2 to 177 lbs. per sq. ft.

If the same size of reinforcing rod is used in both directions, it is obvious that the rod spacing over the short or 20-ft. span of the slab must be considerably closer than that required for the long span. Further analysis of the slab stresses is similar to that for reinforced concrete beams.

Foundations and footings of such size and foot thickness that reinforcement is necessary are designed in a manner similar to that described in sections 12-6, 12-7, and 12-11. Reinforcing rods in mutually perpendicular directions are used; for square bases, the design is usually based upon a set of rods in one direction, and a second set at right angles to the first is used to provide against failure in a perpendicular direction.

12-17. Self-Supporting Tower Foundation. Standpipes, smoke stacks, absorption towers, fractionating columns and vertical cylindrical reaction vessels are classified either as self-supporting or non-self-supporting, depending upon whether they have sufficient stability to maintain a vertical position alone, or whether they require braces or guy wires to stabilize them against the action of overturning forces.

Masonry towers, such as chimneys, are usually integrated with the foundation; metal towers are usually bolted in place. Most towers are of cylindrical shape, but since the horizontal area of the foundation is usually of larger size than the tower itself, the foundation may have any desired shape. Cylindrical foundations are most efficient in the effective use of the material, since the overturning effect of the wind may be considered to act from any direction, but form construction for cylindrical concrete bodies is inordinately expensive. Forms for square bases are easy to construct, but require an excessive amount of material over that necessary to insure stability. For these reasons, the most economical base is one of hexagonal or octagonal shape.

The design of self-supporting tower foundations is similar to the footing analyses described in section 12-6. The soil pressure under the foundation must not exceed the permissible values given in Table 12-2, and may be obtained from the total weight of the loaded tower, the weight of the foundation itself, and the varying load induced by overturning forces. The resistance to overturning afforded by the tower and foundation weights, however, should be based upon the unloaded weight of tower.

Example 12-8. Design a concrete foundation for a petroleum fractionating tower 4 ft. in diameter and 54 ft. high, weighing 30,000 lbs. when empty. Three inches of insulation will be applied to the exterior of the tower after erection; the weight of this insulation, and the various accessory ladles, piping, and pumps that must be mounted on the tower weigh 9000 lbs. The maximum wind pressure will not exceed 25 psf. of projected area of the tower exterior, the frost line is assumed 4 ft. below grade, and the soil is of alluvial nature, with a maximum safe pressure of 1 ton per sq. ft.

Solution. A foundation composed of two octagonal prisms will be employed, since it represents the best compromise between form construction and material cost. The upper prism will be 4 ft. high and will have a short diameter (or the diameter of the inscribed circle of the octagon) of 6 ft., which will allow sufficient area for foundation bolts and necessary attachment media. This portion of the foundation may extend 1 ft. above grade. The lower prism may be assumed as 2 ft. high, with a short diameter of 13 ft. 6 in. The area of an octagon is $0.828d^2$, where d is the short diameter. The area of the upper portion of the foundation is 0.828×6^2, or 29.8 ft., that of the lower portion 0.828×13.5^2, or 151 sq. ft. The total weight of the foundation is $150[29.8 \times 4 + (151 \times 2)]$, or 63,000 lbs. The volume of earth fill above the lower portion of the foundation is $(4 - 1)(151 - 29.8)$, or 363 cu. ft.; if the density of the earth is assumed as 90 lbs. per cu. ft., the weight will be 363×90, or 32,700 lbs. The minimum weight of the unloaded tower, without insulation, is $30{,}000 + 63{,}000 + 32{,}700$, or 125,700 lbs.

The total wind pressure is equal to the product of the tower diameter and length and the unit wind pressure, and is given by $4.5 \times 54 \times 25$ which is equal to 6080 lbs. This force is assumed to be concentrated halfway between the top and bottom of the tower. The

moment of this force, with respect to the base of the foundation, is 6080(54/2 + 6), or 200,000 ft.-lbs. The distribution of pressure on the base, caused by this moment, is given by an application of Eq. 5-16 (see sections 12-5 and 12-6). The section modulus of an octagonal shape differs very little from that of a circular shape, which is equal to $\pi D^3/32$ (Fig. 5-12). The maximum soil pressure, from the flexure formula, is

$$S = \frac{M}{Z} = \frac{200{,}000}{\pi \times 13.5^3/32} = 803 \text{ psf.}$$

The soil pressure due to dead weight is 125,700/151, or 830 psf., giving soil pressures of 830 + 803, or 1633 psf., and 830 − 803, or 27 psf., for opposite edges of the foundation. Since actual compression exists at both edges, the resultant of the dead and overturning loads falls within the region corresponding to the middle third of a rectangular footing, and the tower force system is stable.

The maximum soil pressure, however, is induced when the tower is in actual operation. If the tower, inadvertently or by design, is nearly filled with a petroleum liquid, its weight may be approximated as $\pi \times 4^2 \times 50 \times 47.5$, or 30,000 lbs., where 47.5 is assumed as the density of the liquid. The total dead weight includes this weight plus the weight of the insulation, and is equal to 125,700 + 30,000 + 9000, or 164,700 lbs. The dead weight soil pressure is 164,700/151, or 1088 psf., giving a maximum soil or toe pressure of 1088 + 803, or 1891 psf. Since the maximum bearing strength of the soil is 1 ton per sq. ft., this pressure is within safe limits.

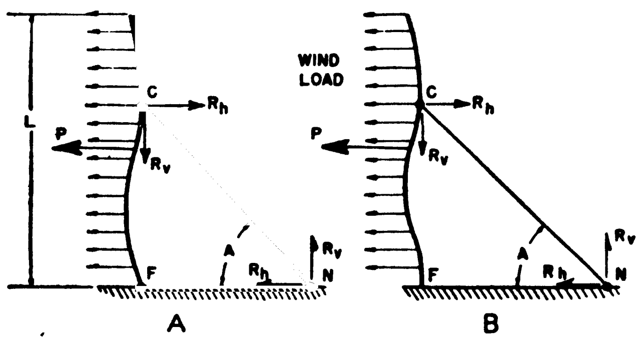


Fig. 12-18. Force Systems on Guyed Stacks.

12-18. Guyed Towers. Non-self-supported towers usually require lighter and smaller foundations than self-supporting units, since the guy wires provide for the overturning effort. Guy wires are used in sets of three or four, spaced 120° or 90° apart, and are usually composed of 6 × 7 cast steel or plow steel wire rope (see Chap. 18). Towers from 18 to 48 in. in diameter up to 50 ft. high are usually provided with one set of guy wires; two sets are used for towers from 36 to 60 in. in diameter up to 75 ft. high; and three sets are used for towers up to 72 in. in diameter and 125 ft. high.

A single set of guy wires is usually attached to a collar placed at the upper third point of the stack. The force system on guyed stacks is shown in Fig. 12-18; the condition at *A* will prevail when the stack material and the attachment to the foundation is light and flexible; the condition at *B* may be anticipated when the base of the stack is rigid, and is firmly bolted to the foundation.

The condition shown at *A* is similar to a single overhung beam hinged or supported at *F*, the foundation, and at *C*, the guy wire collar. For a distance *CF* equal to $2L/3$, the horizontal reaction R_h of the guy wire is

$$R_h = 0.75P \tag{12-23}$$

where *P* is the total wind pressure.

For a distance *CF* equal to $3L/4$, the horizontal reaction is

$$R_h = 0.67P \tag{12-24}$$

The vertical reaction R_v is given by

$$R_v = R_h \tan A \tag{12-25}$$

where *A* is the angle between the guy wire and the ground line. The actual load *R* on the guy wire is

$$R = \sqrt{R_h^2 + R_v^2} \tag{12-26}$$

The condition shown at *B* is similar to an overhung beam built-in at the foundation *F*, and supported at point *C*. From Fig. 5-45, Case 4, the reaction R_h for a distance *CF* equal to $2L/3$ is given by

$$R_h = 0.316P \tag{12-27}$$

For a distance *CF* equal to $3L/4$, the horizontal reaction is

$$R_h = 0.537P \tag{12-28}$$

Since the degree of fixation of the stack bottom is difficult to evaluate, the actual magnitude of the reaction R_h will lie somewhere between the values given by Eqs. 12-23 and 12-27, or 12-24 and 12-28.

The anchorage for the guy wires may be of several types; two forms are shown in Fig. 12-19. In each of these, a sufficiently long eyebolt, with a large washer, is used to guard against possible pullout. Guy wires are usually furnished with turnbuckles so that some initial tension, usually about 5000 psi., can be set up in the supporting cables. The initial tension amounts to about 1000 lbs. for ½-in. guys, and 250 lbs. for ¼-in. guys, and is considered inclusive of the weight of the wires. The foundation or anchorage weight must be sufficiently heavy to take care of the vertical reaction R_v and the initial tension in the guy wires; this weight, however, need not be furnished entirely by the concrete mass, since the pyramidal volume of earth fill shown at the left in Fig. 12-19 can be considered to add to the weight of the anchorage. The foundation should be sufficiently deep so that the soil pressure at its sides, induced by the horizontal reaction R_h, does not exceed the bearing capacity of the soil. For anchorages that are not completely buried, as in the illustration at the left, Fig. 12-19, it may be advisable to have sufficient excess weight in the foundation so that the frictional force at the lower surface of the foundation is greater than the horizontal component of the pull on the guy wires. In such instances, it is imperative

that the foundation have sufficient depth so that the surface in contact with the earth is below the frost line. The coefficient of friction for wet soil is much

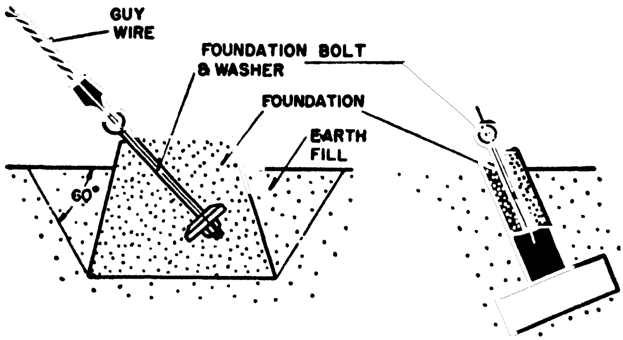


FIG. 12-19. Guy Wire Anchorage.

lower than for dry soil. In dry soils where the anchorage extends below the frost line a coefficient of friction of about 0.3 can be assumed with safety, but the restraining effect of the earth at the sides of the foundation should be disregarded. Guy wire anchors in rock should be of the type shown in Fig. 12-19 (right) and should be undercut to provide a positive lock for the anchorage.

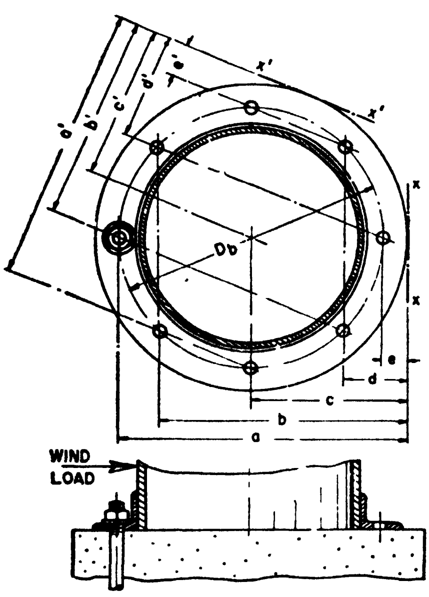


FIG. 12-20. Attachment of Tower to Foundation.

12-19. Foundation Bolts. Guyed towers and other equipment not subjected to the full effects of wind or other overturning loads are connected to the foundation by bolts which need resist only the relatively small lateral forces. Foundation bolts for guyed towers whose base approximates a fixed end beam, as in Fig. 12-18 (right), and installations such as the motor foundation shown in Fig. 12-6, should be checked for the increased tensile stress in the bolts induced by the moment of any horizontal force or overturning couples.

Foundation bolts for self-supporting out-of-door towers should have sufficient stress capacity to resist the overturning effect of wind pressure or other force couples. Fig. 12-20 shows the base of a tower, fastened to a concrete

foundation by means of bolts embedded in the concrete, and passing through an angle foot welded to the tower shell. At least eight bolts, and preferably twelve or more, should be used; common practice calls for a bolt with its embedded end bent into L shape, or supplied with a large washer held by a nut threaded at the embedded end, to provide adequate resistance against possible pullout.

The analysis of the stresses in foundation bolts is similar to that illustrated in Fig. 6-14, and described in section 6-8. The moment on the bolts is equal to the difference between the overturning moment and the effective moment of the weight W of the empty tower and its moment arm c or c', Fig. 12-20. The tensile stresses in the bolts should be evaluated, with respect to both axis $x-x$ and $x'-x'$, and the maximum value combined with the direct shear on the bolt thread roots, to obtain the maximum resultant shearing stress. The permissible stress for unfinished bolts, 10,000 psi., from the AISC Code may be used, but because of the possibility of corrosion in anchor bolts, it is customary to add 1/8 in. to the theoretical bolt diameter required for stress reasons.

PROBLEMS—CHAPTER 12

1. Compute the amount of cement required for the foundation of Example 12-1 if 2 in. aggregate is used.

2. A 25-HP gearmotor has an output speed of 170 r.p.m. and a sprocket 12 in. in diameter used to drive a crusher by means of a horizontal roller chain. Design a suitable concrete foundation, using 3300-lb. concrete, to be mounted on an alluvial soil. Determine the motor dimensions and weight from NEMA[43] data. Select suitable anchor bolts and make the fabrication layout for the base.

3. A beam reinforced with a single 5/8-in. diameter round bar is 3 in. wide and 6 in. deep (to the steel) and resists a moment of 24,000 in. lbs. Determine the controlling stresses.

4. A floor slab made of 3300-lb. concrete has a depth of 7 in. to the steel and is reinforced by 1/2-in. bars spaced 4 in. on centers. Find the permissible unit load if the slab is simply supported and has a span of 11 ft.

5. Find the permissible uniform load for a floor system composed of integral tee beams, using 4250-lb. concrete, each reinforced with three 1-in. diameter deformed bars. The depth of the stem (to the steel) is 12 in., the width is 10 in., the slab is 5 in. thick, the stem spacing is 5 ft. 4 in. on centers, and the floor span is 10 ft.

6. A dished head vessel for oil storage at atmospheric temperature and a pressure of 50 psi. is 8 ft. in diameter and 15 ft. long. The horizontal axis is 6 ft. from the ground. Design the vessel and a suitable pair of footings for this structure.

7. Like Problem 6, but the vessel is 25 ft. long with an axis 12 ft. from the ground Use a center and two end footings.

CHAPTER 13

WOOD AND OTHER NON-METALLIC CONSTRUCTION

Wood

13-1. Timber is one of the oldest building materials known and is still extensively used because of its availability, ease of fabrication, and low cost. Wooden structures, if maintained continuously wet or continuously dry, and if protected from the attacks of termites and marine borers, may have a life of many hundreds of years. The regular application of paint, or other surface treatment, or impregnation with suitable preservatives will aid materially in prolonging the life of the structure.

Structural timbers are usually classified as: joists and planks, 2, 3, and 4 in. thick, in widths up to 16 in. by even inches; beams of widths and depths from 5 to 6 in., up to 20 in. by even inches; and posts or timbers to carry axial loads, in 5- and 6-in. sizes and larger. These figures refer to the rough-sawn or undressed dimensions, dressed timbers with dimensions up to 6 in. are actually 3/8 in. smaller than the nominal size; while nominal dimensions greater than 6 in. are reduced 1/2 in. by finishing. A 2 x 4-in. joist, for example, is actually 1 5/8 x 3 5/8 in. in size; a 4 x 16-in. plank is 3 5/8 x 15 1/2 in.

Table 13-1.—Unit Stresses for Timber

Species	Flexure psi.	Compression psi.		Horizontal Shear psi.	E	Sp.G.	Constant	
		Perpendicular to Grain	Parallel to Grain		$\times 10^{-6}$		K_1	K_2
Cedar	1000	175	730	90	8	0.32		
Douglas fir	1470	275	1070	110	12	0.51	1375	3300
Oak	1870	500	1330	170	15	0.69	1700	4000
Yellow pine	1600	325	1470	140	16	0.60	1375	3300
Redwood	1600	250	1330	90	12	0.42	1125	2700

Timber is commercially available in three grades or designations: clear, select, and common. The safe unit stresses for a few representative types of wood are given in Table 13-1. Knots, checks, and other timber defects such as sloping grain materially affect the strength, and the allowable stresses must be reduced to 75% of those listed for select grades, and to 60% for common grades. Plywood, or laminated timber, is a comparatively recent development. It consists

of thin sheets, usually of yellow pine, glued together under pressure with the grain lines of adjoining sheets perpendicular. Plywood is extensively applied as gusset plates and for other structural details, and is used to some extent for built-up beams and columns, the different sections being held together by bolts, screws, or connectors.

13-2. Fasteners. Wire nails, spikes, leg screws, and bolts are used as fastening media for timber structures. Wire nails and small spikes were originally designated by the cost per hundred and weight per thousand nails. Sizes of ordinary wire nails are given in Table 13-2; for equal lengths, spikes are larger in diameter than wire nails. Boat spikes are of square section with a chisel point; they are available in sizes from ½ to ⅝ in. by sixteenths, and in lengths up to 14 in. Lag screws are available in sizes indicated in Table 13-3.

TABLE 13-2.—DIMENSIONS OF ORDINARY WIRE NAILS, INCHES

Designation	Diam. d	Length
Eight-penny	0.13	2½
Twelve-penny	0.15	3¼
Sixteen-penny	0.16	3½
Twenty-penny	0.19	4
Forty-penny	0.22	5
Sixty-penny	0.26	6

TAPLE 13-3.—DIMENSIONS OF LAG SCREWS, INCHES

Diameter	5/16	3/8	7/16	½	9/16	⅝	¾	⅞
Min. Length	1½	1½	1½	1½	2	2	2½	3
Max. Length	6	6	8	10	12	12	12	12

The approximate safe holding power, or allowable load in tension on nails and spikes, may be found by:

$$P = 1150d\sqrt{G^5} \tag{13-1}$$

where P is the safe load per lineal inch of penetration, G is the specific gravity of oven-dry wood, and d is the diameter of the nail or spike in inches. For yellow pine, the constant 1150 should be replaced by 920 for twelve-penny and smaller nails, and by 805 for larger nails.

The safe holding power of screws is given by:

$$P = 1700dG^2 \tag{13-2}$$

The penetration must be at least two thirds the length of the nail or screw. Nails, spikes and screws driven parallel to the grain have only 60% of the holding power given by Eqs. 13-1 and 13-2. Hanger bolts, Fig. 13-6, are employed instead of lag screws when parts are frequently removed; they are similar in principle to machine studs, but have a wood screw thread at one end.

The safe lateral resistance in shear for nails and spikes is given by:

$$R = K_1\sqrt{d^3} \qquad (13\text{-}3)$$

and for screws

$$R = K_2d^2 \qquad (13\text{-}4)$$

where K_1 and K_2 are constants obtained from Table 13-1. For unseasoned wood only 75% of the values found by these formulae should be used, while for nails and screws driven parallel to the grain the value should be only 60%. For metal to wood connections, the values found from Eqs. 13-3 and 13-4 may be increased 25%.

Bolts are used for timber connections when too many screws or nails would otherwise be required for convenient construction. Bolts are usually inserted in pre-drilled holes, whose diameters are the same as that of the bolts. Even for carefully fitted connections, however, the distribution of bearing pressure over the length of the bolt is extremely variable, and it is customary practice to reduce the allowable bearing pressure, B, per square inch of projected area of the bolt parallel to the grain, to

$$B = 1100 - 66T/d \qquad (13\text{-}5)$$

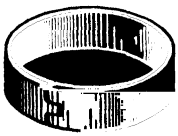
where d is the diameter of the bolt and T is the thickness of the heaviest member through which the bolt passes. The values obtained should be increased 70% for ½-in. bolts, 50% for ⅝-in. bolts, 40% for ¾-in. bolts, 30% for ⅞-in. bolts, 20% for 1- and 1¼-in. bolts, 15% for 1½-in. bolts. Bearing pressures may be increased 25% when metal side plates are used, and must be decreased 25% for unseasoned timber.

If the pressure is perpendicular to the grain of the wood—a condition which exists when the axis of the bolt is parallel to the grain—the allowable unit bearing pressure can be taken as 275 psi. based on the projected area for L/d ratios up to 6. For L/d ratios of 8, 10, and 12, the allowable pressures are 240, 185, and 140 psi. respectively. For the smaller sizes of bolts, these pressures may be increased as indicated in the preceding paragraph. No increase in pressure because of metal side plates, however, is permitted for bolts parallel to the grain.

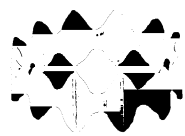
The center distance between adjacent bolts should not be less than $4d$; members in tension may require considerably greater transverse spacing. For tension members, the distance from the end of the member to the center of the first bolt should be at least $7d$ to avoid splitting the end; compression members should be designed with a minimum similar distance of $4d$. The distance from the center

of a bolt to an edge parallel to the load need not exceed $1.5d$; if the bolts bear in the direction of the edge, the minimum distance should be $4d$.

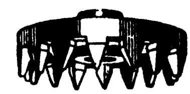
13-3. Connectors. Metal plates or fittings used to develop shear between the interior surfaces of timber joints are known as timber connectors. One typi-



Split Ring Connector. A split ring Teco connector is a smooth ring of steel with a tongue and grooved break or "split" which increases its load capacity. Split rings transmit loads when placed in pre-cut grooves in the faces of adjoining timbers.



Toothed Ring Connector. A toothed-ring Teco connector is a ring of sixteen gauge hot-rolled steel, ribbed to guard against lateral bending, with sharpened teeth on each edge. These rings, imbedded half their depth in the contacting surfaces of adjacent timbers, transmit loads from member to member.



Teco Shear Plate. Teco shear-plate connectors are designed to transmit loads from wood to steel, or vice-versa.

Fig. 13-1. Split Ring Connectors. *Courtesy of the Tinber Engineering Co.*

cal form, called a split ring connector, is shown in Figs. 13-1 and 13-2; dimensions and allowable loads are given in Table 13-4. Grooves are cut into the wood so that the ring fits in place snugly. It should be noted that the safe loads given are for connector rings used in pairs, one on each side of the central member. If these connectors are used for redwood structures, the values should be reduced to 85% of those listed. The minimum spacing for split ring connectors is the ring diameter plus ½ in., but center distances equal to one and one-half the ring diameters are preferred. As indicated in Fig. 13-2, the minimum edge distance, measured along the grain, is 1½ in.; if the end margin is as little as 1 in., the safe load should be reduced to 60% of the values listed in Table 13-4.

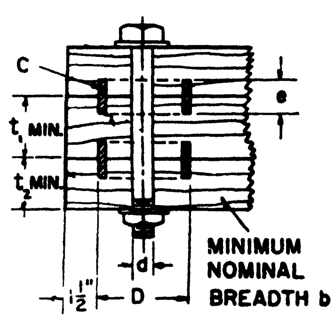


Fig. 13-2. Ring Connector Application.

Toothed connectors are similar in principle to split ring connectors but are of integral ring form, with pointed teeth along both edges; these teeth cut into and imbed themselves in the timbers if pressure is applied by a special high-strength screw furnished for that purpose. After the connector has been installed, the screw is removed and replaced by an ordinary steel bolt. The toothed connector is more readily installed than the split ring connector, since pre-cut grooves are not required. Safe loads and other data on these connectors are given in Table 13-5. For redwood, 90% of the listed values should be used;

for loads perpendicular to, or at an angle greater than 45° with the grain, the safe loads should be reduced to 75% of their listed value. Edge distances and other minimum requirements for toothed connectors are essentially the same as for split ring connectors, except that greater edge distances are preferred because of the possibility of splitting during installation.

TABLE 13-4.—SPLIT RING CONNECTOR DATA
(Refer to Fig. 13-2 for dimensions)

Nominal, Inches							Safe Load (Pair), psi.	
Diameter of Connector	D	d	e	t_1	t_2	b	Parallel to Grain	Perp. to Grain
2½	2.92	½	¾	3	1½	4	5700	4000
4	4.50	¾	1	3	2	6	12,000	8400
6	6.66	¾	1¼	4	2½	8	18,000	10,800
8	8.82	¾	1½	5	3	10	23,000	115,000

TABLE 13-5.—TOOTHED CONNECTOR DATA
(Fig. 13-1)

Diameter of Connector in.	Bolt Diameter in.	Safe Load (Pair) Parallel to Grain: Tight Hole psi.	Safe Load (Pair) Parallel to Grain: Average Hole psi.
2	½	2400	2200
2⅝	⅝	4200	3600
3⅜	¾	5800	5200
4	¾	6900	6300

Flanged plate connectors are used for fastening timbers to steel gusset plates or straps; they are inserted in pre-cut grooves in the timber, but have a hub for bearing against the bolt. Claw-plate connectors are cast and have projecting teeth which are forced into the timber by screw pressure, while an integral hub fits into a hole in the metal strap or gusset.

13-4. Wooden Beam Design. Timber beams and joists are designed for flexure by an adaptation of the flexure formula:

$$bd^2 = \frac{6M}{S} \tag{13-6}$$

Safe values of S are given in Table 13-1. Depth to breadth ratios of from 3 to 6 for joists, and from 2 to 4 for floor girders, are commonly used.

The maximum shear or end reaction R is given by:

$$R = 2Sbd/3 \tag{13-7}$$

where S is the allowable horizontal shear from Table 13-1. If the end of the beam is notched, as indicated in Fig. 13-3, the allowable unit shear S should be reduced to

$$S_1 = \frac{Sd_n}{d} \tag{13-8}$$

where d_n is the depth at the support.

Deep, narrow beams may fail by lateral buckling. For a simply supported beam, the safe total load B for lateral deflection is

$$B = \frac{db^3E}{6L^2} \tag{13-9}$$

where E is the modulus of elasticity from Table 13-1, and L is the beam span in inches.

Beam deflection is computed as in sections 5-22 and 5-23. The actual deflection, however, may be considerably greater than the computed value, particularly if unseasoned timber is used, since wooden beams will yield or "give" after being in place for some time. One method of compensating for this phenomenon is to limit the computed deflection to 1/720 of the span, rather than 1/360 as is common for structural steel.

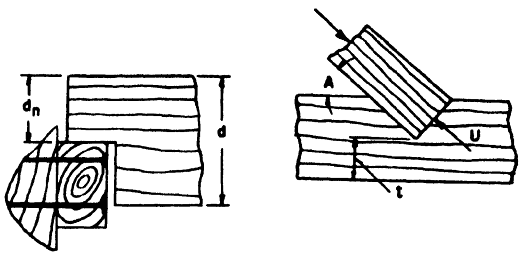


FIG. 13-3. Wooden Beam Seating, Horizontal and Oblique.

Values for compression or bearing resistance, listed in Table 13-1, apply to interior bearing areas greater than 6 in. long, and to end bearing areas. Short interior areas, subjected to compression perpendicular to the grain, may be permitted increases above the listed values of 10%, 30%, and 60% for respective lengths of 4, 2, and 1 in.

For allowable compression or bearing load at an oblique angle A to the grain (Fig. 13-3)

$$U = \frac{PQ}{P\sin^2 A + Q\cos^2 A} \tag{13-10}$$

where P and Q are the allowable unit compressive stresses parallel and perpendicular to the grain, from Table 13-1.

Timber column design depends upon the L/d ratio; when the ratio is less than K,

$$\frac{F}{A} = S_c\left[1 - \left(\frac{L/d}{K}\right)^4 \div 3\right] \tag{13-11}$$

where F is the total load in pounds, A is the cross-sectional area, S_c is the allowable unit compressive stress parallel to the grain, L is the unsupported length in inches, d is the least dimension of the column, and

$$K = \frac{\pi}{2}\sqrt{E/6S_c} \qquad (13\text{-}12)$$

where E is the modulus of elasticity, Table 13-1.

For slender columns, where L/d is greater than K,

$$\frac{F}{A} = \frac{\pi^2 E}{36(L/d)^2} \qquad (13\text{-}13)$$

with a maximum L/d ratio of 50.

The design of timber tension members is based upon the net section, making the necessary deductions for bolt holes, ring grooves, etc., and using the allowable stresses given for flexure. Staggered bolt holes should be deducted from the cross section perpendicular to the direction of stress if the connecting centers of adjacent holes make an angle less than 45° with the cross section. Timbers used as tension members should be free of defects such as knots; if such material cannot be obtained, the allowable stresses should be reduced to 75% of the values in Table 13-1.

Example 13-1. A portion of the space above the lower chords of the roof trusses of an industrial building, having 10-ft. bays, is to be floored over to provide for the storage of six oil drums. The drums are 3 ft. in diameter and 4 ft. high, made of sheet metal approximately 1/16 in. thick, and contain oil weighing 55 lbs. per cu. ft. Design the flooring and supporting beams.

Solution. The weight of the oil in one drum is

$$(\pi \times 3^2/4) \times 4 \times 55 = 1555 \text{ lbs.}$$

The weight of the drum itself is

$$(0.284 \times 0.063)36(48 + 2 \times \pi \times 36^2/4) = 133 \text{ lbs.}$$

The total weight of one drum is

$$1555 + 133, \text{ or } 1688 \text{ lbs.}$$

The usual spacing for floor joists, to eliminate excessive deflection of the flooring itself, is 16 in. on centers. The floor width will necessarily be in excess of 6 ft., or 72 in., to permit some handling of the drums; if five 16-in. joist bays are assumed, the total load can be carried on six joists with a consequent floor width of about 80 in. The load per square inch of floor will be equal to the weight of six drums divided by the floor area, or

$$1688(6/80)120 = 1.05 \text{ psi.}$$

It will probably be advisable to raise this value to 1.25 psi. to allow for localized concentrations.

If yellow pine tongue-and-groove flooring of select quality is specified, the allowable flexural stress is 75% of 1600 psi., or 1200 psi. Since the flooring is securely nailed to the joists, and is further stiffened by the tongue-and-groove construction, it is safe to assume the flooring to approximate the condition of a uniformly loaded continuous beam, over multiple spans, which is analogous to the uniformly loaded beam illustrated in Fig. 5-37.

The maximum bending moment occurs at the support, and is obtained from Table 5-1. Assuming a floor section 1 in. wide, perpendicular to the joists, we have

$$M = 304wl^2/2888 = (304 \times 1.25 \times 16^2)/2888 = 33.6$$

The theoretical thickness is given by Eq. 13-6,

$$d = \sqrt{\frac{6 \times 33.6}{1200}} = 0.41 \text{ in.}$$

Flooring of 1-in. nominal size, which is actually $^{13}\!/_{16}$ in. thick.

The joist design will be based upon one of the intermediate joists, which supports a floor area 10 ft. long and 16 in. wide. The uniform load per inch of length is 16×1.25, or 20 lbs.; the joist is considered to be simply supported, uniformly loaded beam, and the moment is a maximum at the center, or

$$M = 20(120^2/8) = 36{,}000 \text{ in.-lbs.}$$

From Eq. 13-6,

$$d = \sqrt{180/1.625} = 10.5 \text{ in.}$$

The nearest commercial member is nominally 12 in. and actually $11\frac{1}{2}$ in. deep. The maximum deflection will be at the center, and is obtained from Fig. 5-35, Case 1, where I equals $bd^3/12$, or 2060 in.^{4}, and E is 1,600,000 psi., from Table 13-1.

$$y = \frac{5 \times 20 \times 120^4}{384 \times 1{,}600{,}000 \times 2060} = 0.00136 \text{ in.}$$

which is very small in comparison to the span.

If the floor joists are mounted on the flanges of channels serving as the lower chords of the supporting trusses, with spacer bars S, Fig. 13-4, bolted to the channels and placed between adjacent joists, to serve as nailing strips, the full depth of the joist is available to resist horizontal shear. The allowable unit stress in shear is 75% of 140, or 105 psi., and the maximum end reaction is given by Eq. 13-7,

$$R = 2 \times 105 \times 1.625 \times 11.5/3 = 1310 \text{ lbs.}$$

The actual reaction is equal to one half the total weight, or 1200 lbs.

Although considerable lateral support is given by the flooring, it will be advisable to check the beam for lateral buckling by Eq. 13-9 as follows:

If the total load B is equal to the product of the unit load w and the span length L, then

$$L = \sqrt[3]{\frac{db^3E}{6w}}$$

or

$$L = \sqrt[3]{\frac{11.5 \times 1.625^3 \times 1{,}600{,}000}{6 \times 20}} = 87 \text{ in.}$$

This indicates that some form of support at the center of the span is desirable, which may be accomplished by the use of cross-bridging shown in Fig. 13-4. It will probably be advisable to use three sets of bridging, spaced 30 in. apart, and they will also assist in transferring the flooring load from one joist to the adjacent members and thus allow a better distribution of the floor load.

The allowable compression, perpendicular to the grain, is 75% of 325, or 243 psi. The end reaction is 1200 lbs., and the joist breadth $1\frac{5}{8}$ in., so the necessary seating length is

$$(1200/243)1.625 = 3.04 \text{ in.}$$

A seat length of 4 in. is ample, and can be obtained in either of the constructions shown in Figs. 13-4 and 13-5.

In some constructions the floor joists are framed into the channel; in others, the joists may be notched as shown in Fig. 13-5. The latter construction is economical of headroom above the floor, and permits the nailing strip N to be bolted to the channel or angle at fairly infrequent intervals, in contrast to the spacer strips S in Fig. 13-4, which require two bolts for every 16 in. of joist spacing on each support. If the joist depth is cut to 10 in. at the ends, the allowable unit shear S_1, by Eq. 13-8, must be reduced to

$$S_1 = \frac{105 \times 10}{11.5} = 91.5 \text{ psi.}$$

and, by substitution in Eq. 13-7, the maximum end reaction is

$$R = 2 \times 91.5 \times 1.625 \times 10/3$$
$$= 990 \text{ lbs.}$$

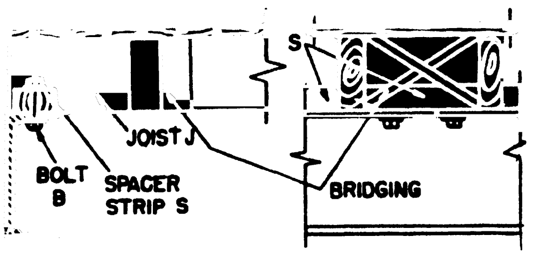


FIG. 13-4. Floor Joist.

This construction will necessitate a joist of greater breadth. The product bd^2 is equal to 180 in.^{3}; for a joist with a nominal width of 3 in., and an actual width of 2⅝ in., the required depth is 8.27 in., and a joist with a nominal depth of 10 in. and an actual depth of 9½ in. will be satisfactory. If the joist is notched to a depth of 1½ in., then the allowable unit shear S_1 is

$$S_1 = \frac{105 \times 8}{9.5} = 88.5 \text{ psi.}$$

and the maximum end reaction is

$$R = 2 \times 88.5 \times 2.625 \times 8/3 = 1240 \text{ lbs.}$$

which is satisfactory.

The maximum unsupported joist span length, to avoid lateral buckling, is

$$L = \sqrt[3]{\frac{9.5 \times 2.625^3 \times 1{,}600{,}000}{6 \times 20}}$$
$$= 131 \text{ in.}$$

indicating that lateral supports are not required. (In the usual construction, however, cross-bridging would probably be added, spaced approximately 3 to 3½ ft. apart.)

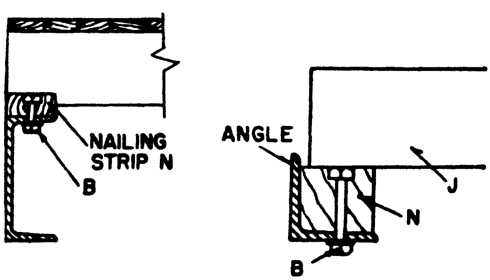


FIG. 13-5. Floor Joist Supports.

Example 13-2. Design a support, as illustrated in Fig. 13-7, for a 20-HP 1150-RPM variable speed motor, for a mixer drive for a research project. The weight of the motor and motor base is 776 lbs.; the belt drives at an angle of 20° with the horizontal as shown; the total belt pull is 511 lbs. The support is to be made of timber because of its temporary character; it is planned to fasten the support to the floor of the laboratory, but it will be moved from place to place as the requirements of the research work dictate.

Solution. The belt pull of 511 lbs., acting at an angle of 20° with the horizontal, may be resolved into horizontal and vertical components as follows:

$$511 \times \cos 20° = 480 \text{ lbs., horizontal}$$

$$511 \times \sin 20° = 175 \text{ lbs., vertical}$$

Fig. 13-8 shows the plan view of the motor feet, with the horizontal component of the belt pull acting at a distance of 17 in. from the centroid M of the bolt holes. The bolts are placed at the vertices of a 17-in. square, and the moments about the centroid M are

$$17 \times 480 = (4 \times 8.5 \times f_m) + (4 \times 8.5 \times f_n)$$

$$\text{and } f_m = f_n = 120 \text{ lbs.}$$

f_m represents each lateral (left to right) force, and f_n represents each transverse (front to back) force.

Fig. 13-9 shows a right side view of the forces acting on the centerline of the motor; axes DC and HG represent the bolt hole centerlines. Taking moments about HG:

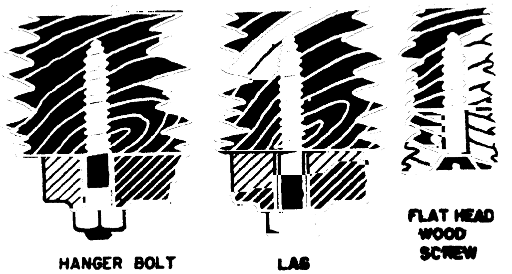


FIG. 13-6. Wood Screws.

$$-(175 \times 25.5) \pm (17 \times R) - (776 \times 8.5) = 0$$

or R, the reaction at bolts D and C, equals 650 lbs. Similarly, the reaction at HG equals 300 lbs.

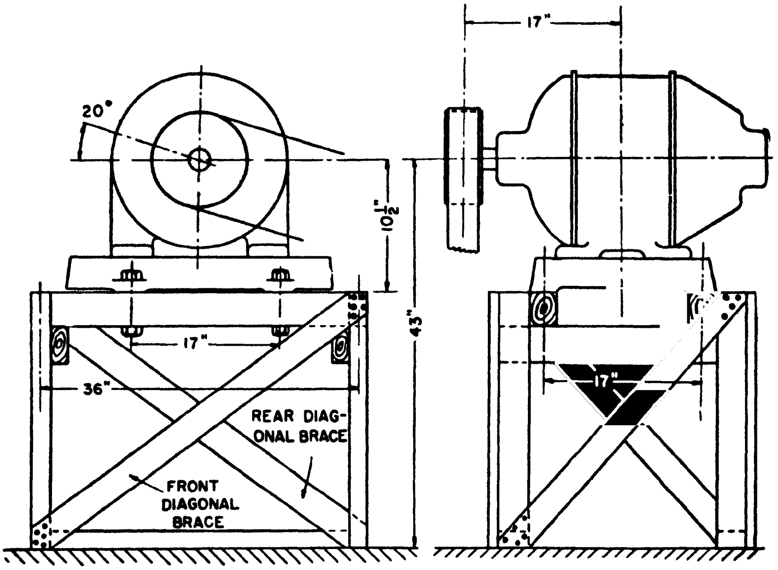


FIG. 13-7. Motor Support.

Fig. 13-10 shows the action of the external forces on the front beam AB of the support. The horizontal force of 240 lbs. is transmitted through AB by bolts C and D to joint B. The external reactions on AB are found from:

$$\Sigma M_{B_R} = \pm (R_L \times 36) + (240 \times 10.5) - (650 \times 18) = 0$$

$$\text{or } R_L = 255 \text{ lbs., upward}$$

$$\Sigma M_{B_R} = +(650 \times 18) + (240 \times 10.5) \pm (R_R \times 36) = 0$$

$$\text{or } R_R = 395 \text{ lbs., upward}$$

Member AB acts as a beam, although that portion of the span between D and B is subjected to a compressive stress of 240 lbs. From Fig. 13-11 the point of maximum moment is at D, and the moment is

$$395(18 - 8.5) = 3750 \text{ in.-lbs.}$$

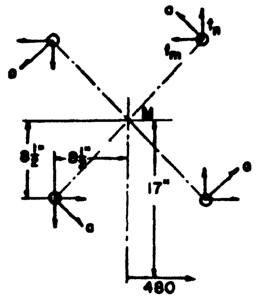


Fig. 13-8. Force Analysis for Motor Support.

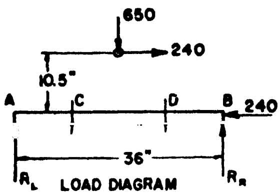


Fig. 13-10. Load Diagram for Front Beam of Motor Support.

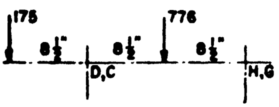


Fig. 13-9. Force Analysis for Motor Support.

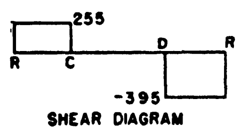


Fig. 13-11. Shear Diagram for Front Beam of Motor Support.

Structures of this character are usually made of any materials at hand; for yellow pine, the design stress should be based upon common grade, for which an allowable flexural unit stress of 60% of 1600, or 960 psi., is permissible. The base of the motor is furnished with ⅞-in. cored holes for hold-down bolts, for which ¾-in. bolts are employed. The projected area of the bolt holes must be deducted in computing the stress in member AB, since the position of the bolt and maximum moment coincide. Using a 2×4-in. member as an initial assumption, the width b of AB will be $(1⅝ - ¾)$, or ⅞ in.; the depth d will be 3⅝ in. From the flexure Eq. 13-6,

$$S = \frac{6 \times 3750}{0.875 \times 3.625^2} = 1950 \text{ psi.}$$

which is too high. Assuming a 3×4-in. member, the width b will be $(2⅝ - ¾)$, or 1⅞ in. The unit stress then becomes

$$S = \frac{6 \times 3750}{1.875 \times 3.625^2} = 910 \text{ psi.}$$

which is satisfactory.

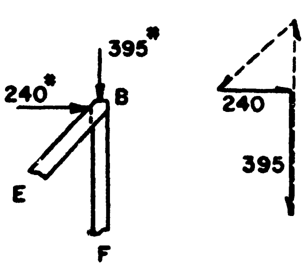


Fig. 13-12. Space and Force Diagrams for Upper Joint of Motor Support.

The additional compressive stress between D and B is equal to $(240/2.625)3.625$, or about 25 psi., which may be neglected, particularly as the flexural stress is 50 psi. below the allowable.

Fig. 13-12 shows space and force diagrams for joint B. The stress in the vertical member BF is 612 lbs. compression; that in the diagonal brace EB is 324 lbs. tension. Member BF acts as a column 32½ in. long. Selecting a 2 × 4-in. member, it is necessary to check the value of K from Eq. 13-13 to ascertain whether the length least-dimension ratio L/D is greater or less than K, so as to select the correct column formula. The allowable unit compressive stress, parallel to the grain, is 60% of 1470, or 882 psi. Substituting,

$$K = \frac{\pi}{2}\sqrt{\frac{1{,}600{,}000}{6 \times 882}} = 27.3$$

The L/d ratio is 32.5/1.625, or 20, which requires the use of Eq. 13-11,

$$\frac{F}{1.625 \times 3.625} = 882\left[1 - \left(\frac{20}{27.3}\right)^4 \div 3\right]$$

or F is equal to 4700 lbs., which is far in excess of the actual load of 612 lbs. On account of nail spacing, however, it is not advisable to use anything smaller than a 2 × 4-in. member

The tension member EB is designed on the basis of the flexural stress from Table 13-1, with an allowance made for possible defects, such as knots, etc. Assuming a "defect percentage" of 30%, the allowable unit tensile stress is 60% of 70% of 1600, or 672 psi. The minimum thickness of member EB will be nominally 1 in., or actually ⅝ in. The sum of the thickness of members EB and BF is ⅝ plus 3⅝, or 4¼ in., permitting the use of eight-penny or twelve-penny nails, respectively 2½ and 3¼ in. long. The safe lateral resistance of an eight-penny nail in seasoned timber is given by Eq. 13-3, as

$$R = 1375\sqrt{0.13^3} = 64 \text{ lbs.}$$

Common "pick-up" timber may be unseasoned, and therefore 75% of the above value, or 48 lbs., may more nearly represent the actual lateral resistance of an eight-penny nail. This will necessitate 324/48 or 7 nails at the joint.

The lateral resistance of a twelve-penny nail in unseasoned timber is

$$R = 0.75 \times 1375\sqrt{0.15^3} = 60 \text{ lbs.}$$

which will necessitate 324/60 or 5 nails at the joint. This nailing is preferred to the former.

For adequate nailing area, the brace EB should be at least of 3 in. nominal width. The unit tensile stress is then equal to 324/(0.625 × 2.625), or 197 lbs., which is far less than the allowable.

It may be of interest to design a bolted joint at B. The smallest standard bolt is ½ in. in diameter, and the allowable unit pressure is found from Eq. 13-5 to be as follows:

$$B = 1100 - (66 \times 3.625/0.50) = 621 \text{ psi.}$$

This value may be increased 70% for a ½-in. bolt, but must then be reduced 25% to allow for unseasoned timber; the allowable bearing stress per square inch of area is 0.75 × 1.70 × 621, or 792 psi. The actual unit load on the brace is 324/(0.625 × 0.50) or 1030 psi., which is too high for the load capacity of the bolt. If bolts are used at B and E, the thickness of member EB must be increased to 324/(792 × 0.50), or 0.82 in. This will necessitate a nominal thickness of at least 1½ in. If bolts are used, it will be necessary to extend the diagonal brace past member BF and AE, so as to obtain the minimum end distance of $7d$, or 3½ in., to avoid end splitting.

The rear frame corresponds to the front frame $ABFE$, and is constructed of members of the same size, although the external loads are appreciably less. It should be noted that the rear diagonal brace is still in tension, because the horizontal components of the belt pull act towards the left, as indicated in Fig. 13-8. The diagonal side bracing is theoretically unnecessary, because all the forces on the side frames are vertical, but it should be made of the same size as the member EB. The footing strip EF will be made of a 2 × 4-in. member, drilled for ¾-in. bolts. The front beam AB and the corresponding rear beam are mounted on transverse 2 × 4-in. members nailed to the columns.

Although the columns and bracing are amply strong, the stresses in the beam *AB* and the joint *B* are close to the maximum allowable values. The designer should, therefore, guard against hasty or incomplete analysis, since supports, brackets, and bases are usually far less expensive than the equipment they carry, and a few additional pennies spent in design will show appreciable dollar economy in the long run.

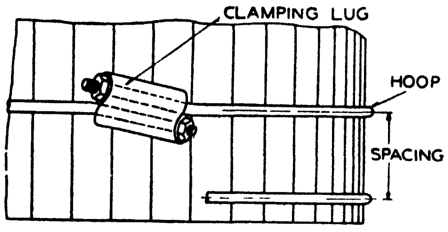


FIG. 13-13. Hoop and Lug for Wooden Tank.

13-5. Wooden Tanks. Tanks made of wood are frequently used for water storage and a wide variety of chemical solution and reaction purposes. They usually cost only about one half as much as steel tanks, particularly in the smaller sizes, and can be constructed easily in out-of-the-way places when other material is not available. They are more readily protected from freezing, and do not deteriorate as rapidly, if neglected, as do steel tanks. The life of a wooden tank, if periodically inspected and repaired, is usually fifteen years; cypress tanks often have a useful life of from 20 to 25 years. Cylindrical wooden tanks should have staves dressed on all four sides to a thick-

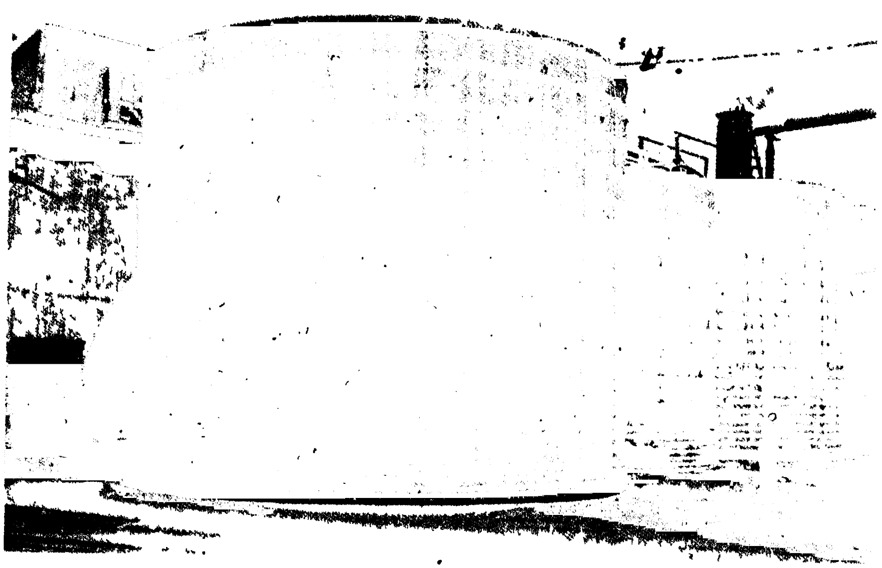


FIG. 13-14. Large Wooden Tank. *Courtesy Mixing Equipment Co.*

ness of not less than 2¼ in. for tanks up to 16 ft. in diameter and depth, or not less than 2¾ in. for greater diameters and depths. Hoops for cylindrical tanks should be of circular section, bent to fit the outer radius of the tank, connected by malleable iron lugs as shown in Fig. 13-13. The hoops should be so located on

the tank that the lugs are arranged in a uniform helical line, as shown in Fig. 13-14. Wooden cylindrical tanks are usually built with flat bottoms, supported on a wooden or steel grill.

The pressure of the fluid in a cylindrical gravity tank is resisted by the hoops alone, the staves serving only as containers for the fluid. From section 3-8, the stress on the longitudinal seam of a cylindrical vessel with a vertical axis is pR, where p is the unit pressure, psi., and R is the inner radius in inches. Since the weight of a column of water 1 sq. in. in area and 1 ft. high is 0.434 lb., the total force on the longitudinal seam at any distance h below the surface of the fluid is

$$F = 0.434gRh \qquad (13\text{-}14)$$

where h is given in feet, and g represents the specific gravity of the liquid.

The permissible unit hoop stress is usually taken as 12,000 psi.; some allowance should be made for initial stress in the threaded ends, and for stresses caused by swelling the wood.

Example 13-3. Determine the hoop size and spacing for a cylindrical tank 22 ft. in diameter and 20 ft. high. The tank is to be used for storage of a 31.7° Bé. alum solution (sp. gr. 1.280).

Solution. The seam stress, from Eq. 13-14, at the bottom of the tank is,

$$F = 0.434 \times 11 \times 12 \times 20 \times 1.280 = 1470 \text{ lbs.}$$

The seam force or stress is zero at the top of the tank. The stress per foot of height is 1470/20, or 73.5, say 75 lbs. Using rounded values, the stress varies from 1470 lbs. at the bottom to 1395 lbs. one foot above the bottom, or an average of (1470 + 1395) 12/2, or 17,196 lbs. over the first foot of height. This stress requires a total steel area of 17,196/12,000, or 1.43 sq. in. The cross-sectional root area of ⅞-in. threaded rods, from Table 6-1, is 0.42; for 1-in. rods the area is 0.55. The first foot of tank height will require 1.43/0.42 or four ⅞-in. hoops, or 1.43/0.55 or three 1-in. hoops. Actually, 1-in. hoops, spaced about 4 in. on centers, would be used for the lower foot of the tank height. The spacing may be increased in proportion to the decrease in the seam stress in the upper portion of the tank, as illustrated by Fig. 13-14.

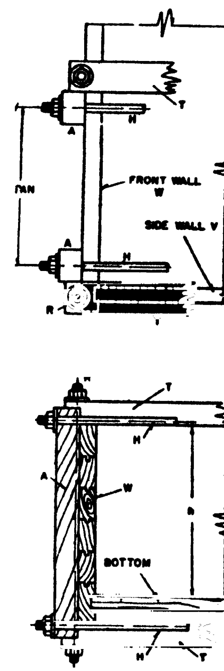


Fig. 13-15. Rectangular Wooden Tank Details.

The construction of rectangular wooden tanks is shown in Fig. 13-15. The side walls V are usually dadoed or rabbetted into the front and rear walls W; horizontal or "buck" stay rods H and vertical timbers A are used to resist the pressure of the liquid. The vertical stay rods R and the timbers T are used to tighten or "cinch" the seams in the wall planks The vertical stays are not subject to water pressure stress, but are usually made of the same size as the horizontal stays to guard against initial or tightening stresses, and loads caused by swelling. For a tank of considerable length, with three or more buck stays H, the planks comprising the tank walls should theoretically be treated as continuous

beams; in actual practice, the plank thickness is based upon the water pressure at the bottom of the tank, and the plank is considered as a simply supported, uniformly loaded beam whose span is equal to the distance between the vertical timbers *A*. In applications where the tank is frequently filled from above, the tank is constructed by using buck stays and timbers at or above the side walls only, so as to leave the entire area clear of any obstruction. Such construction requires considerably heavier wall planking, but prevents corrosion of the rods. If the dimensions of the tank are such that intermediate supports are necessary, it is usually advisable to cover the buck stays with corrosion-resistant tubing.

In the design of the buck stays, it is customary to use a maximum allowable unit stress of 12,000 psi. at the roots of the threads. The area of the washers under the nuts should be checked so that the allowable crushing strength of the wooden timbers is not exceeded. The vertical timbers *A* are subjected to a load varying from zero at the top to a maximum at the bottom, but are usually designed as uniformly loaded simple beams; the load intensity being taken either as the average or as the maximum unit load.

13-6. Piling. Wooden piling is used where foundations are to be placed in soil where the safe bearing load is very low. When ordinary foundations will overload a soil, pilings are driven, and a reinforced concrete cap is constructed on top of them. The pilings should extend about 6 in. into the concrete. Wooden piles can be obtained in various lengths and are specified according to the nature of the soil. For piles less than 25 ft. in length the diameter of the lower end is usually 6 in., and at the top 10 in. For piles longer than 25 ft. the top diameter is 12 in. or more.

The load-carrying capacity of piling is usually dependent upon the frictional resistance between the pile and the ground. Wooden piles have comparatively little strength as columns. The safe load that can be carried with a pile depends upon the soil and varies for different localities. Local building codes sometimes govern the allowable pile loading; usually 20 and sometimes 25 tons per pile is permitted. Usually it is necessary to drive a few piles for test purposes. The safe load can then be determined by either of the two equations:

Driven by a drop hammer

$$P = \frac{2WX}{p + 1.0} \tag{13-15}$$

Driven by a steam hammer

$$P = \frac{2WX}{p + 0.1} \tag{13-16}$$

where P is the safe load in pounds per pile, W is the weight of the hammer in pounds, X is the height of hammer fall in feet, and p is the penetration (or sinking) in inches under the last blow made on sound wood.

Piles must be driven carefully to be sure that they are deep enough to develop full strength. If driven too much or too violently the piles may split

or break and thus destroy most of the load-carrying capacity. Piling is ordinarily driven with a center to center distance of 3 ft. Closer spacing will disturb the ground and reduce the frictional resistance. The top of the piles should be driven or cut off below normal ground water level to prevent rapid decay. Piling in swampy ground must be chemically treated to insure reasonably long life.

Plastics

13-7. Plastics are used in place of metals and other construction materials in order to make use of some unusual physical property of the plastic, or to decrease material or fabrication costs.[20] There are many types of plastics, both natural and synthetic. In general, plastics are classified as either thermoplastic or thermosetting, although some types are of intermediate variety, and cannot be truly placed in either classification. Thermoplastic materials are those in which heat causes a physical change in the material. Their load-carrying capacity is appreciably affected by the conditions of operation, since the material becomes more plastic with an increase in temperature. Thermosetting materials are those in which heat causes a chemical change in the material. When once molded or formed into shape they become rigid, and do not become plastic when reheated or when the operating temperature is allowed to rise. Of the two types, thermosetting plastics have higher compressive strengths than thermoplastic materials, although the flexural strength of the latter is usually greater than that of the former.

Both classifications of plastics are commercially available in rod, sheet, and bar form, either in rolled or extruded form. Innumerable small and medium sized parts are produced in large quantities in metal molds; the design of such parts involves highly specialized data and techniques, and is beyond the scope of this text. In this discussion only those applications in which plastics are used as a major part of fairly large equipment will be considered.

13-8. Strength of Plastics. Table 13-6 shows the physical properties of some of the best known and most widely used plastic materials which are of interest in the design and construction of processing equipment. Plastics are usually selected on the basis of their chemical or electrical resistance. Chemical equipment, such as tanks and vessels, of comparatively large size, can be made entirely of plastic materials provided proper strength can be built into the walls and supporting members to allow for the imposed loads. Plastics can also be used as a lining for metal or wooden tanks and vessels. Plastic lined vessel fabrication is similar to that described in Chap. 11 for non-ferrous construction. When plastics are used as linings, the strength of the plastic is usually disregarded in computing values for wall and shell thicknesses. The necessary resistance to pressure is provided by the metal walls which are used to support or back up the lining. Plastics can be used as lining either in sheet or molded form, or can be applied in the form of a fluid in much the same way that paint is applied. Care must be taken to insure proper polymerization or "setting" of

the plastic which usually involves a heating or possibly a pressure application during installation.

TABLE 13-6.—PHYSICAL PROPERTIES OF PRINCIPAL PLASTICS USED AS ENGINEERING MATERIALS

Materials	Tensile Strength kips per sq.in.	Compressive Strength kips per sq.in.	Flexural Strength kips per sq.in.	Impact Strength Izod (ft.-lb. per in. of Notch ½ × ½-in. bar)
Phenolic:				
Wood-flour filled	4-11	16-36	8-15	0.15-0.25
Mineral-filled	4-10	18-35	8-20	0.13-0.72
Fabric-filled	5-8	20-32	8-13	0.8 -4.8
Cast phenolic	5-12	10-30	9-14	0.3 -0.4
Laminated Phenolic:				
Paper base	7-18	20-40	13-20	0.6 -7.6
Cotton base	8-12	30-44	13-20	1.4 -15
Asbestos base	7-12	18-45	10-35	1.8 -11
Phenol-furfural:				
Wood-flour filled	6-11	28-36	8-15	0.30-0.56
Mineral-filled	5-10	24-36	26-30	0.32-0.74
Fabric-filled	6-8	26-30	10-13	1.20-4.60
Urea-formaldehyde	5-13	24-35	10-15	0.28-0.32
Vinyl chloride	8-10	10-12	12-14	0.6 -1.2
Vinylidene chloride	5-7	7.5-8.5	15-17	2.0 -8.0
Me. methacrylate	4-7	10-15	10-15	0.2 -0.4
Polystyrene	5-9	11.5-13.5	14-19	0.35-0.50
Ethyl cellulose	6-9	10-12	4-12	0.6 -6.5
Cel. acet. butyrate	2.5-7.5	7.5-22	2.8-13	0.8 -5.5
Cellulose acetate	3-10	7-27	3.7-10	0.7 -4.2
Haveg 43	2.5	10.5	5.6	

Unfortunately the data available in manufacturers' literature, or in Table 13-6, does not permit proper evaluation of the loads that can be carried by plastic materials. The data given are useful largely as an indication of the degree or magnitude of the tensile and compressive strengths, especially in small parts. All plastic materials—even thermosetting plastics—will tend to flow under pressure. This phenomenon is known as creep and is similar in all respects to the creep of steel under heavy load at increased temperatures. There are very meager data on the creep and creep resistance of plastics. Plastic impregnated wood and various types of mineral and fabric filled phenolic plastics are designed for relatively small creep, but when subjected to continuous loads practically all plastics tend to distort and assume continuously differing shapes. Whenever heavy loads are continuously placed on plastics, creep will occur and it is essential to design walls of plastic vessels with very high factors of safety (20 and larger) to allow for this phenomenon. The tensile and compressive stresses of

plastics can be used in the strength equations developed in previous chapters and stresses calculated just as in the case of steel and other materials. It is unsafe, however, to depend upon tabular strength data unless some test experience is developed with regard to the aging and exact creep characteristics of the plastic in the general form and shape in which it is to be used. This rather limits the use of plastics as stress members in the design of equipment, supports, or other applications; and common practice, when previous experience is unavailable, is to support all plastic vessels by metal or wood, either as a continuous backing, or in the form of staves or bands, as shown in Fig. 13-17. An alternate method of design to compensate for the creep characteristic is to use the plastic as a lining, bonded to the supporting material.

13-9. Design Limitations. The design of comparatively large plastic equipment is complicated by a number of factors. Because of the lack of availability of creep data, the determination of permanent deformation under normal load conditions, especially at elevated temperatures, is largely a matter of estimate. Physical strength measurements [9] are usually made by short-time tests. and plastic materials are characterized by the fact that short-time testing gives higher strength data than tests over a long continuous period. The physical strength data for most plastics is therefore usually higher than can be depended upon in practice. Because most plastic vessels are molded, fabrication methods and limitations are often the governing factor in design.

For these reasons, the design of plastic molded parts, at the present time, may be considered analogous to the design of cast iron parts. Vessels made of sheet or plate steel may be more accurately designed in accordance with theoretical data and strength characteristics than cast iron equipment. When it becomes possible to fabricate parts of plastic materials in a similar fashion to the fabrication of steel vessels from steel sheet (using rivets or welding to effect joints), it may then be feasible to apply theoretical design to plastics in the same manner as is presently done for steel. Since plastic materials at the present time are usually formed to shape in a manner similar to cast iron parts, the range of the possibilities of the use of plastics is analogous to the limitations as to use and design of iron castings.

Plastic part design is therefore largely empirical, and depends upon judgment and experience. When major pieces of equipment are required, manufacturers and suppliers of plastic materials should be consulted. For further data as to design methods and the possibilities and limitations of molded plastics, references such as "Plastics for Industrial Use" [48] should be consulted.

13-10. Available Forms of Plastics for Engineering Applications. Many commercial plastics may be obtained in sheet form, from which cylindrical, conical, or prismoidal shapes and forms may be made by lapping the ends of the sheets and cementing them by using suitable plastic binding agents or cements. A wide variety of commercial plastics can be procured in the form of pipe, tubing, fittings, and valves. Pipe, fittings, and valves are usually made of thermo-

setting materials. while tubing of either thermosetting or thermoplastic material may be obtained commercially. However, Saran, a vinylidene chloride thermoplastic, is successfully used in piping installations. Several forms of transparent plastics, such as Lucite and Plexiglass (acrylics) are available, and are widely used for gage glasses and similar applications.

Practically all types of pipe fittings, similar in character and proportions to standard iron pipe fittings, may be obtained in plastic form. No extensive data on such fittings are of more than temporary value, since new types of plastics having widely differing working pressures and temperatures appear every month. In general, pipe and fittings are available in nominal sizes from ½ to 12 in.; the pipe lengths and their wall thicknesses vary according to the service. Table 13-7 is a representative example of such data. It should be noted that although the outer diameter of Saran pipe corresponds exactly to that of standard iron or steel pipe, the inner diameter varies to some extent, but corresponds approximately to Schedule 80 specification for iron pipe. Because of this correspondence with iron pipe sizes, some plastic pipes can be threaded with standard pipe dies and joined by using standard (125 lbs.) iron pipe fittings. If pipe fittings made of plastic materials are used, however, the standard pipe thread is not satisfactory, and six-pitch Acme threads are generally employed for threaded connections. Most of the connections between plastic pipe and fittings are made by means of flanges and bolts. A wide variety of fittings is available commercially; the design and proportions of such fittings vary to some extent with the individual manufacturer. One form of plastic pipe connection recommended for Haveg pipe is shown in Fig. 13-16. Tapered grooves are machined near the pipe ends to fit a pair of split steel flanges. A rubber or other plastic gasket is held in place between the pipe ends; care must be taken to tighten the bolts gradually and uniformly so that a tight joint without pipe distortion is obtained. Bolt circles on the flanges are laid out in accordance with the ASA-ASME flange standards (section 10-7) and thus allow direct attachment of plastic piping to equipment conforming to the ASME standards. Various types of expansion joints and valves are also available in plastic materials. For size and dimensional data, manufacturers' catalogs should be consulted when plastic fittings are under consideration.

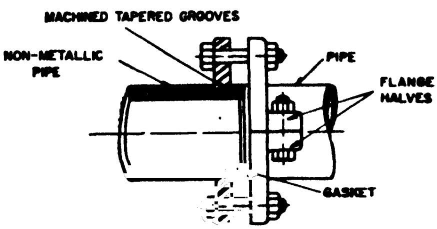


FIG. 13-16. Plastic Pipe Connection.

Plastic molded cylindrical tanks in a wide variety of sizes are available commercially. Flat-bottomed tanks must be fully supported on wood or concrete foundations. Wood foundations should not be used for severe out-of-doors service, or in cases where any overflow of the liquid may cause deterioration

of the foundation. Under such conditions, concrete foundations finished with special acid-resisting grouting cement should be used. Fig. 13-17 shows flat-bottomed plastic molded tanks made of Haveg 43. These tanks are designed for operation at a full head of liquid at atmospheric pressure, and are proportioned as indicated in Table 13-8. Tanks up to 16 in. in diameter are usually supplied without any reinforcing bands, staves or hoops. The wall and bottom thicknesses must therefore be sufficiently great to take care of the stresses induced by the weight of the liquid. Tanks 18 to 36 in. in diameter require reinforcing bands; tanks in excess of 36 in. require staves and hoops, except for those of comparatively small height. If these tanks are used at elevated temperatures or at pressures substantially greater than atmospheric, even the smaller sizes may require the use of bands, or staves and hoops, and the larges sizes should be reinforced by a metal container or shell which fits snugly over the outside of the tank.

TABLE 13-7.—SIZE AND PRESSURE CHARACTERISTICS OF SARAN PIPE

Nominal Size inches	O.D. inches	I.D. inches	Calc.* Burst. Pressure lb./sq.in.
½	0.840	0.546	1300
¾	1.050	0.742	1060
1	1.315	0.957	970
1¼	1.660	1.278	820
1½	1.900	1.500	740
2	2.375	1.939	620
2½	3.875	2.277	570
3	3.500	2.842	510
3½	4.00	3.307	470
4	4.500	3.749	460

* Allowable working pressure may be taken as one fifth this value.

Since pressure vessels of plastic materials can be designed with safety only if experimental data regarding creep are available, the plastic material should in most cases be regarded as a lining, and the supporting bands or reinforcing shell should take care of all the resistance to internal pressure. It is possible to obtain condensers and heat exchangers constructed entirely of molded plastics. The design of such equipment is analogous to that of metal equipment, but the determination of the proper working stresses is so much a matter of experience and unpublished experimental data that the cooperation of manufacturers and

TABLE 13-8.—PROPORTIONS OF CYLINDRICAL TANKS—HAVEG 43

Size (inner diameter)	Maximum Tank Heights Corresponding to the Following Wall Thicknesses						Band Size	Hoop Size
	½	⅝	¾	1	1¼	1½		
10	15'0"	15'0"						
12	10'6"	15'0"						
*14	8'10"	15'0"						
16	7'7"	15'0"						
18	6'7"	15'0"					⅛" × 1½"	
20	5'11"	15'0"					⅛" × 1½"	
24	4'7"	15'0"					⅛" × 1½"	
†30	3'5"	10'6"	15'0"				⅛" × 1¾"	
36	2'4"	8'7"	14'6"				⅛" × 1¾"	
42	1'11"	7'3"	14'6"				⅛" × 1¾"	⅝"
48		6'2"	12'2"	14'6"			3/16" × 2"	⅝"
‡60		4'7"	9'5"	14'0"			3/16" × 2"	⅝"
72		3'6"	7'8"	14'0"			3/16" × 2"	⅝"
84		2'10"	6'4"	10'6"	13'6"		3/16" × 2½"	¾"
96		2'3"	5'3"	8'10"	13'0"		3/16" × 2½"	¾"
108		1'10"	4'6"	7'9"	12'6"		3/16" × 2½"	¾"
120			4'0"	6'10"	10'2"	12'0"	3/16" × 2½"	¾"

* No bands, staves, or hoops required.
† Bands only required.
‡ Staves and hoops required. Tank heights equal to or less than 2 feet require bands only.

NOTE: Thickness of bottom is approximately ⅛ in. greater than the wall thickness for ½-in. and ⅝-in. thick tanks, and ¼ in. greater for ¾-in. wall thicknesses and over.

consultants be obtained for the design and construction of plastic molded equipment of any importance.

A brief analysis of the stresses in the walls and bottoms of the flat-bottomed cylindrical tanks illustrated in Fig. 13-17 and listed in Table 13-8 will illustrate the preceding discussion.

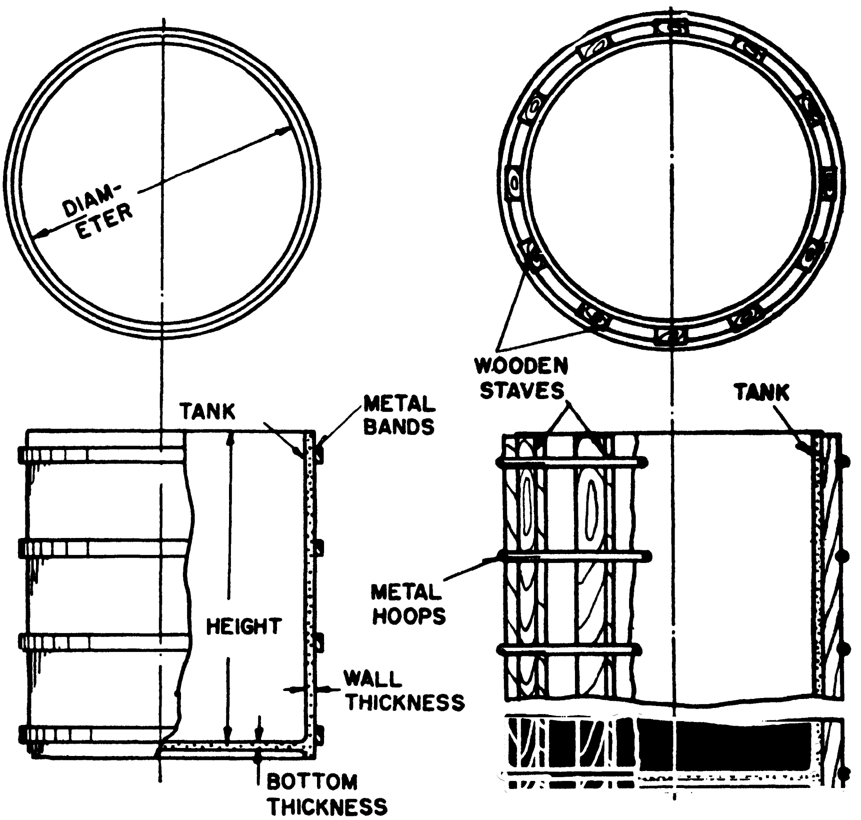


Fig. 13-17. Plastic Tank Details.

If it be assumed that an unreinforced tank 16 in. in diameter and 15 ft. high be filled with commercial hydrochloric acid (35 to 37% HCl), with a specific gravity of 1.178, the force acting along a longitudinal element at the bottom of the vessel, from Eq. 13-14, is

$$F = 0.434 \times 1.178 \times 8 \times 15 = 61.4 \text{ lbs.}$$

The force varies from a maximum of 61.4 lbs. at the bottom of the tank to a minimum of zero at the top; the stress per inch of length at or near the bottom is approximately 60 lbs. Since the wall thickness is ⅝ in. (Table 13-7), the resisting area is 0.625 × 1, or 0.625 sq. in., and the resultant unit wall stress is 60/0.625, or 96 psi. From Table 13-6, the ultimate tensile strength of Haveg 43 is 2.5 kips or 5000 psi., which is far in excess of the actual stress.

The tank bottom is ¾ in. thick, and is subjected to a unit pressure p induced by the weight of the fluid, or

$$p = 0.434 \times 1.178 \times 15 = 7.7 \text{ psi.}$$

Since the bottom is integral with the walls, an approximation of the stress may be obtained by a transposition of Eq. 4-10, as follows

$$S = \frac{0.25\,pd^2}{t^2}$$

Substituting

$$S = \frac{0.25 \times 7.7 \times 16^2}{0.75^2} = 910 \text{ psi.}$$

Although the use of Eq. 4-10 is not rigorous in this case, and may result in a computed value for S considerably in excess of the actual, the magnitude of the unit stress is still appreciably greater in the tank bottom than in the walls. The manufacturer's recommendation that these tanks should be fully supported should obviously be followed. The wall thickness is, of course, appreciably greater than required for stress resistance because of handling and shipping strength requirements.

If a similar vessel 120 in. in diameter and 12 ft. high be considered, the force on a longitudinal element is

$$F = 0.434 \times 1.178 \times 60 \times 12 = 368 \text{ lbs.}$$

For this size of tank, a 1½-in. wall thickness is specified in Table 13-8. For a length of 1 in. at or near the bottom of the tank, the unit wall stress is 368/1.5, or 245 psi. This stress is materially greater than the stress in the walls of the 16-in. tank, but a reference to Table 13-8 will show that both staves and hoops are required for the 120-in. tank. The staves and hoops carry practically all of the wall stress.

The 120-in. tank has a bottom thickness of 1¾ in., and the unit pressure on the bottom is

$$p = 0.434 \times 1.178 \times 12 = 6.1 \text{ psi.}$$

The unit stress is

$$S = \frac{0.25 \times 6.1 \times 120^2}{1.75^2} = 7160 \text{ psi.}$$

The comparatively high value of this stress indicates that full support for the bottom is imperative, although the stress is considerably below the ultimate compressive strength of the material.

Cylindrical tanks with a conical bottom are frequently specified for applications where suspended solids are present and where rapid drainage may be required. A steel shell for supporting the bottom is necessary, into which the tank is grouted with acid-resisting cement. The steel shell is usually provided with brackets or legs to support the vessel. Dished bottom tanks of similar construction are also available, although dished bottom tanks less than 18 in. in diameter are usually supplied with steel supporting shells. Support for these sizes usually consists of a wooden false-work supplied by the user.

Plastic molded tanks for internal pressure or vacuum service are available in a range of sizes from 18 in. to 5 ft. inner diameter and in maximum lengths up to 14 ft. Vacuum tanks are provided with internal integrally molded stiffening ribs spaced about 1 ft. apart. Drainage slots are provided at the bottom to permit complete removal of the contents of the vessel. Both types of vessels are provided with dished heads; one or both heads may be molded integrally with the vessel, but a manhole is required if the heads are non-removable. Both

heads are protected by steel shells grouted in place with acid-resisting cement. Tie rods running through the shell flanges from head to head, parallel to the vessel axis, provide longitudinal support against internal pressure, and are used as fastening media for the removable head. These vessels are reinforced by longitudinal wooden staves held in place by steel hoops, designed so that practically all the bursting stress is taken by the hoops, the plastic serving essentially as a lining material only.

An analysis of the actual shell stress may be of interest. A Haveg tank 5 ft. in diameter, with a length of 14 ft., has a wall thickness of 1½ in., and is rated at a maximum pressure of 15 psi. From Eq. 3-3, the actual stress in an unsupported wall is

$$S = \frac{pR}{t}$$

Substituting

$$S = \frac{15 \times 30}{1.5} = 300 \text{ psi.}$$

This stress is only one sixteenth of the ultimate tensile strength of Haveg 43, as shown in Table 13-6, but the manufacturers of these vessels still consider stave and hoop reinforcement necessary. The result of this computation emphasizes once more the necessity for very careful analysis and stress value selection in plastic molded design.

Rectangular plastic molded tanks in a range of sizes varying from 6 in. to 6 ft. deep, and in lengths up to 15 ft., may be obtained commercially Such vessels are widely used in the chemical industry as storage and reaction tanks, filters, and for pickling, cleaning, and electroplating applications.

Almost any type of outlet or other fitting can be molded into tank bottoms or sides. The design and construction of removable heads or covers is essentially similar to that employed for metallic construction. Through bolts are, however, used in preference to cap screws or tap bolts, unless the molded plastic is furnished with molded metal inserts into which the screws may be threaded.

Radial and step or thrust bearings are often subjected to severe abrasive and corrosive action in chemical processing equipment such as mixers and centrifuges, and many plastic bearing materials are available for such service. Such bearings are usually designed and constructed for each specific application and few generalities can be given, except that it is now nearly always possible to obtain some form of plastic for a definite bearing job providing the temperature of the bearing does not exceed about 200° F., or operate at very high surface speed. Usually, plastic bearings can be lubricated with the liquid being handled or by suitable oils or greases.

13-11. Joining of Plastic Parts. Thermoplastic materials offer possibilities for fusion joining corresponding to the welding of metal parts. Some plastic pipes can be joined together by heating the ends of the pipe to a fairly high temperature by applying them to a chromium-plated hot surface. The pipe ends are placed against the heated surface until a molten bead of material appears.

whereupon they are pressed together firmly and allowed to cool. The strength of the joint should be equal to that of the pipe itself. Plastics dissolved in solvents can be used to cement parts together. The bonding strength of some plastic cements is fairly high but, in general, there must be considerable overlap to provide sufficient joint strength. Plastic cemented joints are not recommended for general use unless the overlapping joints are held together firmly by some form of clamp which will maintain its pressure through the whole life of the joint. Flanged connections are sometimes used in effecting properly cemented plastic parts.

Glass

13-12. Glass is used to considerable extent in large scale chemical equipment, and may be employed either by itself or as a glass lining. Boro-silicate glass has a tensile strength of approximately 10,000 psi., a modulus of elasticity of 98×10^5 psi., and a softening point of about 1500° F. Such a glass can be obtained as tubing and piping as well as in other forms. Tubing and pipes of different internal diameters and wall thicknesses are available from small I.D.'s up to at least 12 in. These pipes can be obtained in various lengths up to at least 10 ft. and are made, if desired, with standard end connections as illustrated in Fig. 13-18. Fittings are available for angles, return bends and valves.

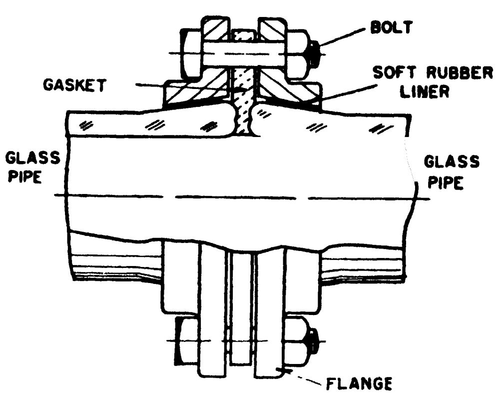


Fig. 13-18. Flanged Connection for Glass Pipe.

Glass can be applied as lining for tanks and, by recent developments, it is possible to make repairs to glass linings which may become cracked by spraying molten glass on the damaged part.

Carbon

13-13. Carbon can be used for structural parts. Both very dense and rather porous forms of carbon can be obtained in block and brick shapes and also in the form of small cylindrical vessels, pipes, pipe fittings and valves. Carbon parts, such as pipe fittings, are usually joined by bolting flanges; great care must be exercised in tightening or drawing up such pieces. The strength of various forms of carbon is usually quite low, although compressive strengths

are very much greater than tensile and shear strengths. The extreme chemical inertness of carbon to many corrosive atmospheres is the greatest factor in its industrial use. The strengths of carbon, however, are such that great care must be taken in fabricating material from it and it must not be allowed to be subjected to shock or continued vibration. Table 13-9 gives a few of the more common forms of industrial carbon available, together with the most useful physical properties. Karbate tubing is one form that has been developed for use in heat exchangers and evaporators and has proven to be a satisfactory structural element for such use.

TABLE 13-9.—REPRESENTATIVE PHYSICAL CHARACTERISTICS OF CARBON AND GRAPHITE PRODUCTS

	Density		Strength, lb. per sq. in.			Elastic Modulus lb./sq.in. $\times 10^{-6}$
	g./cc.	lb./cu.ft.	tensile	compressive	transverse	
Carbon cylinders 10-14-in. dia., inc.	1.525	95.2	470	2120	950	5.4
Carbon beams and blocks 6 × 6 in. to 20 × 20 in., inc.	1.55	96.7	500	2140	990	7.1
Carbon pipe and tubes ½-4-in. I.D., inc.	1.51	94.2	885	10,200	2700	21.0
Carbon brick Standard sizes	1.55	96.7	1530	8320	3070	10.3
Graphite cylinders 6-12-in. dia., inc.	1.55	96.7	610	3420	1810	8.0
Graphite beams and blocks To 5 in. thick, inc.	1.56	97.3	700	3050	1750	8.8
Graphite pipe and tubes ½-4-in. I.D., inc.	1.68	104.8	780	4550	2820	14.0
Graphite brick Standard sizes	1.56	97.3	700	3050	1750	8.8
"Karbate" pipes and tubes 10 Series ½-2-in. I.D., inc.	1.77	110.0	1700	10,500	4170	29.0
Over 2-in. I.D.	1.76	110.0	2000	10,500	4640	26.0
"Karbate" pipe and tubes, 20 Series ½-2-in. I.D., inc.	1.86	116.0	2600	8900	4650	23.0
Over 2-in. I.D.	1.91	119.0	2350	10,500	4980	21.0
"Carbocell" (porous carbon) Grade 50	1.05	65.5	180	830	500	>1.2
"Graphicell" (porous graphite) Grade 50	1.05	65.5	110	500	250	....

PROBLEMS—CHAPTER 13

1. A portion of the space above the lower chords of the trusses of section 8-9 is to be floored over to provide for storage. The storage space is to extend between joints *L* and *W* and between adjacent trusses. The maximum weight to be carried is 200 lbs. per sq. ft., and yellow pine flooring and beams of select quality are to be used. The entire flooring should be carried by two beams connected to the trusses at joints *L* and *W*, although aluminum alloy beams may be used for this purpose if necessary. Design the flooring, beams, and connections.

2. Design a wooden support, similar to that of Fig. 13-7, for a 25-HP, 1725-RPM explosion proof motor weighing 650 lbs. The distance from the centerline of the motor to the floor is 7 ft. and the belt drive is at an angle of 40° to the horizontal. A good quality of oak may be used.

3. A storage shed for Portland cement has brick walls 8 in. thick with a center to center span of 9 ft. The building is to be provided with either a flat roof, supported by rectangular wooden beams, or with a roof having a pitch of about ⅓. The roof is to be of 1-in. yellow pine sheathing, covered with felt and asphalt. Design the supporting member for both types of construction and compare the material costs.

4. Investigate the possibility of using a Haveg tank 12 in. diameter, 10 ft. 6 in. high, with a ½-in. wall, for storing metallic mercury.

5. What stress is developed in a ½-in. diameter Saran pipe under a working pressure of 250 psi.? Same for a 4-in. diameter and 90 psi.

6. Design an open top rectangular wooden plating tank 20 ft. long, 12 ft. wide, and 6 ft. deep, filled to within 3 in. of the top with a plating solution whose specific gravity is 1.3. If possible, arrange the design so that the entire area of the upper surface of the tank is clear of any obstruction.

CHAPTER 14

BELT AND CHAIN DRIVES

14-1. The electric motor is the most convenient medium by which electrical energy, generated by prime movers, is applied to the power demands of industry. High-speed motors are usually more economical from the standpoint of initial cost and efficiency of operation than low-speed units; in order to provide suitable operating speeds for such equipment as machine tools, rotary driers, crushers, and other processing units, and to permit variation in speed when required, some form of intermediate power transmitting mechanism must be employed. When the driving motor and the driven shaft rotate at the same speed, are in axial alignment, and are expected to operate without disconnection except for maintenance or repair, some form of coupling is usually employed. For similar drive conditions where the driven shaft is to be disconnected from the motor during operation, some form of clutch may be used. Couplings and clutches are described in Chap. 16.

For non-coincident shaft axes, both positive and non-positive driving media are employed. Each of these may be further divided into drives that connect adjacent shafts, and drives in which the shafts are comparatively far apart. Positive drives allow exact speed and power control and are often used for transmitting heavy loads where space is limited; non-positive drives are more versatile, transmit little shock, are usually inexpensive to install and maintain, and operate with little noise.

14-2. Friction Drives. Friction gearing is used to transmit light loads between parallel shafts, or between shafts with intersecting axes. If the cylindrical friction wheels in Fig. 14-1 are assumed to operate without slip, the surface speeds of both wheels must be equal. The velocity ratio of such a pair must be inversely proportional to their diameters, or, expressed as an equation:

$$\frac{\text{Diameter of Wheel } D}{\text{Diameter of Wheel } d} = \frac{\text{RPM of wheel } d}{\text{RPM of wheel } D} \qquad (14\text{-}1)$$

The same equation will hold for bevel friction wheels if the diameters are taken at corresponding points on both wheels.

The power transmitted by a pair of friction wheels depends upon the tangential force at their peripheries, and is a function of the normal force or pressure between the wheels and the coefficient of friction of the wheel face material. Friction wheels may be made of any suitable material but are usually made of cast iron. The periphery of the driving wheel is generally faced or covered with a material having a high coefficient of friction; the face of the

driven wheel is usually cast iron or aluminum. Table 14-1 gives recommended values of the coefficient of friction and the allowable pressures per inch of face width of the wheels, for various fibrous facing materials on the driving wheel in contact with a metal faced driven wheel. If the drive is started frequently under full load, the coefficient of friction should be reduced by one half to

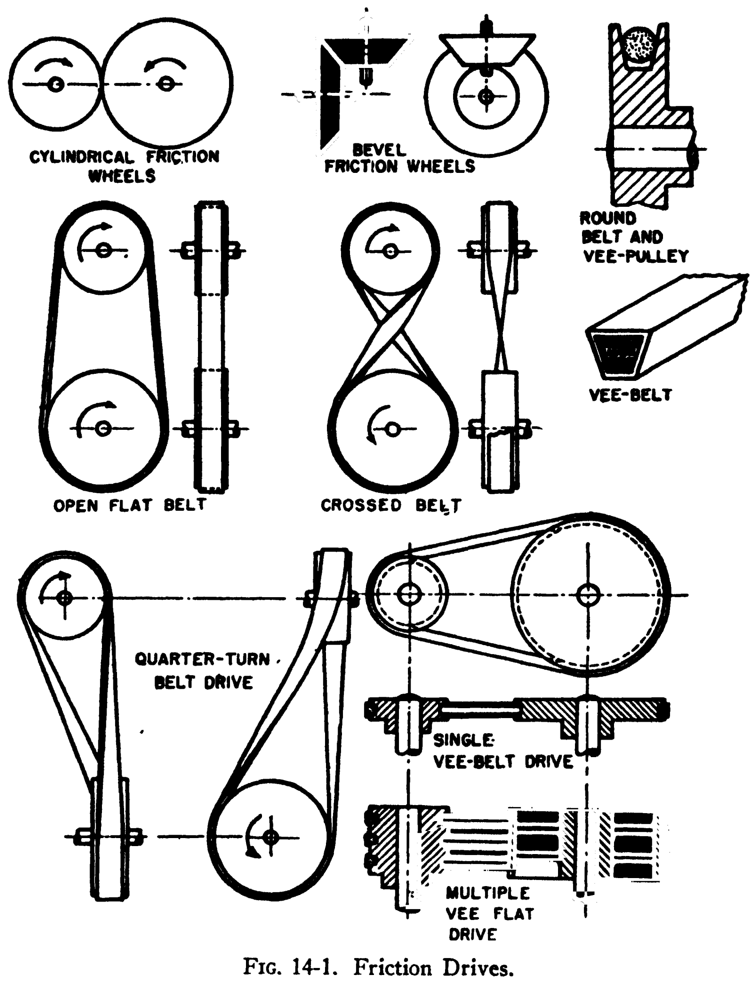


FIG. 14-1. Friction Drives.

two thirds of the values given in the Table 14-1, because of the inevitable slip at starting. Leather fiber subjected to slippage becomes glazed and hard, which causes a decided reduction in the coefficient of friction. Tarred fiber is preferred when the drive is started and stopped frequently. The driving wheel should be made of the softer material on account of the slippage that is unavoidable in friction drives. If the driven member is made of the softer material,

and if it should stop because of an overload while the harder driver is turning, the soft material would wear away at one place and thus form a flat spot or groove in the periphery of the driven wheel.

TABLE 14-1.—DATA ON FRICTION WHEEL DRIVES

Driving Wheel Material	Coefficient of Friction f	Allowable Pressure per Inch of Face, lbs.
Leather	0.135	150
Cork composition	0.210	50
Tarred fiber	0.277	250
Wood	0.150	150
Leather fiber	0.300	300

Commercial friction wheels are obtainable in face widths in increments of one inch. The soft facing used for driving wheels is held on a drum between two metal side plates so that uniform facing thickness can be maintained. The soft-faced wheels are usually 1 in. wider than the metal-faced driven wheels to allow proper wheel face engagement without metal to metal contact between the face of the driven wheel and the sideplates of the driving wheel.

The peripheral force E at the surface of the wheels is equal to the product of the coefficient of friction f and the normal pressure, N, or

$$E = fN \qquad (14\text{-}2)$$

For a given horsepower and speed, consider a force E acting tangent to a circle whose radius is r inches. The work done by this force in moving once around the circle is $2\pi rE$ in.-lbs., or $2\pi rE/12$ ft.-lbs. If the force rotates n times per minute, the work done per minute is $2\pi rEn/12$ ft.-lbs. Since 33,000 ft.-lbs. of work per minute is equivalent to 1 HP,

$$\text{HP} = \frac{2\pi rEn}{12 \times 33{,}000} \qquad (14\text{-}3)$$

or

$$E = \frac{63{,}025\ \text{HP}}{rn} \qquad (14\text{-}4)$$

Another useful form of this equation is obtained by noting that the peripheral velocity V, in feet per minute, of either wheel is equal to $2\pi rn/12$; by substitution, the driving force is given by

$$E = \frac{33{,}000\ \text{HP}}{V} \qquad (14\text{-}5)$$

Example 14-1. The velocity ratio of a pair of friction wheels is 3, and the wheels are to operate at a center distance of 16 in. The driver is to be made of tarred fiber and rotates at 300 RPM; the driven wheel is of cast iron, and ten horsepower is to be transmitted. Design a set of wheels for these conditions.

Solution. Let d and D represent the diameters of the driving and driven wheels. If the velocity ratio is 3, then $3d$ equals D. The center distance, which is equivalent to the sum of the wheel radii, is equal to 16 in., and since $(d + D)/2$ is equal to 16 in., and $(3 + 3d)/2$ is 16 in., the diameter d is equal to 8 in. Substituting in Eq. 14-4, the peripheral force at the wheel surface must be:

$$E = \frac{63{,}025 \times 10}{4 \times 300} = 525 \text{ lbs.}$$

From Table 14-1, the coefficient of friction for tarred fiber is 0.277, and the permissible pressure per inch of face is 250 lbs. From Eq. 14-2,

$$N = E/f = 525/0.277 = 1900 \text{ lbs.}$$

The required face width is 1900/250 or 7.6 in.

It would be advisable to use an 8-in. face, 24-in. diameter cast iron driven wheel, and a 9-in. face, 8-in. diameter tarred fiber driving wheel. The excess face width is desirable since the coefficient of friction is reduced at high rate of slip, and also because the pressure between the wheels at starting may be greater than the normal running pressure. If the wheels are to start and stop frequently, it may be advisable to increase the face widths to 10 in. and 11 in.

14-3. Belt Drives. Friction wheels are not adapted to drives where the distance between the shaft axes is large, because wheels of excessive size would be required. For such service, flexible connectors or belts are usually employed. Belt drives fall into two classifications: flat-belt drives, in which a connector whose width is appreciably greater than its thickness operates on cylindrical wheels or pulleys, and vee- or round-belt drives in which rope or shaped belting materials operate in grooved wheels or sheaves. Varied forms of these connectors are shown in Fig. 14-1.

Open-belt drives and crossed-belt drives are used for power transmission between parallel shafts; quarter-turn belts, or any other fractional turn, can be used when the shaft axes are not parallel. For satisfactory flat-belt operation, the centerline of the belt approaching the pulley must be in a plane perpendicular to the axis of rotation. This rule, which is known as the Law of Belting, if applied to the quarter-turn belt of Fig. 14-1 will show that the system will operate satisfactorily for the indicated directions of rotation; the belt will not stay on the pulleys, however, if the directions of rotation are reversed. By the same rule, open- and crossed-belt drive shafts should lie in parallel planes with the pulleys in alignment. Crowned and flanged pulleys, shown in Fig. 14-2, will compensate for the effects of slight misalignment. Narrow flat belts are often kept on pulleys by means of flanges on the pulley face; belts wider than 2 in. are usually run on "crowned" pulleys having a slightly greater diameter at the center than at the edges; the belt tends to seek the highest point on the pulley face, and thereby centers itself on the crown of the pulley.

14-4. Materials for Flat-belt Drives. The most common flat-belt material is vegetable-tanned leather, although mineral-tanned leather and other materials are used extensively. Leather and other flat belts can be endless or jointed. All leather belts are made of sections of the material (usually 40 to 50 in. long) cemented together; a long diagonal splice or lap is used, and the joint is practically as strong as the belt itself. Double or two-ply belts are made by cementing together two strips of leather with the hair side out. The average ultimate tensile strength of vegetable or oak-tanned leather belts is about 4000 psi.; mineral or chrome-tanned leathers usually have higher strengths. Belts with joints can be taken down readily to compensate for excessive slack; wire lacing applied either by hand or by machine is in wide use; rawhide lacing and metal fasteners are also common. Endless or cemented-joint belts require some mechanical means for adjusting center distances to compensate for the stretching that takes place after the drive has been in service for some length of time. Leather is affected by moisture and the length of a belt will change appreciably under varying atmospheric conditions. Vegetable-tanned leather belts will increase in length rather uniformly as the per cent humidity is increased. The magnitude of the change is roughly 3 to 4% decrease in length for a change of humidity from 100% to zero. The shrinkage in mineral-tanned leather belts may be from two to three times this value. Such a change in length may be very serious in cases where belts are subjected to alternate dry and moist conditions because of proximity to equipment where process steam may escape at intervals, or where the belt dries out over shut-down periods. For example, a chrome-tanned leather belt 100 ft. long when operating satisfactorily in air at 80% humidity will shorten to about 96 ft. if subjected to a 10% humidity. Unless care is taken to compensate for this change in length, which has an effect of about 3 ft. on the center distance of the drive, the belt will be overstressed or it may even pull down or loosen the line shaft.

Fabric belts are made from canvas or cotton duck folded to three or more layers or plies and stitched together. They are usually impregnated with a filler, principally linseed oil, to render them waterproof. These belts are cheap and are widely used for intermittent service under hot or dry conditions and in installations where little attention is given to their maintenance. They are used to some extent as conveyor belts. Flat rubber belts are made from layers of canvas duck or other woven fabrics which are impregnated with compounded rubber, and then vulcanized. These belts are used where exposure to moisture or outside weather conditions are anticipated. Rubber belting is less expensive than leather belting, but is affected by light, heat, and oil. Balata belting is similar to rubber belting, but unvulcanized balata gum is substituted for the rubber. This material does not oxidize in air, is waterproof, and is not affected by animal oils or alkalies. It is seriously affected by mineral oils, however, and may become soft and sticky at temperatures in excess of 120° F. It is about 25% stronger than rubber belting. Synthetic rubber belting is a comparatively new

power transmitting medium that will probably have many applications in the future.

14-5. Flat-belt Pulleys. Representative types of pulleys for flat belts are shown in Fig. 14-2. Solid cast iron pulleys with a single row of arms, or with two rows for heavy-duty service, are probably the least expensive; split pulleys permit attachment or removal of the pulley without removing the units which may be already in place. Pulleys of pressed steel are used extensively for modern countershafting, since they are considerably lighter than cast iron pulleys, and consequently easier to install. Paper or fiber pulleys are usually fabricated with a metal hub; in some instances, metal pulleys are made up with cork or rubber facings.

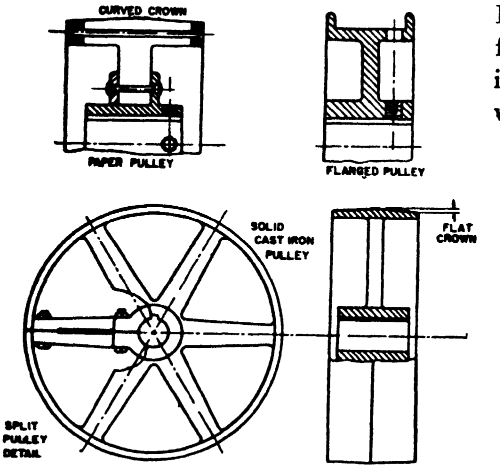


FIG. 14-2. Flat Belt Pulleys.

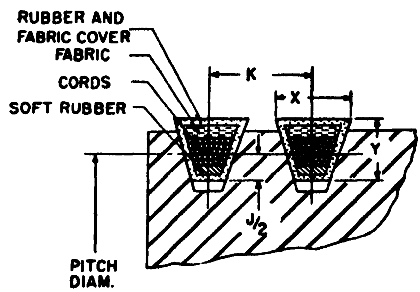


FIG. 14-3. Proportions and Pitch Diameter of Vee-Belt Sections.

14-6. Grooved Sheave Drives. Modern vee-belts are made of fabrics and cords molded in rubber and covered with fabric, as shown in Fig. 14-3. Originally used as single belts in grooved vee-pulleys or sheaves, they are now employed in multiple form for short-center industrial drives for medium and heavy power demands, as shown in Fig. 14-1. They can be run at high or low speeds on sheaves with velocity ratios up to 10:1, and at comparatively short center distances. If one belt should break, the remaining ones in the drive will carry the load until it is convenient or feasible to shut down for repairs. Vee-belt drives, on account of the wedging effect of the belt in the sheave groove, cause less pull on the shaft (R in Fig. 14-6) than flat belts of the same general characteristics. Speed ratios for vee-belt drives are calculated by using the pitch diameters of the sheaves; these may be obtained from Table 14-2 and Fig. 14-3.

Vee-belts are sometimes used with a small grooved driving sheave and a flat-faced (cylindrical) driven pulley, and are then termed vee-flat drives; the larger and more expensive wheel is a plain pulley, which is cheaper than a

grooved sheave of similar size. In some instances, flat-belt drives can be replaced by vee-flat drives by purchasing and installing a motor sheave and the necessary belts, and using the original driven pulley on the machine. The speed ratio is found to determine the belt length. For an open belt, the length L is given by:

$$L = 2\sqrt{C^2 - \left(\frac{D-d}{2}\right)^2} + \pi\left(\frac{D+d}{2}\right) + (D-d)\ \text{arc sin}\left(\frac{D-d}{2C}\right) \tag{14-6}$$

For crossed belts:

$$L = 2\sqrt{C^2 - \left(\frac{D+d}{2}\right)^2} + (D+d)\left[\frac{\pi}{2} + \text{arc sin}\left(\frac{D+d}{2C}\right)\right] \tag{14-7}$$

where C is the center distance, and D and d the diameters of the large and small pulleys, in inches.

TABLE 14-2.—VEE-BELT SECTION DATA

(Refer to Fig. 14-3)

Section	X	Y	J	K
A	½	11/32	0.35	5/8
B	21/32	7/16	0.44	¾
C	7/8	5/8	0.60	1
D	1¼	¾	0.75	1 7/16
E	1½	1	1.00	——

Equations 14-6 and 14-7 show that the length of a crossed belt is essentially a function of a constant center distance C, and the sum of diameters D and d. If a set of stepped pulleys be designed for a crossed belt, the operation will be satisfactory if the sum of every corresponding pair of diameters remains constant. For an open-belt drive, however, Eq. 14-6 shows that the belt length is a function of both $D + d$ and $D - d$, and the determination of the required sets of diameters for a stepped cone depends upon satisfying Eq. 14-6 instead of obtaining equal $D + d$ values for each set of steps.

In the form in which it is presented, Eq. 14-6 requires trial-and-error solution. If, however, the speed ratio between any set of steps is denoted as S, the

diameter of the smaller pulley d of the pair may be found from a transposition of Eq. 14-6 to be:

$$d = \frac{\sqrt{(4CL - 8C^2)(S-1)^2 + \pi^2(S+1)^2C^2} - \pi(S+1)C}{(S-1)^2} \tag{14-8}$$

where S is the pulley diameter ratio or velocity ratio, and is always greater than unity. The nature of the foregoing expression is such that slide rule computations are not sufficiently accurate for a solution.

14-8. Countershafting. Fig. 14-4 shows an application of a stepped cone drive, serving as a variable-speed drive for a machine tool. The driven machine has a three-step cone pulley Z, each step of which may be driven by a corresponding step from the cone pulley C on the countershaft S. (Machine tool drive speeds usually vary in a geometrical ratio which may have a value from 1.2 to 2.0.) The countershaft is driven through the medium of tight-and-loose pulleys T and L and the belt from the driving pulley on the line shaft which is coupled or directly connected to the motor. The function of the tight-and-loose pulleys is to permit the countershaft (and machine) to be stopped while the line shaft continues to rotate. When the belt is on the tight pulley T, the countershaft rotates; when the belt is shifted to the loose pulley L (which is free to rotate on C), the rotation of the countershaft ceases. The belt shifter is arranged to permit moving the belt from L to T from the floor, and will hang vertically for either belt position.

Fig. 14-5 illustrates a reversing drive in which two loose pulleys on countershaft C are driven by open and crossed belts from a line-shaft pulley. Each pulley rotates freely on the shaft when the sleeve S is in a central position between the clutches. The clutches themselves are keyed to the shaft but are free to move axially. (Springs between the inner collars and the clutches prevent involuntary engagement.) When the sleeve is moved to the right as illustrated, the clutch arms A force the conical portion of the clutch into the conical clutch face in the pulley and the frictional force between these two surfaces causes the pulley to rotate the countershaft C.

The forward and reverse speeds of the countershaft C may be varied by using different diameters of pulleys for both forward and reverse drives. The pulleys in Fig. 14-4 are crowned because the belts remain in continual contact with them, but the tight and loose pulleys and the driving pulley in the drive of Fig. 14-2 are flat so that belt shifting may be accomplished easily. Pulley crown is unnecessary in this case, since the shifter pins P serve as guides to prevent the driving belt from slipping off the pulley.

14-9. Belt Theory and Design. The power transmitted by a belt depends upon the tension or pull in the belt, the belt speed, the coefficient of friction between the belt and the pulley surface, and the arc of contact between the

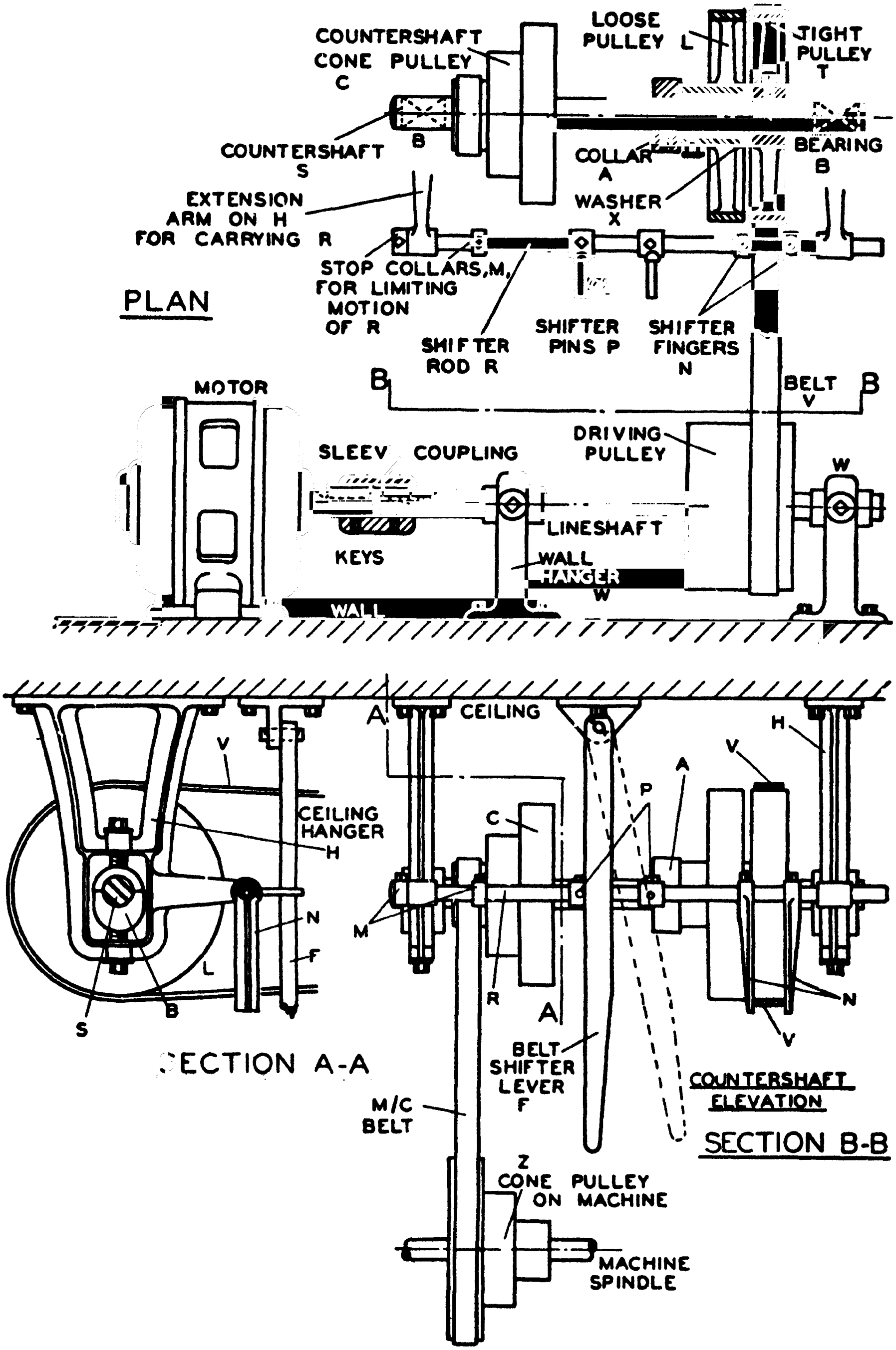


Fig. 14-4. Tight and Loose Pulley Countershaft Drive.

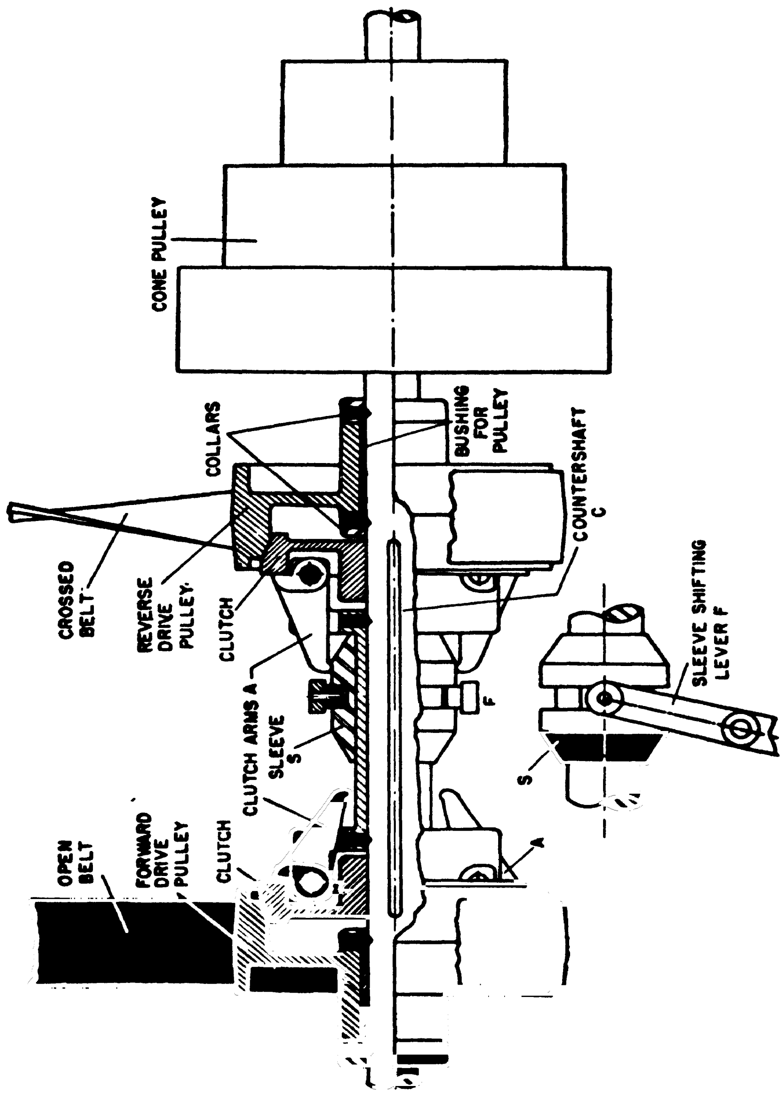


FIG. 14-5. Reversing—Open and Crossed Belt—Countershaft Drive.

belt and the pulley. Belts are rarely stressed to a point greater than from 5 to 10% of their ultimate strength because continuous operations under greater tension will seriously reduce the life of the belt. A belt operating at a unit stress of 400 psi. has a very short life, whereas one operating at 200 psi. will give long service. Power magnitudes are almost directly proportional to belt speeds up to 5000 ft. per min. Higher speeds are seldom economical on account of the centrifugal force of the belt, which tends materially to reduce the contact between the belt and the pulley. Belt speeds between 3000 and 4500 ft. per min. will generally give most satisfactory results.

Fig. 14-6 shows the force relation in belt drives; the upper illustration shows a horizontal drive with the slack side of the belt on top; the lower illustration shows the slack side of the belt underneath the horizontal centerline. The actual effective turning force is E, and is equal to the difference between the tight side pull T and the slack side pull L. The ratio between T and L is called the belt tension ratio p, and

$$T/L = p = e^{f\theta} \qquad (14\text{-}9)$$

where e is the base of natural logarithms (2.718), f is the coefficient of friction between the belt and the pulley, and θ is the angle of contact between the belt and the pulley. (The derivation of this equation may be found in any standard text in Mechanics. For simplicity, any consideration of the centrifugal force has been omitted.)

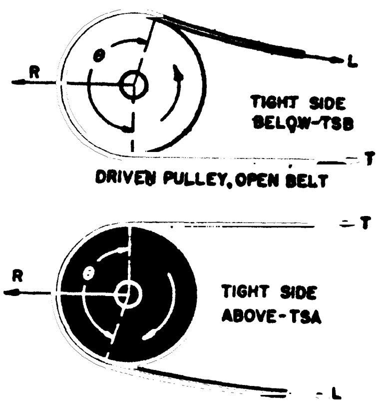


FIG. 14-6. Relation of Belt Tensions in Open Belt Drives.

The stress in the belt is dependent upon the tight side pull T, and since the latter (from Eq. 14-9) varies with the coefficient of friction and the contact angle, their importance may be readily appreciated. Fig. 14-6 also shows how the angle of contact is affected to some extent by the belt position; with the tight side below the drive centerline, any increase in belt length tends to increase the angle of contact.

The value of the tension ratio materially affects the pull or load that the belt may exert on the shaft. This force is represented in Fig. 14-6 by the reaction R. For a drive in which the belt sides are approximately parallel, R is equal to $T + L$. As E equals $T - L$, the value of R, in terms of the tension ratio p, is

$$R = \left(\frac{p+1}{p-1}\right) E \qquad (14\text{-}10)$$

Values of the tension ratio for various types of belts can be obtained from Fig. 14-7.

The tension ratio in a vee-belt drive is given by

$$T/L = p = e^{f\theta/\sin d} \tag{14-11}$$

where d is one half the groove angle. In standard vee-belt sheaves, the groove angle is usually 40°.

The tension ratio in a vee-flat drive is taken as the smaller of the values obtained from Eq. 14-10 or 14-11, and is usually about the same for the vee-belt sheave and the flat pulley, since the large angle of contact of the flat pulley compensates for the wedging effect obtained by the use of the grooved sheave.

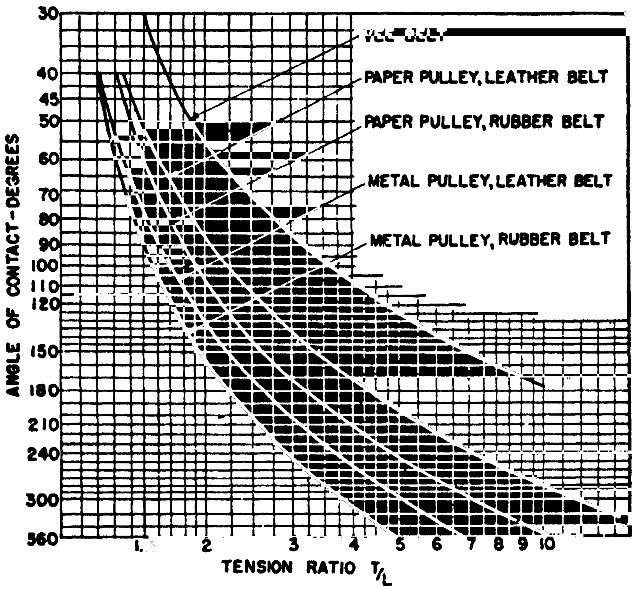


FIG. 14-7. Belt Tension Ratios.

The angle of contact between a belt and pulley may be found from the following, where D and d are the diameters of the large and small pulleys and C is the center distance, in inches:

$$\theta = 180° \pm 2 \text{ arc } \sin\left(\frac{D-d}{2C}\right) \tag{14-12}$$

An approximate expression often used is

$$\theta = 180° \pm \frac{60(D-d)}{C} \tag{14-13}$$

This equation gives values with 1° of the theoretical for arcs of contact between 180° and 110°, but is 3° too high for an arc of 100° and 5° too high for a 90° arc.

14-10. Flat Leather Belt Selection. The American Leather Belting Association has correlated performance data for leather belting; these data are summarized in Tables 14-3 and 14-4, and Figs. 14-8 and 14-9. The data may be

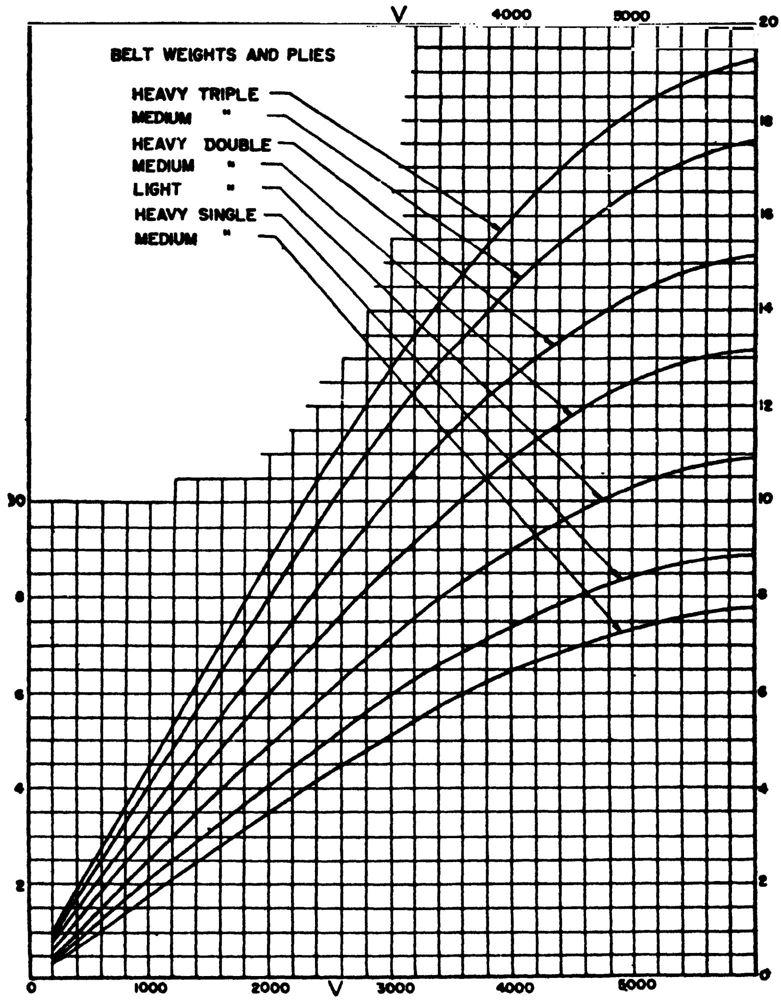


Fig. 14-8. Power Transmitting Capacities of Flat Leather Belts.

used with the following empirical equation to calculate belt capacities for life expectancies of from five to seven years.

$$H = JKANPMU \qquad (14\text{-}14)$$

where H is the horsepower that may be transmitted, with reasonable belt economy, per inch of belt width. J is the theoretical horsepower, per inch of

width, from Fig. 14-8. K is the correction factor for pulley diameter and center distance, and is obtained from Fig. 14-9. In this figure CD represents the center distance in feet, while TSA and TSB indicate the relation of the tight side of

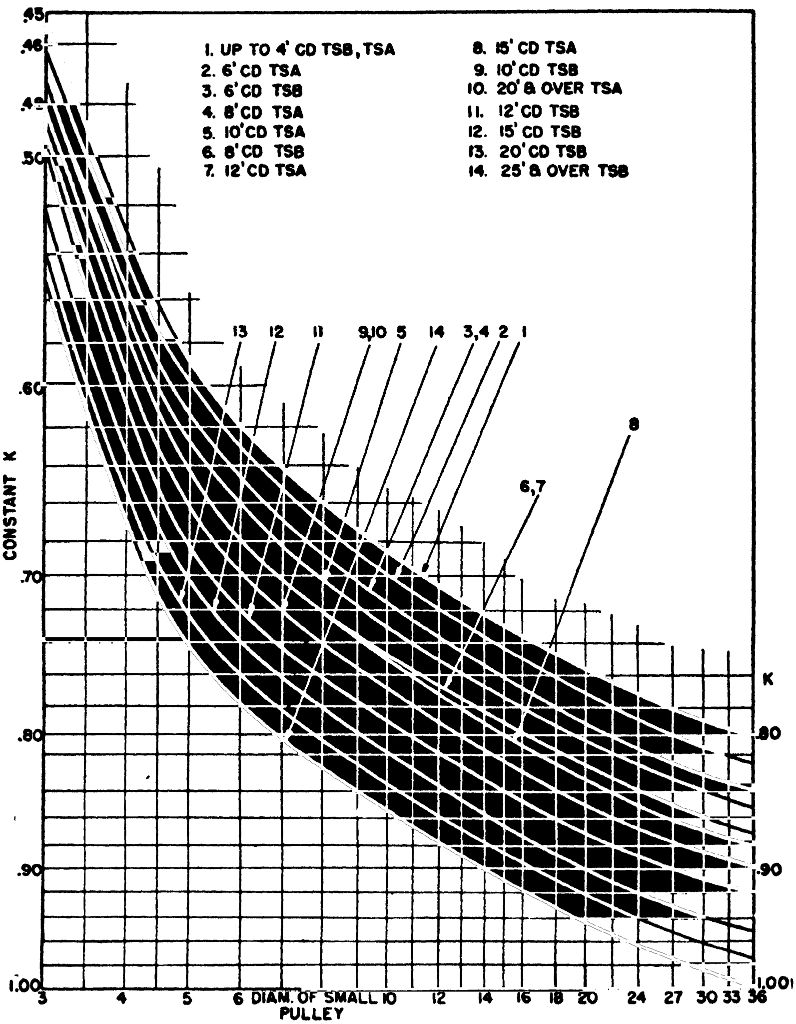


Fig. 14-9. Constant K for Flat Leather Belts.

the belt respectively above and below the horizontal centerline of the drive, as shown in Fig. 14-6. Drives with vertical centerlines should be considered TSA; gravity idlers and pivoted motor drives with the tight side of the belt next to the pivot point should be designed using factors represented by $25'CD$-TSB. Constants A, N, P, M, and U are atmospheric condition, angle of centerline

TABLE 14-3.—CONSTANTS FOR LEATHER BELT DESIGN

Atmospheric conditions	*A*	Pulley materials	*P*
Clean, scheduled maintenance	1.2	Small pulley, fiber, or paper	1.2
Normal	1.0	Cast iron or steel	1.0
Oily, wet, or dusty	0.7		
Service	*M*	Angle of centerline of drive	*N*
Temporary or intermittent	1.2	Horizontal to 60° from horizontal	1.0
Normal	1.0	60 to 75° from horizontal	0.9
Important, or continuous	0.8	75° from horizontal to vertical	0.8

PEAK LOADS *U*

All electric motor drives, motor pulley diameters	
3 to 3½″	0.5
4 to 4½″	0.55
5 to 5½″	0.58
6 to 10″	0.6
11 to 13″	0.63
14 to 17″	0.65
18 to 23″	0.68
24 to 30″	0.7
All other drives	
Steady belt loads	1.0
Jerky belt loads	0.8
Shock and reversing belt loads	0.6

TABLE 14-4.—MINIMUM PULLEY DIAMETERS FOR FLAT LEATHER BELTS

(All figures are in inches)

Belt Width	Belt Thickness						
	Single		Double			Triple	
	Medium 11/64″	Heavy 13/64″	Light 9/32″	Medium 5/16″	Heavy 23/64″	Medium 15/32″	Heavy 17/32″
Under 8	3	5	6	8	12	20	24
8 and over	5	7	8	10	14	24	30

Commercially obtainable belt widths:

½ to 1″ in increments of ⅛″.
1 to 4″ increments of ¼″.
4 to 7″ in increments of ½″.
7 to 12″ in increments of 1″.
Width greater than 12″ made up to order.

of drive, pulley material, type of service, and peak load factors, and may be obtained from Table 14-3. Limitations as to commercially available belt widths, and minimum pulley diameters for belt thicknesses, may be obtained from Table 14-4. Data on standard motor pulleys are given in Table 14-5. Belt widths should be from 15 to 25% less than the face width of the pulley, although belts ½ to 1 in. narrower than the pulley faces are often found in practice.

TABLE 14-5.—STANDARD MOTOR PULLEY SIZES

Pulley				Horsepower at Various Motor Speeds			
Diam Inches	Face Inches	Bore Inches	Max. Belt Width Inches	3500 RPM	1750 RPM	1175 RPM	870 RPM
3	3	¾	2½	1½	1	¾, 1	½
4	3½	1	3	3, 5	1½, 2, 3	1½, 2, 3	1
4½	4½	1⅛	4	7½		5	1½, 2
5	4½	1¼	4	10	7½	5	3
6	5½	1⅝	5	15	10	7½	5
8	6¾	1⅝	6	20	15	10	7½
9	7¾	1⅞	7	25, 30	20, 25	15	10
10	7¾	2⅛	7	40, 50	30	20, 25	15, 20
11	9¾	2⅜	9		50	30, 40	25, 30
12	11	2⅝	10			50	40
14	13	2⅞	12				50

An empirical relation known as the Millwright's Rule can also be used to estimate the approximate capacity of lineshaft and countershaft belt drives for normal service:

$$\text{Single Belts} \quad W = 800\ \text{HP}/V \tag{14-15}$$

$$\text{Double Belts} \quad W = 500\ \text{HP}/V \tag{14-16}$$

where W is the belt width in inches and V the belt speed in feet per minute.

Example 14-2. A line shaft with a 36-in. diameter cast iron pulley is driven by a medium double-ply leather belt 5 in. wide from an 870-RPM electric motor with a 10-in. paper pulley. The center distance between the shaft axes is 6 ft., and the angle of inclination of the drive is about 70° to the horizontal. Service and atmospheric conditions, and starting torque, are normal.

Solution. The belt speed $= \dfrac{\pi dn}{12} = \dfrac{\pi \times 10 \times 870}{12} = 2280$ ft. per min.

From Fig. 14-8, the theoretical horsepower J, per inch of width for this speed, is 6.6. The correction factor K (from Fig. 14-9) for a small pulley 10 in. in diameter and a center distance of 6 ft. is 0.7, for the worst possible condition, occurring when the tight side of the belt is above the centerline. From Table 14-3, factor A is taken as 1.0, as the atmospheric conditions are normal; N as 0.9, as the angle of the centerline of the drive is at 70° to the horizontal; P as 1.2, because the smaller pulley is of paper; M as 1.0 for normal service, and U as 0.6 for a 10-in. pulley electric motor drive. Substituting in Eq. 14-14:

$$H = 6.6 \times 0.7 \times 1.0 \times 0.9 \times 1.2 \times 1.0 \times 0.6 = 3.0\text{/in. of width}$$

For a belt 5 in. wide, the total power that can be transmitted is 3.0×5, or 15 HP.

The Millwright's Rule for a double belt, Eq. 14-16, may be used to check this result, as follows:

$$HP = \frac{WV}{500} = \frac{5 \times 2280}{500} = 22.8$$

Example 14-3. A gyratory crusher, with a capacity of 100 tons per hour of ¾-in. rock discharge, is powered through a 36-in. diameter, 13-in. face cast iron driving pulley which is to rotate at about 300 RPM. A horizontal leather belt is used between this pulley and a fiber pulley on the shaft of a 50-HP, 870-RPM motor. The distance between the pulley centers is about 8 ft. Select a suitable leather belt for this drive.

Solution. From Table 14-5, it is found that an 870-RPM, 50-HP motor is usually equipped with a standard 14-in. diameter, 13-in. face pulley. For this motor pulley, the crusher speed will be:

$$\text{Crusher RPM} = \frac{(\text{Motor Pulley Diam.})(\text{Motor Speed})}{\text{Crusher Pulley Diam.}} = \frac{14 \times 870}{36} = 338 \text{ RPM}$$

A 12-in. wide belt may be used with a 13-in. face pulley; the required horsepower per unit of face width will therefore be 50/12 or 4.167. For a 14-in. pulley diameter, with a center distance of 8 ft., and with the tight side of the belt below, the correction factor K from Fig. 14-9 will be 0.787. (The motor may be placed so that this condition prevails.)

The motor pulley is of fiber and, by reference to Table 14-3, P is equal to 1.2. The angle of the drive is horizontal, so N is 1.0. The material to be crushed comes to the machine in periodic lots, so the service factor M may be taken as 1.2. The air in the vicinity of the crusher will probably be filled with dust particles; the factor A may therefore be assumed as 0.7. There is considerable likelihood of peak loads, particularly as foreign bodies may be included with the material to be crushed, and the size of rock as it comes to the crusher is very likely to vary considerably; it will therefore be advisable to choose factor U as 0.65, for motor drive. A transposition of the horsepower expression gives:

$$J = H/KANPMU = 4.167/(0.787 \times 0.7 \times 1.0 \times 1.2 \times 0.65) = 8.08$$

The belt speed V is $\pi dn/12$, or $\pi \times 14 \times 870/12$, or 3200 ft. per min. From Fig. 14-8, for a speed of 3200 ft. per min., a light double-ply belt will transmit 7.6 HP per inch of width, a medium double-ply belt 9.2 HP per inch of width. The latter is required and will consequently be selected. Checking the horsepower per inch of width that this belt will carry, by Eq. 14-14:

$$H = 9.2 \times 0.787 \times 0.7 \times 1.0 \times 1.2 \times 1.2 \times 0.65 = 4.69$$

The required width is therefore 50/4.69, or 10.7, say 11 in. A reference to Table 14-4 indicates that the minimum pulley diameter for a medium double-ply belt with a width over 8 in. is 10 in.; since the minimum pulley size in this drive is 14 in., the required width of 11 in. is satisfactory. The pulley face is 2 in. wider than the belt, and is satisfactory. The belt capacity, by Eq. 14-16, is

$$HP = \frac{11 \times 3200}{500} = 70$$

This comparatively high value is accounted for by the fact that the actual belt is subjected to severe service, while the belt capacity from Eq. 14-16 is based upon normal service.

14-11. Flat Rubber Belt Selection. Flat rubber belts are usually selected from manufacturers' catalogs. As an illustration of the type of data available, Fig. 14-10 was prepared from manufacturers' data for flat rubber transmission

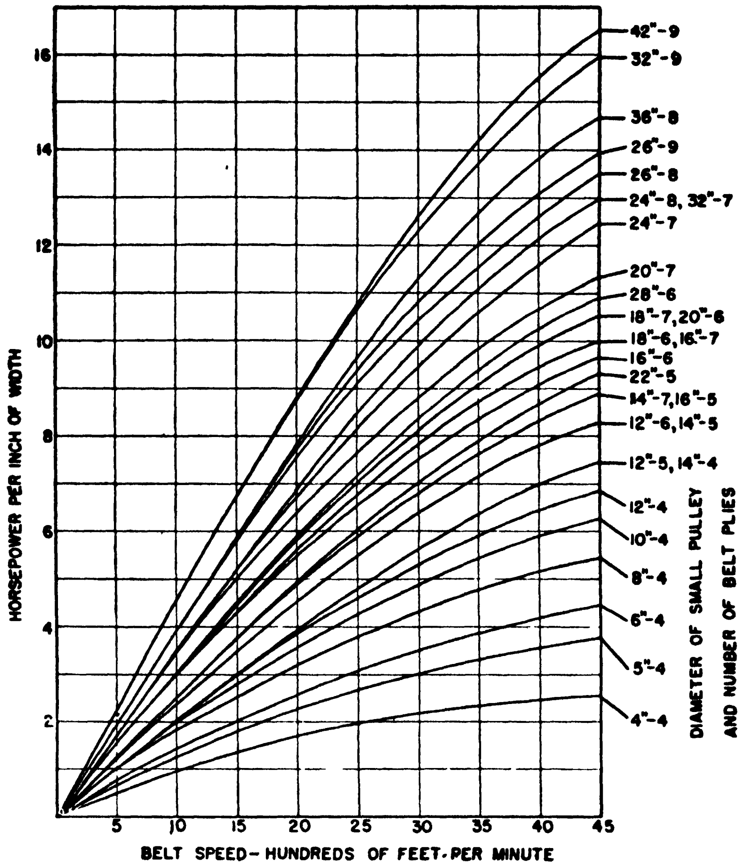


FIG. 14-10. Power Transmitting Capacities of Flat Rubber Belts.

belts, and may be used for estimating purposes where a belt for general industrial and heavy-duty service is required. The curves give the power transmitted per inch of width of these belts for a given pulley diameter and a given number of plies. The curves represent the maximum load for the minimum number of plies. For example, the maximum power transmitted over a 14-in. pulley occurs when a 7-ply belt is used. An 8-ply belt used with the same pulley will transmit less power, and is consequently not shown on the plot. The same relation holds

for all other curves given. The power data obtained from Fig. 14-10 must be multiplied by the arc of contact correction factor obtained from Fig. 14-11. Recommended proportions for rubber belts are given in Table 14-6.

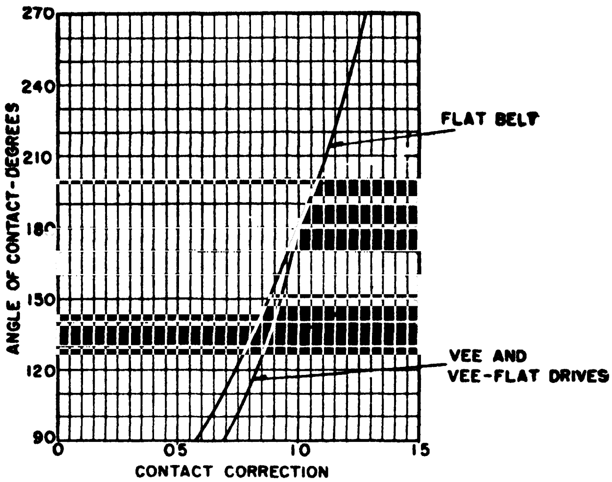


FIG. 14-11. Correction Factor for Arc of Contact—Rubber Belting.

TABLE 14-6.—RUBBER BELT PROPORTIONS

Belt Width, Inches	Number of Plies	
	Min.	Max.
2	3	5
3, 4	3	6
5, 6, 8	4	6
10, 12	4	7
14, 16, 18	5	8

Example 14-4. Select a flat rubber belt for the gyratory crusher of Example 14-3.

Solution. The belt speed, from Eq. 14-4, is 3200 ft. per min. The arc of contact θ, from Eq. 14-13, is

$$\theta = 180° - \frac{60(36 - 14)}{8 \times 12} = 166°$$

From Fig. 14-11, the correction factor for the arc of contact is 0.94. If we assume a belt 12 in. wide, the required horsepower per inch of belt width will be (50/12)0.94, or 4.4.

From Fig. 14-10, a 4-ply belt on a 14-in. pulley will transmit 5.8 HP per inch of width. Checking, we have

$$5.8 \times 0.94 = 5.45 \text{ HP per inch of width}$$

which necessitaes a final width of 50/5.45, or 9.2 in., say 10 in. From Table 14-6, a belt 10 in. wide has a minimum thickness of four plies, and the design is therefore satisfactory.

14-12. Vee-belt Selection. Vee-belts are obtainable in five standard sections, as shown in Table 14-2, ranging from Section A which has a depth of $^{11}\!/_{32}$ in. and a maximum width of $\frac{1}{2}$ in., to Section E, with a depth of 1 in. and a maximum width of $1\frac{1}{2}$ in. Vee-belts are made without joints, in a wide variety of pitch lengths. Stock drives are designed for three ranges of center distances, i.e., A short, B average, C long, as shown in Table 14-7. For example, A-31 indicates a belt of "A" section with an inside length of 31 in.; C-144 indicates a belt of "C" section, with an inside length of 144 in. which corresponds to a pitch length of 146 in. Vee-belt drive selection is usually made from manufacturers' catalogs, as most of the belts and sheaves are standardized. Tables 14-7 to 14-10, inclusive, are representative of catalog data that are available.

Example 14-5. Select a vee-belt drive for the gyratory crusher of Example 14-3.

Solution. From Table 14-9 it is seen that a C section belt should be used for a 50-HP, 870-RPM drive. From the data for Example 14-3, the speed ratio is 870/338, or 2.57, and the center distance is 8 ft. From Table 14-7, the belt that most nearly fulfills these conditions is C-210, which may operate at a center distance of 79.5 in. for a speed ratio of 2.55, or at a center distance of 79.7 for a speed ratio of 2.61. This belt is classed under center distance *C*, and from Table 14-8 it is seen that a heavy-duty drive for a speed ratio of 2.55 and a motor speed of 865 RPM may transmit 3.9 HP per belt. The required number of belts is equal to 50/3.9, or 12.8, say 13 belts, to operate on sheaves with pitch diameters of 9.4 and 24 in., as given in Table 14-8. From Table 14-9, 13 belts are within the maximum limit.

Consideration should also be given to a design involving a somewhat shorter center distance. If it is feasible to move the motor sufficiently close to the crusher to use belt C-96, a reference to Table 14-7 indicates that the center distance is 21.6 in., corresponding to center distance *A*. From Table 14-8 the capacity of one belt is 3.61 HP; the number of belts required would be 50/3.61, or 13.8, and necessitates 14 belts. Shorter belts are obviously less costly than long belts, but an additional belt is required, and there is some increase in sheave cost because of the additional groove. A cost analysis of the two drives (made on the basis of data from a manufacturer's price list) follows:

Drive with C-210 Belts		Drive with C-96 Belts	
1—9.4" × 13 groove sheave	$103.40	1—9.4" × 14 groove sheave	$110.00
1—24" × 13 groove sheave	212.40	1—24" × 14 groove sheave	226.00
13—C-210 Belts	243.10	14—C-96 Belts	120.40
	$558.90		$456.40

The short center distance with fourteen belts is therefore more economical in first cost. It will also save space, and may be regarded as the more desirable drive. Even if the fourteen belts do not last as long as the thirteen longer ones, replacement per belt will be less expensive.

Example 14-6. Analyze Examples 14-3, 14-4, and 14-5 on a comparative basis to determine the relative magnitudes of the pull on the shaft.

TABLE 14-7.—TYPICAL PORTION OF A CENTER DISTANCE TABLE FOR SHEAVE AND VEE-BELT COMBINATIONS. VEE-BELT "C" SECTION, WITH CAST IRON STOCK SHEAVES FOR 3 TO 14 GROOVES ONLY. SEE ALSO TABLE 14-8

(For complete data see manufacturers' catalogs)

CD—Center Distance, inches. *BS*—Belt Serial Number.

Speed Ratio	Center Distance *A*				Center Distance *B*				Center Distance *C*									
	CD	*BS*	*CD*	*BS*	*CD*	*BS*	*CD*	*BS*	*CD*	*BS*	*CD*	*BS*	*CD*	*BS*	*CD*	*BS*	*CD*	*BS*
2.40	21.3	*C*-96	25.9	*C*-105	29.1	*C*-112	33.5	*C*-120	37.9	*C*-128	45.7	*C*-144	54.8	*C*-162	63.9	*C*-180	79.0	*C*-210
2.50	21.5	*C*-96	25.9	*C*-105	29.4	*C*-112	33.9	*C*-120	38.1	*C*-128	46.1	*C*-144	55.0	*C*-162	64.3	*C*-180	79.4	*C*-210
2.55	21.6	*C*-96	26.3	*C*-105	29.6	*C*-112	34.0	*C*-120	38.2	*C*-128	46.3	*C*-144	55.1	*C*-162	64.5	*C*-180	79.5	*C*-210
2.61	21.7	*C*-96	26.4	*C*-105	29.8	*C*-112	34.1	*C*-120	38.3	*C*-128	46.5	*C*-144	55.2	*C*-162	64.6	*C*-180	79.7	*C*-210
2.83	27.4	*C*-120	31.5	*C*-128	40.0	*C*-144	47.0	*C*-158	49.0	*C*-162	54.7	*C*-173	65.8	*C*-195	87.7	*C*-240		

TABLE 14-8.—TYPICAL PORTION OF A TABLE FOR THE HORSEPOWER PER BELT, VEE-BELT "C" SECTION. FOR CAST IRON STOCK SHEAVES OF 3 TO 14 GROOVES ONLY

(For complete data see manufacturers' catalogs)

Service: *HD*—Heavy Duty, *ND*—Normal Duty, *LD*—Light Duty

Speed Ratio	Sheave Pitch Diameters		Suggested HP Range 20-75, Motor Speed 1750 RPM								Suggested HP Range 15-50, Motor Speed 865 RPM							
			Driven Shaft Speed RPM	Center Distance *A*		Center Distance *B*		Center Distance *C*			Driven Shaft Speed RPM	Center Distance *A*		Center Distance *B*		Center Distance *C*		
	Driver	Driven		*HD*	*ND*	*HD*	*ND*	*HD*	*ND*	*LD*		*HD*	*ND*	*HD*	*ND*	*HD*	*ND*	*LD*
2.40	10.0	24.0	730	4.79	5.75	5.06	6.07	5.24	6.28	8.1	361	4.05	4.86	4.24	5.09	4.38	5.24	5.5
2.50	9.6	24.0	700	4.75	5.70	4.96	5.95	5.13	6.15	8.1	346	3.79	4.55	3.96	4.75	4.09	4.90	5.3
2.55	9.4	24.0	686	4.58	5.50	4.78	5.74	4.94	5.93	8.1	339	3.61	4.33	3.77	4.53	3.90	4.68	5.2
2.61	9.2	24.0	671	4.40	5.29	4.59	5.50	4.78	5.73	8.1	332	3.52	4.23	3.68	4.22	3.80	4.56	5.1
2.83	10.6	30.0	619	5.40	6.49	5.60	6.72	5.82	6.98	7.8	306	4.69	5.63	4.88	5.85	5.04	6.05	5.5

TABLE 14-9.—DRIVE SELECTION TABLE FOR VEE-BELTS. (TABLE GIVES RECOMMENDED SECTION AND MAXIMUM NUMBER OF BELTS)

HP	Speeds RPM 1750	1175	870	695
½ to 3	A, 6	A, 6	A, 6	
5	A, 6	A, 6	A, 6	
7½	A, 6; B, 6	B, 6	B, 10	
10	B, 10	B, 10	B, 10	
15	B, 10	B, 10	C, 14	
20	B, 10; C, 14	C, 14	C, 14	
25, 30, 40	C, 14	C, 14	C, 14	
50	C, 14	C, 14	C, 14	
60	C, 14	C, 14; D	D	D
75	C, 14	D	D	D
100		D	D	D

TABLE 14-10.—PORTION OF A TABLE GIVING HP RATING, 180° CONTACT, PER BELT, C SECTION, HEAVY DUTY

Belt Speed ft./min.	Pitch Dia. Small Sheave, in. 9"	10"	12" and Over	Max. Rating, and Duty
1000	2.0	2.5	2.7	3.0
2000	3.8	4.3	5.2	5.5
2400	4.2	4.7	6.1	6.3
3000	4.7	5.5	7.1	7.5
3200	4.8	5.7	7.4	7.9

Solution. It has been pointed out that the actual pull on the bearings of the motor and driven machine depends upon the tension in the two sides of the belt rather than on the effective belt pull. The effective pull E for the flat leather or rubber belt drives, from Eq. 14-5, is:

$$E = \frac{33{,}000 \times 50}{3200} = 516 \text{ lbs.}$$

The angle of contact, from the solution to Example 14-4 for the smaller pulley, is 166° The angle of contact for the larger pulley is 360 — 166, or 194°. From Fig. 14-7, for the leather belt and paper pulley, with θ equal to 194°, p equals 2.75. From Eq. 14-10 the bearing pull is the greater of the following

$$R = \left(\frac{3.8 + 1}{3.8 - 1}\right) 516 = 885 \text{ lbs.}$$

$$R = \left(\frac{2.75 + 1}{2.75 - 1}\right) 516 = 1106 \text{ lbs.}$$

and the total bearing pull is, therefore, 1106 lbs. Similarly, for the flat rubber belt, for a value of θ of 166°, and a fiber pulley, p is equal to 2.9; for a value of θ of 194°, and a cast iron pulley, p is equal to 2.4. The latter value governs the magnitude of the bearing pull, or

$$R = \left(\frac{2.4 + 1}{2.4 - 1}\right) 516 = 1255 \text{ lbs.}$$

If the center distance is decreased to 4 ft., to increase the value of θ for the metal pulley, the bearing pull R will be somewhat smaller.

For the vee-belt drive based upon a center distance of 21.6 in., the angle of contact θ may be obtained from Eq. 14-13, and is found to be 137°. For this angle, from Fig. 14-7, p is 5.8.

The pitch line velocity V is

$$V = \frac{\pi D n}{12} = \frac{\pi \times 9.4 \times 870}{12} = 2120 \text{ ft. per ft.}$$

and the effective pull is

$$E = \frac{33{,}000 \times 50}{2120} = 778$$

and

$$R = \left(\frac{5.8 + 1}{5.8 - 1}\right) 778 = 1100 \text{ lbs.}$$

practically the same as the flat leather belt drive. If sheaves comparable in diameter to the flat belt pulleys, operating at the extended center distance, were used, the pull R would be reduced materially.

RECAPITULATION

Belt	Effective Pull, lbs.	Bearing Pull, lbs.
Leather	516	1106
Rubber	516	1255
Vee	778	1100

Example 14-7. Investigate the possibility of applying a vee-flat drive to the gyratory example crusher of Example 14-3, and compare it with the other drives selected.

Solution. If the 36-in. flat pulley with which the crusher is equipped is to be used, a sheave with a 14-in. pitch diameter is required. For a vee-flat drive, the center distance should be as short as possible, approaching a limit where the two wheels almost touch; in

no case should the center distance be greater than the diameter of the larger wheel. The length of the belt selected will determine the exact center distance. The theoretical pitch diameter of the large pulley, from Table 14-2, is $(36 + 0.60)$, or 36.6 in.; the maximum center distance is 36 in.; if a center distance of 30 in. be assumed, and the diameters of 14 and 36.6 in. be substituted in Eq. 14-6, the belt length L will be 144.01 in. A C-144 belt has an inside length of 144 in.; the pitch length will be $144 + \pi J$, or about 146 in., and will probably be satisfactory. The actual center distance will be about 31 in. The angle of contact, from Eq. 14-13, is 136° for the small pulley or sheave, and 224° for the flat pulley. From Table 14-10, we find that a C-section belt, for heavy-duty service with 180° contact, at a speed of 3200 ft. per min. will transmit 7.4 HP per belt. From Fig. 14-11, the correction factor for an arc of contact of 136° is 0.88. The horsepower per belt is 7.4×0.88, or 6.52, and 50/6.52, or 7.7, say 8 belts, are required. From Table 14-6, for a C-section belt, the distance between the centerlines of adjacent sheave grooves, dimension K, is 1 in., and the 12-in. face of the crusher pulley will be amply wide to carry the required 8 belts. The contact angle of the belt on the sheave is 136°, and the tension ratio p is 3.8; the contact angle of the belt on the pulley is 224°, and the tension ratio p is 2.75, considering a rubber vee-belt on a metal pulley. The bearing pull R is based upon the latter, and

$$R = \left(\frac{2.7 + 1}{2.7 - 1}\right) 516 = 1105 \text{ lbs.}$$

which is practically the same as the bearing pull on the leather belt or the vee-belt drive in Example 14-6. The vee-flat drive will prove less expensive than a vee-belt drive with two sheaves, because of the fewer belts required and also because only one sheave need be purchased to change over from flat-belt to vee-flat drive. The actual cost will be \$103.40 for the motor sheave, and \$102.40 for the belts, or a total of \$205.80, which is less than that of any of the other drives considered.

14-13. Chain Types and Application. Chains are used for three classes of service: hoisting, conveying, and power transmission. Coil chain is used for hoisting and hauling purposes. It is usually made of welded wrought iron or steel links. Twisted link coil chain is used for general utility purposes, but not for dangerous lifting. Stud link chain is used to prevent stretching and distortion of the links. The presence of the studs does not materially affect the chain strength. Common coil chain may be loaded to a working value equal to $12{,}300\ D^2$ psi., where D is the chain size, in inches, or the size of the rod from which the links are made. The breaking load is approximately four times this value and the chains are usually subjected to a proof test of twice the working load.

There are five types of chains used for power transmission as illustrated in Fig. 14-12. Detachable link chain is used for low-speed and light-load power transmission, and for conveyors and elevators of moderate capacity and length. The links can easily be detached and replaced as illustrated. Where the drive is exposed to grit, pintle chains are preferred to the detachable link type. Both types of chain are usually composed of unmachined malleable iron links that can be supplied with integral pin, plate, or scraper attachments. These two types of chain should operate at speeds not exceeding 300 to 400 ft. per min.

Steel block, roller, and silent-link chain are used where an exact overall speed ratio is desired and where the center distance of the shaft axes is too great for the economical or feasible use of toothed gearing. Block chain consists of blocks connected by links, or side plates, and pins, and is run at com-

paratively slow speeds, usually limited to a maximum of 600 ft. per min. Roller chain consists of alternating links L and M held by pins which are locked by cotters. The pins also serve to carry the rollers which bear on the sprocket teeth. Roller chain is used for speeds up to 1500 ft. per min. For power requirements too great for single chain, double-, triple-, or quadruple-

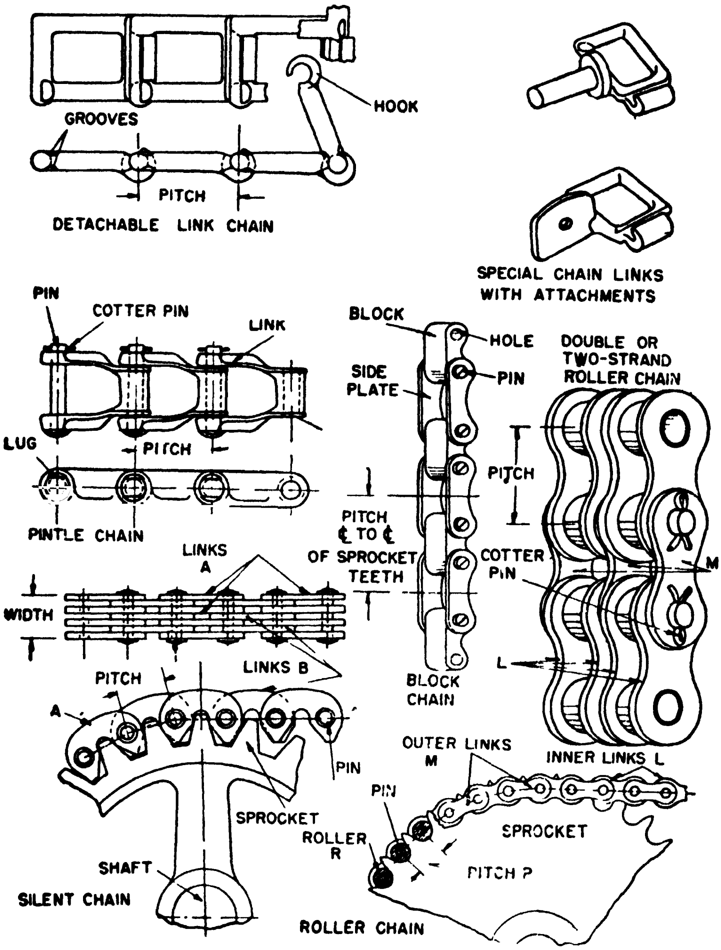


FIG. 14-12. Chain Types and Nomenclature.

strand roller chain may be employed. Both block and roller chain operate on toothed sprockets as illustrated.

14-14. Polygonal Effect in Chain Drives. The centers of adjacent pins of roller chain are connected by straight lines rather than circular arcs. As a consequence the pair of sprockets have an action essentially similar to multi-sided polygons, rather than circles. This is shown diagramatically in Fig. 14-13,

in which a 4-tooth sprocket is shown driving a 6-tooth sprocket by means of a roller chain. For a uniform rotation of the driver, the velocity of the driven member is variable because the driving sprocket radius, as well as the driven sprocket radius, varies throughout the cycle of rotation. This action is referred to as the "polygonal effect" of chain drives and induces a non-constant angular velocity of the driven sprocket, although the total number of revolutions of the driver and driven sprockets are constant per unit of time. The polygonal effect is appreciable only when sprockets with very few teeth rotate at high speeds.

14-15. Roller Chain Selection. The speed ratio of power transmission chain depends upon the number of teeth in the driving and driven sprockets; velocity ratios up to 7:1 are satisfactory. Short-center drives with high-velocity ratios are usually more economical if fine-pitch chain is employed, while narrow large-pitch chain is better adapted to low-ratio long-center drives. The small sprocket should have a minimum of 15 teeth if it serves as the driver; 19 teeth if it is driven. The angle included between the two strands of chains should not exceed 45°. When the speed ratio is less than 2.5, the minimum center distance may be equal to one-half the sum of the sprocket diameters plus the pitch (which provides for tooth clearance); for speed ratios greater than 2.5, the minimum center distance should be equal to the sum of the sprocket diameters. On the other hand, the maximum center distance should not exceed eighty times the pitch.

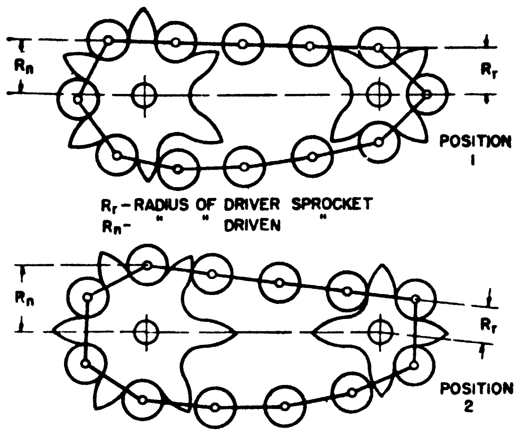


Fig. 14-13. Polygonal Effect in Chain Drives.

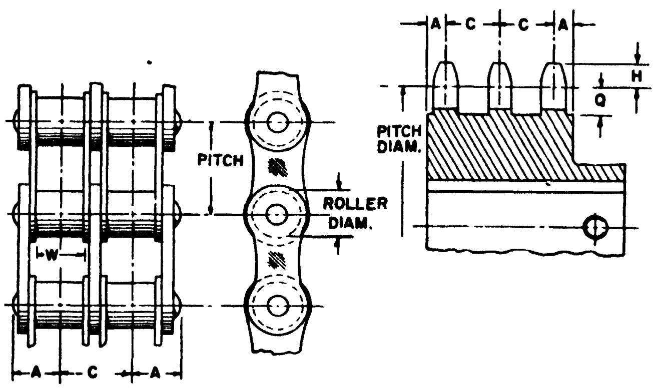
Drives in which the axes of the driving and driven shafts lie in approximately the same horizontal plane are preferred to sharply inclined or vertical drives; the latter require the chain to be run taut, which may involve frequent adjustment of center distances as the chain elongates on account of wear. For horizontal drives with comparatively short centers, the slack portion of the chain should be below the centerline; with the slack on the upper strand there is some tendency to push the chain out of proper engagement with the teeth of the driven sprocket.

Fig. 14-14 shows the essential dimensions of representative roller chain drives; Table 14-11 shows suggested selections of chain pitch, number of strands, and minimum sprocket tooth numbers for various power requirements for three standard motor speeds. If possible, sprockets with the greatest number of teeth

should be used, although any increase in the size of the driving sprocket means a larger driven sprocket, and a longer and more expensive chain. The length of a roller chain, in terms of the number of chain pitches, is approximately given by:

$$L = 2C + \left(\frac{N + n}{2}\right) + \left[\frac{0.0257(N - n)^2}{C}\right] \tag{14-17}$$

where L is the length, and C the center distances, in pitches, and N and n are the numbers of teeth in the large and small sprockets. The next even integral number greater than L should be used to obtain an even number of pitches in



DIMENSIONS—INCHES

Pitch P	D	W	C	A	H	Q
½	0.312	0.312	0.563	0.313	0.150	0.318
⅝	0.400	0.375	0.707	0.384	0.188	0.406
¾	0.469	0.500	0.892	0.493	0.230	0.465
1	0.625	0.625	0.762	0.643	0.300	0.611

FIG. 14-14. Proportions of Roller Chain.

the chain length, since an odd number of pitches will require an offset link in the chain and is undesirable.

The pitch diameter of a power transmission chain is given by

$$D = \frac{Pn}{\pi} \tag{14-18}$$

where D is the pitch diameter, P the pitch, and n the number of teeth. The maximum allowable pitch P_m for a given speed in revolutions per minute may be found from:

$$P_m = \sqrt[3]{\left(\frac{900}{\text{RPM}}\right)^2} \tag{14-19}$$

Table 14-11.—Stock Roller Chain Drives

Motor Speed—RPM

	1750-1800				1160-1200				870-900			
	Chain		Sprocket		Chain		Sprocket		Chain		Sprocket	
HP	Size	Strand*	No. Teeth	Max. Bore	Size	Strand*	No. Teeth	Max. Bore	Size	Strand*	No. Teeth	Max. Bore
10	1/2	*T*	18	1 5/8	5/8	*D*	17	1 13/16	5/8	*D*	21	2 3/8
15	1/2	*T*	23	2 1/4	5/8	*D*	21	2 3/8	3/4	*D*	17	2 1/4
20	5/8	*T*	18	2	5/8	*T*	21	2 3/8	3/4	*D*	19	2 5/8
25	5/8	*Q*	20	2 1/4	5/8	*Q*	22	2 9/16	3/4	*T*	21	2 7/8
30	5/8	*Q*	22	2 9/16	3/4	*T*	21	2 7/8	3/4	*T*	21	2 7/8
40	5/8	*H*	22	2 9/16	3/4	*T*	20	2 3/4	3/4	*Q*	22	3 1/8
50	5/8	*H*	22	2 9/16	3/4	*T*	22	3 1/8	1	*T*	19	3 1/2
60	5/8	*J*	23	2 13/14	3/4	*H*	23	3 3/8	1	*T*	21	3 7/8
75	5/8	*J*	25	2 15/16	3/4	*H*	24	3 1/2	1	*Q*	21	3 7/8
100	5/8	*K*	26	3 1/8	3/4	*J*	24	3 1/2	1	*H*	22	4 1/4

* Number of strands: $D = 2$, $T = 3$, $Q = 4$, $H = 6$, $J = 8$, $K = 10$.

Table 14-12 gives the transmission capacities of several sizes of single-strand roller chain; double- and triple-strand chain will transmit approximately two and three times the power, respectively, that single-strand chains will carry safely. The listed capacities are suitable for drives based upon average 10-hour day operation; for average 24-hour service, the actual load for which the chain is selected should be 25% greater than the transmitted load. For abnormal service and moderate shock loads, the transmitted load should be increased 25% for 10-hour day, and 50% for 24-hour day operation. For heavy shock loads, the load should be increased 50% for 10-hour day, and 75% for continuous operation. Properly designed casings, to retain the lubricant and to protect the drive from dust and grit, should be provided. The drives indicated by an asterisk * should be equipped with rapid drip or with pump lubrication, and therefore require an oil-tight casing; the others are designed for drip lubrication, and a casing that serves principally as a guard will be satisfactory. Data on commercial casings are available in manufacturers' catalogs.

TABLE 14-12.—POWER TRANSMISSION CAPACITIES OF ROLLER CHAIN, PER STRAND, AT MOTOR SPEEDS OF 870, 1160, AND 1750 RPM.

No. Teeth	½" Pitch			⅝" Pitch			¾" Pitch		1" Pitch
	870	1160	1750	870	1160	1750	870	1160	870
15	3.1	3.6	4.2	5.4	6.2	7.0	9.0	10.1	17.3
16	3.3	3.7	4.3	5.7	6.4	7.1	9.4	10.4	17.7
17	3.5	4.0	4.6	6.1	6.8	7.5*	10.0	11.0	18.7
18	3.7	4.2	4.8	6.4	7.2	7.9*	10.6	11.5	19.7
19	3.9	4.4	5.0	6.8	7.5	8.2*	11.2	12.1	20.7
20	4.1	4.6	5.3	7.1	7.9	8.6*	11.6	12.7	21.7
21	4.3	4.9	5.5*	7.5	8.3	8.9*	12.2	13.3*	22.6*
22	4.5	5.1	5.7*	7.8	8.7	9.2*	12.7	13.8*	23.5*
23	4.7	5.3	5.9*	8.2	9.0	9.5*	13.3	14.4*	24.4*
24	4.9	5.5	6.2*	8.5	9.4	9.7*	13.8	14.9*	25.3*
26	5.2	6.0	6.6*	9.2	10.1*	10.2*	14.8	15.9*	26.9*
28	5.6	6.4	6.9*	9.8	10.7*	10.6*	15.9*	16.8*	28.5*
30	6.0	6.8	7.3*	10.4	11.4*	11.0*	16.9*	17.7*	29.9*
32	6.4	7.2*	7.6*	11.1	12.0*	11.2*	17.8*	18.5*	31.2*
36	7.2	8.0*	8.1*	12.3*	13.1*	11.4*	19.7*	19.9*	33.4*

Sprockets are commercially obtainable with the tooth numbers given in Table 14-12; in addition, stock sprockets with 40, 42, 45, 48, 52, 54, 60, 68, 70, 76, 80, 84, 96, 102 and 112 teeth are available. Sprockets may be made of cast iron or forged steel, or may consist of a cast iron hub with a steel sprocket plate bolted to it. Split hub sprockets, to permit easy removal of the sprocket from the shaft, are also available.

Block and roller chain in standard pitches are also obtainable in 18-8 chrome-nickel stainless steel and in bronze, for service where resistance to corrosion is of importance. Stainless steel chain will withstand alternate drying and spraying with 3% brine solution for 200 hours, and is extensively used on food-handling machinery and for marine applications; it also has excellent heat-resisting qualities, since the scaling temperature is in the neighborhood of 1650° F. Bronze chain is lower in cost than stainless steel, but does not have as effective corrosion-resistance and wear characteristics.

Example 14-8. Select a roller chain for the gyratory crusher, of Example 14-3.

Solution. The drive nominally requires 50 HP at 870 RPM, but is subjected to shock loads. Since the material comes to the crusher in periodic lots, the service is more or less intermittent. For these reasons, it may be advisable to increase the transmitted load about 20%, and select chain on the basis of a 60-HP, 870-RPM drive requiring a 1-in. pitch, triple-strand chain with a 21-tooth sprocket. The speed ratio of the motor and crusher is 870/300, or 2.9. The number of teeth in the driven sprocket is then 2.9 × 21, or 60.9, say 60 teeth. From Eq. 14-19, the maximum pitch for a speed of 870 RPM is:

$$P_m = \sqrt[3]{\left(\frac{900}{870}\right)^2} = 1 \text{ in. (approximately)}$$

A reference to Table 14-12 indicates that fast drip or pump lubrication is necessary for a 21-tooth 1-in. pitch sprocket operating at 870 RPM.

The minimum center distance should be equal to the sum of the pitch diameters of the sprockets, or

$$C = \frac{21 \times 1}{\pi} + \frac{60 \times 1}{\pi} = 25.8 \text{ in.}$$

The maximum center distance should not exceed 80 pitches, or 80 in. The chain length, for any center distance within this range, may be obtained from Eq. 14-17.

The chain pull will be equal to the required effective pull E at the pitch line of the sprocket; since there is no appreciable tension in the slack side of the chain, the bearing reaction R can be taken as equal to E. The velocity V of the chain is 21 × 1(870/12), or 1520 ft. per min., and the effective pull, from Eq. 14-5, is

$$E = \frac{33{,}000 \times 50}{1520} = 1086 \text{ lbs.}$$

Since the bearing pull R in a chain drive is equal to the effective force E, the value of R for this drive very closely approximates the bearing pull R of 1106 lbs. given for the leather belt.

14-16. Silent Chain Selection. So-called silent chain is composed of alternate flat steel links A and B, Fig. 14-12, connected by pins. The links have straight faces, which are in contact with the faces of the sprocket teeth. The links rotate slightly on the pins as the chain bends around the sprocket. Present-

day commercial silent chains have pins either bushed in the links, or else have one flat-faced pin and one crowned pin; these roll on each other as the chain bends around the sprocket. Silent chain is specified in inches of width. For a given pitch a chain 2 in. wide will transmit approximately twice as much power as a chain 1 in. wide.

TABLE 14-13.—STOCK SILENT CHAIN DRIVES ¾-IN. PITCH 6-IN. WIDE

Motor Speeds						Drive Data			
1160 RPM		870 RPM		690 RPM					
Driven RPM	HP	Driven RPM	HP	Driven RPM	HP	Min. Center Dist.	No. of Teeth Driver	No. of Teeth Driven	Chain Length Inches
533	54	396	44	317	38	14½	23	50	57
513	42	381	35	305	31	13	19	43	51
487	50	361	40	290	34	14½	21	50	57
483	60	358	48	288	42	17	25	60	67½
465	47	344	38	276	33	14½	20	50	57
444	54	330	44	265	38	17	23	60	67½
441	42	327	35	262	31	14½	19	50	57
427	60	316	48	254	42	19	25	68	75
406	50	301	40	242	34	17	21	60	66
392	54	291	44	233	38	19	23	68	75

Silent chain can be operated at speeds up to 2500 ft. per min., although speeds appreciably below this value are usually more economical from the standpoint of chain life. Pitch diameters and chain lengths may be obtained from Eqs. 14-18 and 14-17. Table 14-13 is a portion of a catalog table showing available stock drives which may be obtained on short notice; it is advisable to make stock selections whenever possible to facilitate delivery. Table 14-14 gives the transmission capacities of several sizes of silent chain 1 in. wide. Chains with ⅜-in. pitch are available in widths from ⅜ to 2½ in.; ½-in. pitch chains from ½ to 3 in.; and ¾-in. chains from ¾ to 6 in.

Example 14-9. Select a silent chain for the gyratory crusher of Example 14-3.

Solution. From the discussion given in the solution to Example 14-8, the chain should have a capacity of 60 HP at 870 RPM. A reference to Table 14-13 shows that the maximum size of stock chain is ¾ × 6 in. for a driven speed of 301 RPM, and has a capacity of only 40 HP. This capacity, however, is underrated about 20%, so the actual capacity, as compared to the overrated load requirement, is about 50 HP. The difference of 10 HP is too great to be ignored, and a stock drive cannot be used. A ¾-in. pitch chain

and a 25-tooth sprocket (from Table 14-14) has a capacity of 10.1 HP at a driving speed of 870 RPM from a 1-in. wide chain, and a chain 6 in. wide will suffice. The number of teeth in the driven wheel is equal to the product of the number of teeth in the driver and the velocity ratio, or 2.9 × 25, or 72.5, say 72 teeth. The pitch-line velocity of the chain is (25 × 0.75)870/12, or 1360 ft. per min., and the chain pull is

$$E = \frac{33{,}000 \times 50}{1360} = 1210 \text{ lbs.}$$

It would probably be advisable to consult the manufacturer or supplier for suggestions regarding this drive; it is probable that a 1-in. pitch chain will be more efficient and result in less bearing load. It should be carefully noted that Tables 14-11 and 14-14 are typical samples, and include only a small fraction of the chain drives commercially available; manufacturers' and suppliers' catalogs should be consulted for actual application.

TABLE 14-14.—POWER TRANSMITTING CAPACITIES OF SILENT CHAINS 1-IN. WIDE

No. Teeth	Motor Speed, RPM							
	3/8" Pitch			1/2" Pitch			3/4" Pitch	
	1800	1200	900	1800	1200	900	1200	900
17	2.9	2.2	1.7	5.2	4.0	3.3	7.8	6.5
18	3.1	2.3	1.8	5.6	4.3	3.5	8.4	7.0
19	3.3	2.5	2.0	6.0*	4.6	3.7	8.9	7.4
20	3.5	2.6	2.1	6.4*	4.8	3.9	9.5*	7.9
21	3.7	2.8	2.2	6.7*	5.1	4.1	10.1*	8.3
23	4.1	3.1	2.4	7.5*	5.7	4.6	11.2*	9.3
25	4.5*	3.3	2.6	8.2*	6.2	5.0	12.2*	10.1

* Requires rapid drip or pump lubrication.

PROBLEMS—CHAPTER 14

1. Find the diameter and face width of two friction wheels transmitting motion between parallel shafts, at a center distance of 8 in. The velocity ratio is 5:3. One wheel is faced with leather, the other is of cast iron. The high speed driver rotates at 200 RPM. Which of the two should be leather faced?

2. A pair of shafts rotate at speeds of 160 and 240 RPM, and their axes are at right angles. The slow-speed shaft carries a bevel friction wheel whose maximum diameter is 12 in., whose face width is 3 in., and which drives a corresponding friction wheel on the high-speed shaft. Determine the diameter of the high-speed wheel.

3. Using a 90° shaft angle, will the slow-speed bevel wheel of Problem 2 operate satisfactorily with a high-speed wheel rotating at 300 RPM? Explain the reason for your answer.

4. Select a leather belt for a 5-HP, 870-RPM electric motor if the center distance is 7 ft., the operation and maintenance normal, and the speed of the driven shaft approximately 400 RPM. A paper pulley is used on the motor and the drive is vertical. What is the total

bearing pull based upon the load applied by the belt? If the relative humidity is changed from 70% to 30% what will be the effect on the bearing pull?

5. The distance between the bearing centerlines of a 10-HP, 1200 RPM AC squirrel cage motor is 20 in. The right overhanging end of the shaft carries a 10-in. diameter pulley. The distance from the centerline of the right bearing to the centerline of the pulley face is 5 in. The rotor of the motor is midway between the bearings, and the total load due to its weight, the winding weight, the shaft weight, and the unbalanced magnetic pull is 500 lbs. The right bearing is 1½ in. diameter and 3¾ in. long, and the left bearing is 1¼ in. diameter and 3 in. long. The tension ratio of the belt is 3. The motor is bolted to the ceiling, so that the belt pull is vertically downward. The right bearing of this motor shows signs of heating, and it is believed that this is due to too high a unit pressure.

a. What is the unit bearing pressure, in lbs. per sq. in., of projected area?

The foreman in charge of the section of the shop that uses this motor and drive suggests that a roller chain, ⅞-in. pitch, with a 32-tooth sprocket, be substituted for the present belt drive.

b. What is your opinion of this substitution?

c. What effect would this substitution have on the bearing pressure?

The maintenance engineer, in charge of the plant, does not concur entirely with the suggestion of the foreman, but advises instead the substitution of a silent chain using a 32-tooth sprocket, ⅝-in. pitch, for the present drive.

d. What effect would this substitution have on the bearing pressure?

e. What is your opinion as to this substitution?

f. Have you any other suggestions to offer?

6. A medium double belt 7 in. wide is used to transmit power from the flywheel of a 60-HP reciprocating engine to an electric generator. The flywheel has a 5 ft. 6 in. diameter, and the generator pulley has a 13-in. diameter and a 12-in. face. The generator speed is 900 RPM, and the center distance is 28 ft. The engine shafts lie in approximately the same horizontal plane. Investigate the power transmitting capacity of the belt. The service is continuous and the maintenance will be excellent. The generator pulley is made of cast iron.

7. Substitute a vee-belt drive for the leather belt of Problem 6. Make any changes in center distance, sheave diameters, etc., as deemed feasible.

8. Substitute a vee-flat drive for the leather belt of Problem 6, using the flywheel on the engine.

9. Select suitable sprockets and chain for the drive of Problem 6.

10. A countershaft rotating at 300 RPM is to drive a machine spindle at speeds of 100, 200, 500, and 700 RPM, by means of a set of stepped pulleys. No step is to have a diameter less than 10 in. or more than 36 in. The shaft axis center distance is 30 in. Find the diameters for a set of four step-cone pulleys for: (a) a crossed belt, using integral inch diameters for the cone steps; (b) an open belt, using diameters to two decimal places. Give the theoretical belt length for each step.

11. Select a flat rubber belt for the drive of Problem 4.

12. A pair of shafts rotate in the same direction and have parallel axes. The speeds are 690 and approximately 240 RPM. The small driving sprocket has 18 teeth and drives a 1½-in. pitch, single strand, roller chain at a center distance of about 2 ft.

a. Find the number of teeth in the driven sprocket.

b. What is the chain speed in feet per minute?

c. Is this considered satisfactory for a roller chain? Explain, giving references from the text to substantiate your answer.

d. What horsepower may be transmitted?

13. A silent chain transmits power from an 1160-RPM motor to a 400 RPM fan which is in service 24 hours per day. The motor sprocket has 21 teeth, and the chain is ¾-in. pitch, 4 in. wide, with a 24-in. center distance.

a. Is the chain velocity excessive? Explain.

b. What horsepower may be transmitted?

CHAPTER 15

TOOTHED GEARING

15-1. Toothed gearing is often used in preference to belting, friction drives, or chain drives, where moderate or large amounts of power must be transmitted at a constant velocity. Three general types of toothed gearing are classified with respect to the relative position of the axes of the shafts on which the gears are mounted. The first type includes gearing for shafts whose axes are parallel, consisting of spur gearing, internal gearing, rack and pinion, helical gearing, and herringbone gearing. The second type includes gearing for shafts whose axes intersect if prolonged, consisting of straight bevel gearing and spiral bevel gearing. The third type includes gearing for shafts whose axes are neither parallel nor intersecting, consisting of worm gearing, hypoid gearing, and spiral gearing.

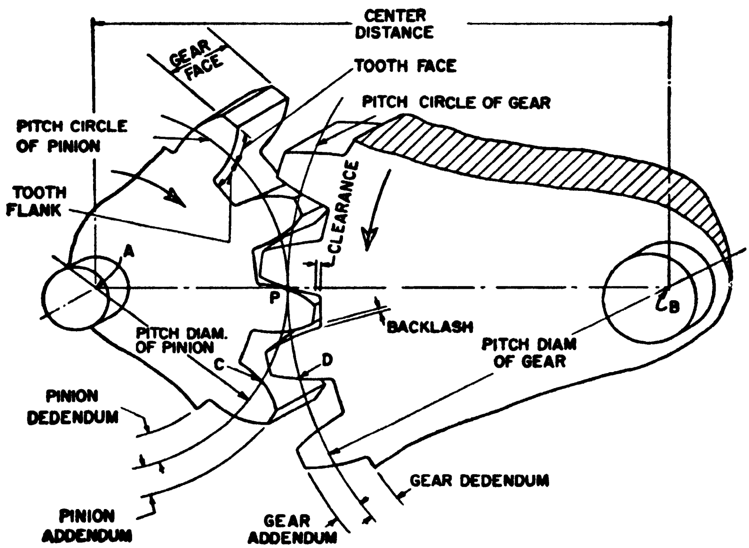


FIG. 15-1. Gear Nomenclature.

PARALLEL AXIS GEARING

15-2. Gear Nomenclature. Spur gearing nomenclature is illustrated in Fig. 15-1. The pitch circles of a pair of spur gears are imaginary circles equiva-

lent to the peripheries of a pair of friction wheels which would operate (disregarding slippage) at the same center distance and velocity ratio as the gears themselves. The velocity ratio of a gear set is the same as that of the corresponding friction wheels, and Eq. 14-1 is applicable. In Fig. 15-1, if the center distance AB is 6 in., and the velocity ratio is 2:1, the pitch circles of the pinion and the gear will have pitch diameters of 4 and 8 in., respectively.

15-3. Pitch. To operate satisfactorily and efficiently, the teeth of mating gears of a set must be of corresponding size, and must be of such shape as to transmit smooth and continuous motion. Two methods are commonly used for tooth size measurement. Circular pitch P_c is the distance from a point on the profile of one tooth to a corresponding point on the profile of the next tooth, measured on the pitch circle, and shown by distance PC on the pinion, or PD on the gear, in Fig. 15-1. Circular pitch is commonly employed for gears having teeth cast to shape. A range of pitches from ½ to 1½ in., inclusive, by ⅛-in. increments, as well as 1¾- and 2-in. pitch gears, are commercially available. The relation of circular pitch and pitch diameter is as follows:

$$D_g = \frac{N_g P_c}{\pi} \tag{15-1}$$

where D_g is the pitch diameter and P_c the circular pitch, in inches, and N_g is the number of teeth in the gear.

If the gear of Fig. 15-1 has 24 teeth and a circular pitch of ⅞ in., the pitch diameter is

$$D_g = \frac{24 \times 0.875}{\pi} = 6.685 \text{ in.}$$

If the pinion has 12 teeth, its pitch diameter will be

$$D_p = \frac{N_p P_c}{\pi} = \frac{12 \times 0.875}{\pi} = 3.342 \text{ in.}$$

(using the subscript "p" to denote the pinion, or smaller member of the set). The center distance for this gear set is one-half the sum of the pitch diameters, or $(6.685 + 3.342)/2 = 5.014$ in.

In many instances, design and construction may be facilitated by specifying center distances in integral inches or in commonly used fractions, thus another method of measuring tooth size, known as diametral pitch, has come into use. Diametral pitch P_d is the ratio of the number of teeth in the gear to its pitch diameter, and is usually used for gears with cut or machined teeth. The relation of diametral pitch and pitch diameter is

$$D_g = \frac{N_g}{P_d} \tag{15-2}$$

Spur gears can be obtained commercially in the following diametral pitches: 1 to 2 by fourths; 2½ to 4 by halves; and 5, 6, 8, 10, 12, 16, 20, 24, 32, and 48.

Gear tooth cutters for special gears are stocked by tool manufacturers for the diametral pitches listed above, and also for the following: 7, 9, 11, 14, 18, 22, 26, 28, 30, 36, and 40.

From Eqs. 15-1 and 15-2, the relationship between circular and diametral pitch is seen to be

$$P_c P_d = \pi \tag{15-3}$$

15-4. Law of Gearing. In order that toothed gears may operate at a constant angular velocity of the driven member for each increment of rotation of the driving member, the tooth curves must be such that the common perpendicular to the profiles at the point of contact will at all times pass through the point of tangency of the pitch circles, or pitch point P of the set, shown in Fig. 15-1. This is known as the Law of Gearing, and governs the shape of the gear tooth profiles. Theoretically, almost any curve may be the basis for the profile of one gear; if the tooth profile of the mating gear is constructed so that the common normal to the point of contact passes through the pitch point, the gear set will operate at a constant angular velocity. Such profiles are said to be conjugate. In practice, only one curve, the involute of a circle, is extensively employed for gear tooth profiles.

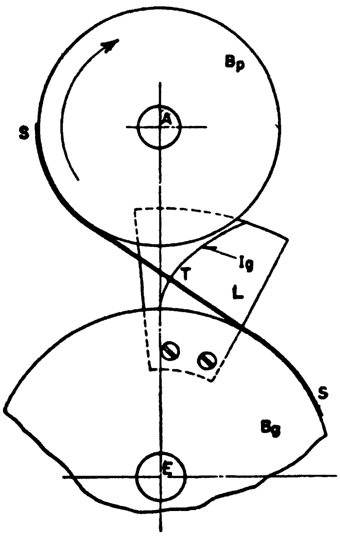


FIG. 15-2. Generation of an Involute of a Circle.

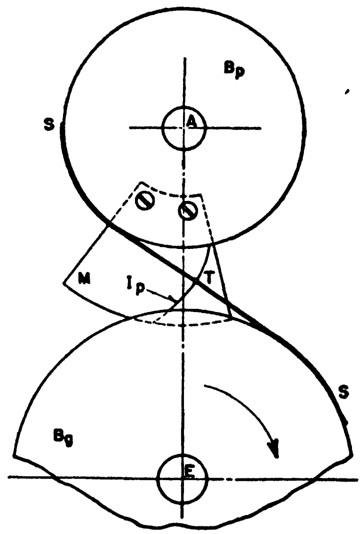


FIG. 15-3. Generation of an Involute of a Circle.

15-5. Involute Tooth Profiles. The involute of a circle is the curve traced by the end of a string or line as it is unwound from the periphery of a circle; this circle is called the base circle of the involute. In Fig. 15-2, the

string SS is attached to disks B_p and B_g, so that B_g turns at a constant angular velocity proportional to the velocity of B_p. The string carries a tracing point T which describes an involute, I_g, on a leaf L attached to and rotating with B_g. In Fig. 15-3, the same tracing point describes another involute I_p on a similar leaf M, attached to and rotating with the disk B_p, which in this case is driven by the disk B_g by means of the string. Fig. 15-4 shows the leaves M and L attached to the fronts of disks B_p and B_g, with the involute I_p and I_g in contact. Although the string SS is removed, the profile I_p will drive the profile I_g at a constant angular velocity, and the action is exactly analogous to the generation of the two involutes. The point of contact T of the involutes is always on the generating line S, and this is called the line of action. The point P, where the line of action cuts the centerline of the two disks, is the pitch point of the gearing; the radii AP and BP are the pitch radii of the pinion and gear, respectively.

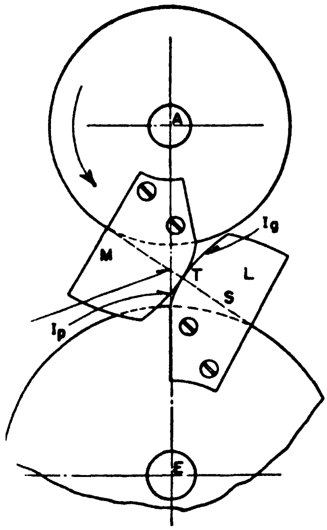


FIG. 15-4. Application of an Involute to Gear Teeth.

Fig. 15-5 shows the application of the involute to spur gearings of standard proportions. The base circle diameters B_p and B_g are obtained by drawing the line of action EPF at an angle θ to a perpendicular through the pitch point P, and drawing the base circles corresponding to diameters B_p and B_g tangent to EPF at E and F. Angle θ is termed the angle of obliquity, or pressure angle of the gearing. The dedendum of either gear of the set is equal to the addendum of the mating gear plus the clearance. The thickness MQ of the tooth measured along the pitch circle of one member, must be equal to the width of the space measured along the pitch circle of the other.

The actual involute may be generated by laying out distances 2-2′, 3-3′, 4-4′, etc., respectively equal to the arc lengths 2-1, 3-1, 4-1, etc., along tangents to the base circles, as illustrated in Fig. 15-5, and drawing the involute through points 1′, 2′, 3′, 4′, etc.

15-6. Tooth Action. Figures 15-6, 15-7, and 15-8 show a complete cycle of involute gear tooth action between a driving pinion and a driven gear. Fig. 15-6 shows tooth A of the pinion entering engagement with tooth B of the gear. The actual contact begins at the point X, where the addendum circle of the gear intersects the line of action, and it is seen that the initial contact takes place near the base of the pinion tooth and at the tip of the gear tooth. Fig. 15-7 shows tooth A and tooth B in contact at the pitch point P. Fig. 15-8 shows the termination of engagement of teeth A and B, at point Y, where the addendum

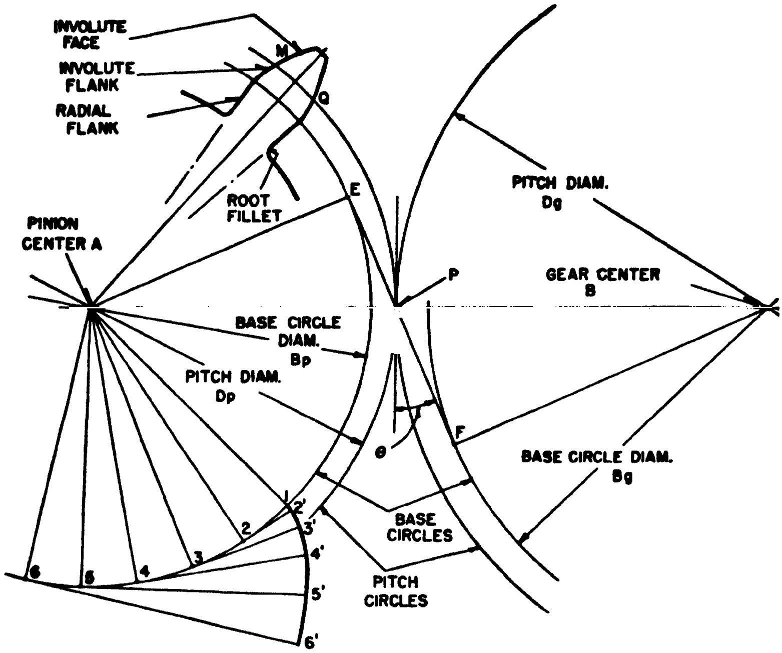


FIG. 15-5. Involute Gear Tooth Construction.

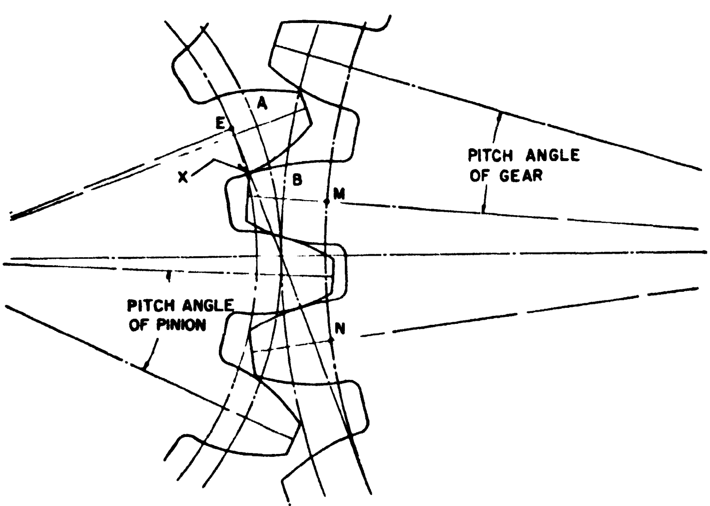


FIG. 15-6. Gear Tooth Engagement—Beginning of Action.

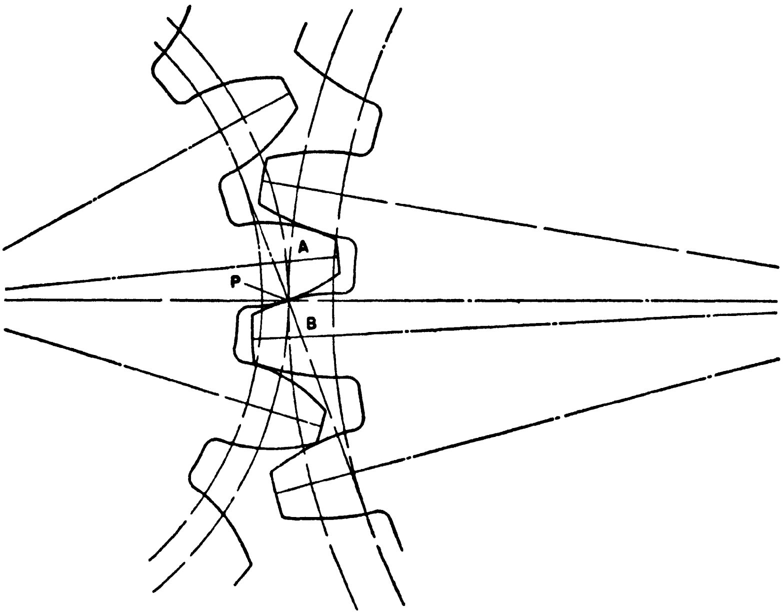


FIG. 15-7. Gear Tooth Engagement—Action at Pitch Point.

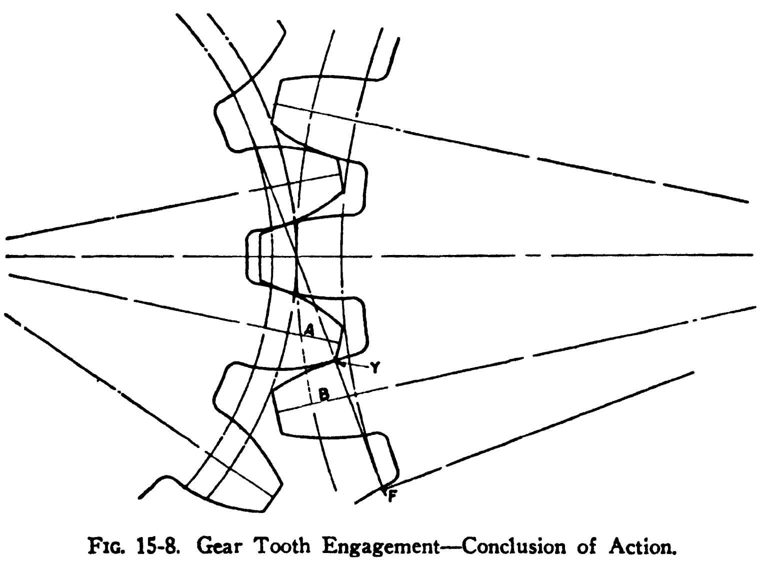


FIG. 15-8. Gear Tooth Engagement—Conclusion of Action.

circle of the pinion intersects the line of action; and it is seen that the contact ends at the tip of the pinion tooth and near the base of the gear tooth. The duration of tooth engagement is determined by the points X and Y in Figs. 15-6 and 15-8, where the addendum circles of the gear and pinion cut the line of action; XY is therefore the effective length of the line of action.

In Figures 15-6 and 15-8, E and F represent the points of tangency of the line of action with the base circles of the pinion and gear. If the gear tooth has a length such that its addendum circle extends past the point E on the pinion, as shown in Fig. 15-9, the gear tooth tip will interfere with or cut into the non-involute portion of the pinion tooth base at the termination of contact.

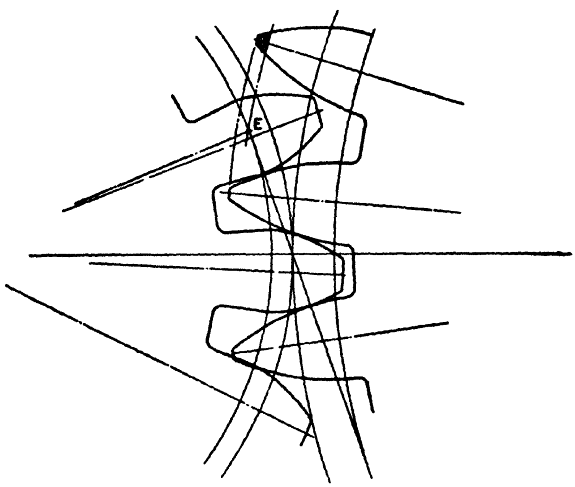


FIG. 15-9. Gear Tooth Engagement—Interference.

Theoretically correct action can begin only at or inside E, and must end at or inside F. The distance from point E on the pinion to the center of the gear is the maximum outer radius of the gear, from the standpoint of correct action; similarly, the distance from point F on the gear to the center of the pinion is the maximum outer radius of the pinion. In Fig. 15-9, the cross-hatched tip of the gear tooth is useless for contact purposes, and, if left unrelieved, will cut into the flank of the pinion tooth and cause interference.

Gear tooth action should be continuous, that is, the second tooth of the driver should engage a tooth on the driver unit at or before the time the first tooth of the driver leaves contact with its driven tooth. Figs. 15-6 and 15-8 show that there are two teeth in engagement at the initial and final stages of contact, although the entire transmitted load is carried by one tooth as it passes

the pitch point, as shown in Fig. 15-7. The engagement ratio may be found by dividing the angle through which one tooth moves in its cycle of engagement, by the angle between the centerlines of adjacent teeth, or, expressed as an equation:

$$\frac{\text{Angle of action}}{\text{Pitch angle}} = \text{Contact ratio} \tag{15-4}$$

The contact ratio may also be obtained, without any layout of tooth profiles, by the following:

$$\text{Contact ratio} = \frac{\text{Length of line of action}}{\text{Base pitch}} = \frac{XY}{MN} \tag{15-5}$$

where the base pitch MN is equal to $P_c \cos \theta$, and the length XY obtained from Figs. 15-6 and 15-8. A contact ratio of 1.5 means that there are two teeth in contact about half the time, with one tooth carrying the load the balance of the time. The contact ratio must be at least 1.00 for continuous action.

There is a certain amount of sliding contact between gear tooth profiles during their cycle of action, beginning with the tooth profiles moving towards each other at the inception of action, decreasing to zero sliding as the profiles meet at the pitch point, and increasing to a second maximum with the tooth profiles moving away from each other at the conclusion of their cycle. Sliding action in which the profiles move towards each other is called approaching action, and that in which they move away from each other is termed receding action. Approaching action is definitely more injurious to the tooth surfaces than receding action, and the ratio of approaching to receding action should therefore always be equal to or less than 1.00.

Action ratios are obtained by the following:

Pinion driving:

$$\frac{\text{Approaching}}{\text{Receding}} = \frac{XP}{PY} \tag{15-6}$$

Gear driving:

$$\frac{\text{Approaching}}{\text{Receding}} = \frac{PY}{XP} \tag{15-7}$$

in which XP and PY are obtained from Figs. 15-6, 15-7, and 15-8. For equal addendum gearing, the action ratio is 1.00, and as a result, the set may be operated with either the gear or the pinion as the driving member. Unequal addendum gearing, similar to that shown in Figs. 15-6, 15-7, and 15-8, is termed irreversible, because the long-addendum element only (in this case the pinion) should serve as the driving member; the short-addendum element should always be the driven member, except in cases where the reversed load is comparatively light.

15-7. Tooth Proportions. In spur gearing of standard proportions, the tooth flanks from the case circle are radial, and the root fillet has a radius equal

to the clearance. The tooth thickness and the tooth space, measured along the pitch circle, are equal. Three types of teeth are commercially important—classified by their angle of obliquity and addendum length.

The 14½° involute gear tooth has an addendum length of 1 in./P_d, a clearance of 0.157 in./P_d, and a consequent dedendum of 1.157 in./P_d. For full involute action, the smallest gear that will operate with a rack is one of 32 teeth. If the teeth are slightly modified at the tips to correct for interference, 12-tooth pinions will operate satisfactorily, although two 12-tooth pinions in contact have a contact ratio of slightly less than 1.0. An 18-tooth pinion is usually considered a minimum for 14½° gearing.

The 20° full length involute gear tooth has an addendum length of 1 in./P_d and a clearance of 0.2 in./P_d. For full involute action, the smallest gear is one with 18 teeth. With slight tip modifications 15-tooth gears operate satisfactorily.

The 20° stub involute gear tooth has an addendum length of 0.8 in./P_d, and a clearance of 0.2 in./P_d. For full involute action, the smallest gear is one with 14 teeth. Pinions with 12 teeth of full involute form operate with but slight interference—not enough to have any appreciable effect upon their satisfactory functioning.

15-8. Materials. Metallic gears with cut teeth are commercially obtainable in cast iron, steel, brass, and bronze, in many sizes and styles, and are generally stocked by manufacturers and supply houses.[23] Cast iron gears with cast teeth may be obtained in the larger tooth sizes, but are used only where the velocity is low, and where smooth action is not particularly important. They are used to a considerable extent for such uses because the larger sizes are much less costly than cut tooth gears.

The limiting pitch-line velocity of commercially cut metallic spur gearing is about 1000 ft. per min.; vibration and noise become excessive beyond this point, but can often be eliminated by using a non-metallic pinion as one unit of the gear set. Non-metallic gears are made of various materials, such as treated cotton pressed and moulded at high pressure, synthetic resins of the phenol type; and rawhide. The first two are oil- and water-resistant; rawhide is affected to some extent by moisture. Rawhide and the treated cotton materials are not entirely self-supporting, and gears of these materials are made with metal shrouds or sideplates at both ends of the teeth, as shown in Fig. 15-12. When using this type in conjunction with a metal gear, a pinion should be selected with a face width large enough to permit the gear to contact only the non-metallic portion of the pinion face, thus avoiding the possibility of metal-to-metal contact. Gears made of phenolic resins require no supporting plates.

15-9. Forces on Gear Teeth. In calculating the strength of gear teeth, a gear or pinion tooth is considered to be subjected to a load L acting along the line of action, as shown in Fig. 15-10. This load L may be resolved into two components: a rotational component F, tending to break the tooth by bending, and a force $F \tan \theta$ tending to separate the gears of the set and to produce a

compressive stress in the tooth. The compression induced by $F \tan \theta$ is usually neglected when the ordinary pressure angles are used.

An inspection of Fig. 15-1 will show that the pinion tooth, although it has the same thickness at the pitchline and the same height as the gear tooth, is substantially smaller at the root, and is consequently not as strong. This difference in strength is a by-product of the change in shape required for satisfactory operation. In determining the tooth strength, therefore, the tooth shape must be considered since it is affected by the number of teeth on the gears.

15-10. Beams of Uniform Strength. In order to analyze the stresses in gear teeth, it is necessary to develop the theory regarding beams of uniform strength. From Chap. 5, for a rectangular beam of width b and depth d, the flexural moment M, in terms of the unit stress S, is $Sbd^2/6$. For a cantilever

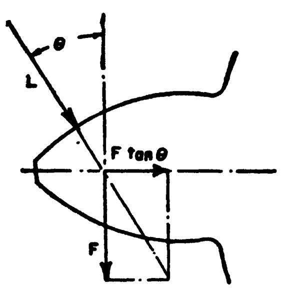


FIG. 15-10. Forces on a Gear Tooth.

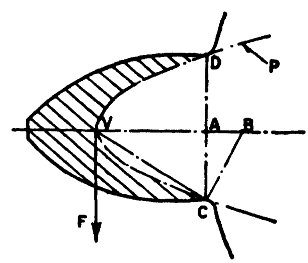


FIG. 15-11. Gear Tooth Strength Determination.

beam with a concentrated load F at the free end, the moment at a distance x from the free end is Fx, and

$$Fx = Sbd^2/6$$

For a beam of uniform section, the stress S varies directly as the distance x; in order to maintain a constant stress S, the width b of the beam section may be varied along the span, to give

$$b = 6Fx/Sd^2 = Kx$$

A beam of uniform strength corresponding to this expression would have a constant depth d; the beam shape in a horizontal plane would be triangular, varying from zero at the point of application of the load, to a maximum width b at the point of support.

Similarly, a cantilever of uniform strength may be developed by holding the stress S and the width b constant, in which case

$$d^2 = 6Fx/Sb = Kx$$

A cantilever beam corresponding to this would have a parabolic shape in a vertical plane, varying from zero at the point of application of the load, to a

maximum depth d at the point of support. If the beam has a shape other than parabolic, any excess of depth d beyond the profile or boundary of the parabola has no effect on the strength. This principle is used in computing the strength of gear teeth.

15-11. Strength of Gear Teeth. A parabola drawn tangent to the tooth profile of a gear tooth with its vertex passing through the point of application of the rotational load will represent a beam of uniform strength within the tooth. This is illustrated in Fig. 15-11, in which the parabola P has its vertex at V and is tangent at points C and D to the root fillets of the tooth. Since the stress in this beam is constant, the weakest portion of the tooth outside the parabolic beam outline (shown cross-hatched in the figure), is essential for the correct tooth action, but it adds nothing to the tooth strength because failure would occur at CD irrespective of the strength of the rest of the tooth.

The moment M at section CD is equal to Fm, where m is the distance or moment arm VA. If b represents the face width of the gear, and d the distance CD, in Fig. 15-11, the flexure equation becomes:

$$Fm = Sbd^2/6$$

or

$$6F/Sb = d^2/m$$

This expression contains two variables, d^2 and m, which may be eliminated as follows: In Fig. 15-11, BC is drawn perpendicular to VC, and distance AB is equal to n. Triangles VAC and CAB are similar, and

$$AB:AC = AC:VA$$

or

$$n \times \frac{d}{2} = \frac{d}{2} \times m$$

from which

$$m = 4n/d^2$$

Substituting for m in the flexure equation,

$$6F/Sb = d^2/(d^2/4n) = 4n$$

For tooth profiles of gears of the same number of teeth, but of different pitches, distance n will vary in proportion to the pitch. If the distance AB in Fig. 15-11 is 0.370 in., representing a 1-pitch gear, the profile of a 2-pitch gear of the same number of teeth will have a distance AB equal to 0.185. If both sides of the above expression are multiplied by P_d,

$$FP_d/Sb = 2nP_d/3$$

For all tooth profiles of gears of the same number of teeth in a particular system, the right hand member of this expression is a constant. This constant is known as the Lewis Factor Y. The expression can then be written as

$$S = \frac{FP_d}{bY} \qquad (15\text{-}8)$$

where F is the transmitted rotational load, b the face width of the gear, Y the Lewis Factor, P_d the diametral pitch, and S the induced unit tensile stress.

The Y factor for any type of tooth may be found by constructing the tooth profile to scale, drawing the inscribed parabola, measuring distance n, and then finding Y equal to $2nP_d/3$. While this procedure may be necessary for special tooth forms, the Y factors for the three standard forms of teeth can be obtained from the following, where N represents the number of teeth:

$$14\tfrac{1}{2}^\circ \text{ Involute} \qquad Y = 0.39 - 2.15/N \qquad (15\text{-}9)$$

$$20^\circ \text{ Full length involute} \qquad Y = 0.484 - 2.85/N \qquad (15\text{-}10)$$

$$20^\circ \text{ Stub involute} \qquad Y = 0.525 - 2.64/N \qquad (15\text{-}11)$$

15-12. Design for Strength. In design, the safe strength of the tooth should be equal to or greater than the transmitted load. The transmitted rotational load is found from Eq. 14-5, which may be rewritten as

$$F = \frac{33{,}000\,\text{HP}}{V_m} \qquad (15\text{-}12)$$

where V_m is the pitch line velocity, in feet per minute. Design stresses for gearing can be obtained from Table 15-1, and are based upon the ultimate tensile strength of the materials used, and a factor of safety of 3 or more. In addition, some allowance is made for the gear velocity, for it is recognized that small inaccuracies in tooth shape, which are almost unavoidable in commercial gearing, give rise to accelerations and decelerations which have a decided effect upon the induced stress. It is customary to use the values for S given in Table 15-1 for safe stresses at zero pitch line velocity, and to modify these values by a suitable velocity factor K for higher speeds.

TABLE 15-1

Material	S = Allowable Stress, psi.
Non-metallic materials	6,000
Cast iron	8,000
Semi-steel and bronze	12,000
Cast steel	15,000
Machine and forged steel	16,000
Hardened steel	30,000
Case-hardened alloy steel	50,000

For pitch-line speeds of 1000 ft. per min. and under, which is the ordinary range of speed for commercially cut metallic gearing:

$$K = 600/(600 + V_m) \qquad (15\text{-}13)$$

For non-metallic gearing:

$$K = 0.25 + 150/(200 + V_m) \qquad (15\text{-}14)$$

Gear face widths of considerable size are likely to be subjected to a non-uniform pressure distribution and the strength equation may give misleading results. Very narrow faces, however, are not economical of material. Face widths for stock gears of cast iron and steel, with cut teeth, are shown in Table 15-3. The effective face width of non-metallic pinions with metallic

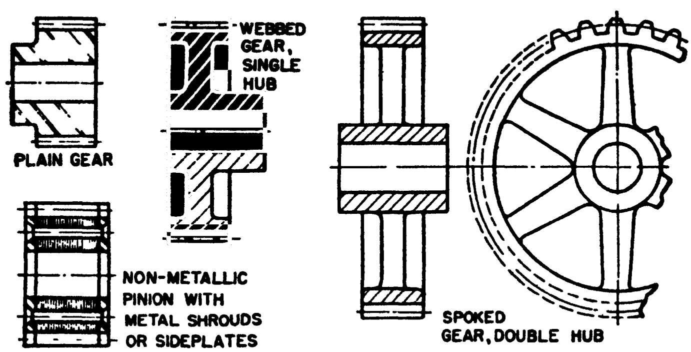


FIG. 15-12. Spur and Helical Gear Construction.

shrouds, shown in Fig. 15-12, is usually ¼ in. greater than the face width of a metallic gear of the same pitch, so that contact with the shrouds is eliminated. The total length of a non-metallic pinion is from ½ to 1 in. greater than the effective face width. For special design, where stock gears are not used, it is recommended that the face width b of gears should lie between $9.5/P_d$ and $12.5/P_d$.

The safe strength of a gear may be expressed as:

$$F_s = SKbY/P_d \qquad (15\text{-}15)$$

where F_s must be equal to or greater than the transmitted load F from Eq. 15-9.

15-13. Design for Wear. Gear sets subjected to continuous service may lose their utility because of excessive wear rather than tooth breakage. Gears may wear excessively because of improper or insufficient lubrication, or because of the presence of foreign particles in the lubricant causing abrasion of the

tooth surfaces. Excessive pressure on the surfaces may cause flaking or pitting at or near the middle of the profile. Properly applied clean lubrication may eliminate some of this trouble, but the teeth must be of such proportions that compressive fatigue failure of the material will not occur.

The limiting load for wear is the allowable load beyond which comparatively rapid wear may be expected to take place, and is given by

$$F_w = D_p b Q W \tag{15-16}$$

where F_w is the limiting load for wear, in pounds, D_p the pitch diameter of the smaller gear, inches, Q the velocity ratio factor, W the material combination constant, and b the face width of gear, inches. This expression has been developed on the basis of the load that may be carried by tooth contact surfaces of various degrees of hardness and curvature. The factor Q is introduced to take care of the degree of curvature of the mating teeth, and is found from:

$$Q = \frac{2N_g}{N_g + N_p} \tag{15-17}$$

where N_g and N_p are the number of teeth in the gear and pinion.

The material combination factor W depends upon the degree of hardness of the tooth materials, and takes into account the cold-working received by the more plastic material from the harder mating material. Representative values of W for $14\frac{1}{2}°$ involute gearing are obtained from Table 15-2.

TABLE 15-2

Cast iron pinion and gear Semi-steel pinion and gear Non-metallic pinion, metal gear	$W = 190$
Machine steel pinion, cast iron or semi-steel gear	$W = 110$
" " " phosphor bronze gear	$W = 90$
" " " machine steel gear	$W = 75$
" " " cast steel gear	$W = 50$
Hardened steel pinion, cast iron or semi-steel gear	$W = 150$
" " " phosphor bronze gear	$W = 135$
" " " machine steel gear	$W = 110$
" " " hardened steel gear	$W = 250$
" " " cast steel gear	$W = 95$

For 20° involute gearing, the values of W should be increased by ⅓.

15-14. Commercial Spur Gears. Cast iron and steel stock gears may be obtained from manufacturers in plain, webbed, and spoked types with either single or double hubs, as shown in Fig. 15-12. Gears with the following numbers of teeth are available: 12, 13, 14, 15, 16, 18, 20, 22, 24, 25, 28, 30, 32, 35,

36, 40, 42, 45, 48, 50, 54, 56, 60, 64, 72, 80, 84, 90, 96, 100, 108, and 120. Gears with any integral number of teeth can be obtained on special order.

TABLE 15-3.—PROPORTIONS OF COMMERCIAL SPUR GEARS

Pitch	Face Width, Inches	Min. Bore, Inches
16	½	⅜
12	¾	½
10	1	⅝
8	1¼	¾
6	1½	1
5	1¾	1¹⁄₁₆
4	2	1⅛
3	3	1⁵⁄₁₆
2½	3½	—
2	4½	—
1½	6	—

Example 15-1. A pair of 14½° involute 4-pitch gears have 18 and 48 teeth, with a face width of 3 in. The pinion is made of semi-steel, with cut teeth, and rotates at 720 RPM. The gear is made of cast iron. Find the horsepower that can be transmitted if the service is intermittent.

Solution. As the service is intermittent, the strength of the teeth need be the only consideration. The pitch line velocity V_m is

$$V_m = \frac{\pi \times N \times \text{RPM}}{12 P_d} = \frac{\pi \times 18 \times 720}{12 \times 4} = 848 \text{ ft. per min.}$$

From Table 15-1, the allowable stresses at zero velocity are 12,000 psi. for the semi-steel pinion, and 8000 psi. for the cast iron gear. The velocity factor K, from Eq. 15-3, is

$$K = 600/(600 + 848) = 0.414$$

The tooth shape factor, from Eq. 15-9, for the 18-tooth pinion, is

$$Y = 0.39 - (2.15/18) = 0.271$$

and for the 48-tooth gear

$$Y = 0.39 - (2.15/48) = 0.345$$

With a face width b of 3 in., and a pitch P_d of 4, the safe strength F_s, from Eq. 15-15, for the pinion, is

$$F_s = 12{,}000 \times 0.414 \times 3 \times 0.271/4 = 1010 \text{ lbs.}$$

and for the gear

$$F_s = 8000 \times 0.414 \times 3 \times 0.345/4 = 855 \text{ lbs.}$$

The gear is thus the weaker of the two elements as the greater strength provided by its tooth shape is not sufficient to compensate for the weaker material. The power that can be transmitted is found from Eq. 15-12 to be

$$HP = \frac{F_s V_m}{33{,}000} = \frac{855 \times 848}{33{,}000} = 22, \text{ approximately}$$

Example 15-2. A pump rotates at about 190 RPM, and requires about 4½ HP for operation. The unit is to be driven by a 5-HP 870-RPM motor through the medium of 14½° involute spur gearing. Select the gearing, specifying stock gears if possible.

Solution a. The required velocity ratio is 870/190, or 4.57. By selecting gear tooth numbers from section 15-14, it is found that the following will closely approximate this ratio: 64/14, 72/16, or 84/18. Since a 15-tooth pinion is usually considered a minimum for 14½° involute gearing, the 72/16 combination will be the first tentative selection. If both gear and pinion are to be made of cast iron, the latter will be the weaker of the two and will serve as the basis for the design.

It is possible to devise a rational equation for the selection of a suitable pitch, but when stock gears are to be employed, it is usually easier to assume a pitch and check the power transmitting capacity of the unit. In this case assume a 6-pitch, 1½-in. face, cast iron pinion and gear. For cast iron, S is equal to 8000 psi., from Table 15-1. The pitch-line velocity of the pinion is found from

$$V_m = \frac{\pi \times 16 \times 870}{6 \times 12} = 607 \text{ ft. per min.}$$

The velocity factor K, from Eq. 15-13, is

$$K = 600/(600 + 607) = 0.498$$

The tooth-shape factor, from Eq. 15-9, is

$$Y = 0.39 - (2.15/16) = 0.256$$

The safe load which can be transmitted, from Eq. 15-15, is

$$F_s = \frac{8000 \times 0.498 \times 1.5 \times 0.256}{6} = 255 \text{ lbs.}$$

The allowable horsepower this set will transmit, from Eq. 15-12, will be

$$\text{HP} = \frac{255 \times 607}{33{,}000} = 4.7$$

which will be satisfactory from the standpoint of strength.

If the pump is to operate continuously, it will be advisable to check the drive for wear. The pitch diameter D_p of the pinion is 16/6 or 2.667 in.; the tooth shape curvature factor, from Eq. 15-17, is found to be

$$Q = \frac{2 \times 72}{72 + 16} = 1.64$$

The material combination factor W, from Table 15-2, is assumed as 190 for cast iron units. The limiting load for wear, from Eq. 15-16, is

$$F_w = 2.667 \times 1.5 \times 1.64 \times 190 = 1250 \text{ lbs.}$$

indicating that an ample margin of safety exists as far as wear is concerned.

The drive velocity, 607 ft. per min., is not excessive, and the unit will probably operate with comparatively little noise; for illustrative purposes, however, the drive will be re-designed for a non-metallic pinion and a cast iron gear.

Solution b. Assume a 5-pitch, 2-in. face, 16-tooth fabrol pinion. For a non-metallic pinion, S is equal to 6000 psi., from Table 15-1. The pitch-line velocity is

$$V_m = \frac{\pi \times 16 \times 870}{5 \times 12} = 730 \text{ ft. per min.}$$

The velocity factor K, from Eq. 15-14, is

$$K = 0.25 + 150/(200 + 730) = 0.411$$

The tooth-shape factor Y is the same as in Solution *a*; the effective face width of the pinion is taken equal to the face width of the gear, or 1¾, although its actual face width, from Table 15-4, is equal to 2 in. The safe load which can be transmitted, from Eq. 15-14, is

$$F_s = \frac{6000 \times 0.411 \times 1.75 \times 0.256}{5} = 221 \text{ lbs.}$$

The allowable horsepower this set will transmit will be

$$\text{HP} = 221 \times 730/33{,}000 = 4.9$$

which is satisfactory from the standpoint of strength. A check of the limiting load for wear will show that its value is higher than that of Solution *a*, and therefore satisfactory. While the drive with this non-metallic pinion has a higher pitch-line velocity than the cast iron gearing, it will probably operate much more quietly, and reduce any shock tendencies.

15-15. Internal Gearing. An internal gear, or annular, and a pinion are shown in Fig. 15-13. An annular may be described as a spur gear turned inside out; the addendum extends from the pitch line to the internal diameter or addendum circle, and the dedendum lies outside the pitch diameter. The internal diameter is obtained by subtracting twice the addendum from the pitch circle. Standard spur gear addenda and dedenda are usually used for annular gearing.

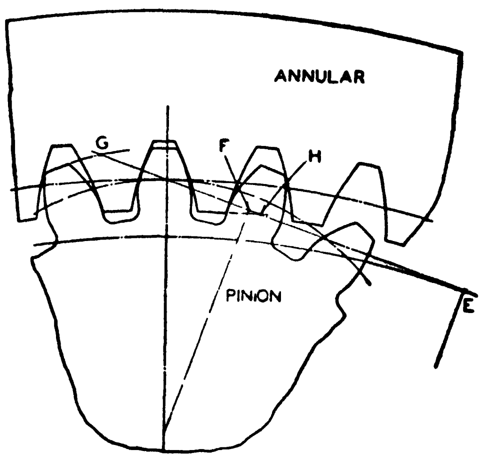


FIG. 15-13. Internal Gearing.

Compared to external spur gearing, the internal gear drive has several advantages. Shafts can be made to rotate in the same direction without the use of an idler gear, annular teeth are stronger than spur gear teeth of the same pitch and tooth number, and an annular gear will provide its own guard, as illustrated in Fig. 15-14. Internal gearing can operate at a much smaller center distance than spur gearing of the same ratio and proportions, since the distance between the axes of the annular and the pinion is equal to the difference of their pitch radii.

For satisfactory operation without considerable modification of tooth shape, the smallest permissible difference between the number of teeth in the annular and the number of teeth in the pinion is 12 for the 14½° involute form, and 7 for the 20° stub-tooth involute form. Even with these differences, however,

some interference may result, as illustrated in Fig. 15-13. F is the point of tangency of the base circle of the pinion and the line of action, and it is seen that the addendum circle of the gear intersects the extension of the line of action at H, outside F, indicating some interference. G represents the other end of the line of action, which can theoretically be extended without limit. Interference of the annular teeth is removed automatically if the teeth are cut on a gear shaper.

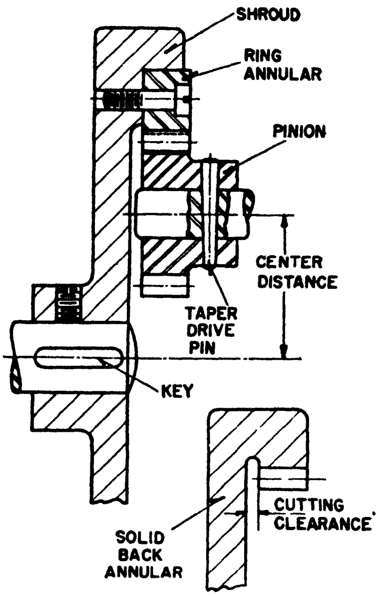


FIG. 15-14. Internal Gearing Construction.

If both members of an internal gear drive are made of like materials, the strength is limited by the pinion. The strength of the annular teeth may be determined by Eq. 15-15, using the following values of Y:

14½°	0.40
20° Full length	0.50
20° Stub	0.55

The durability of annular gearing is determined by calculating the limiting load for wear, F_w, as in spur gearing. Computed values, however, will be on the safe side since there is less relative sliding between the teeth in an internal gear set than in the external spur type.

Internal gears may be obtained in either the "ring" or the "solid-back" types shown in Fig. 15-14, in various pitches from 1 to 16, and in a variety of tooth numbers from 24 to 200. The ring type gear should be seated in a machined recess in a shroud, and screwed and pinned, or keyed, in place. The solid-back type should have a machined recess at least ⅛ in. deep, as illustrated, to provide cutting clearance for gear shaper cutters.

15-16. Spur Gearing for Reciprocatory Motion. A rack, Fig. 15-15, is a spur gear with a pitch circle whose radius approaches infinity as a limit. It is employed for converting rotary to reciprocating motion, or vice versa, and is extensively used for adjustable sliding members. Racks of rectangular section can be purchased commercially and attached to frames in either of the two ways shown (screw A or screw B). Some manufacturers also supply cast iron racks with lugs for bolting to supports or frames.

15-17. Helical Gearing. For high pitch-line velocities and heavy loads, some form of "twisted-tooth" gear is generally used. Two important types are helical gears and double-helical or herringbone gears. These units are frequently employed in geared speed reducers, as shown in Figs. 15-28 and 15-29. Both helical and herringbone gears are essentially spur gears with teeth twisted across the face in the form of a helix about the axis of rotation.

When spur gear teeth engage, the contact extends across the entire tooth on a line parallel to the axis of rotation, and may result in noise and shock at high speeds. In helical gear engagement, contact begins at one end of the entering tooth and gradually extends along a diagonal line across the tooth face as the

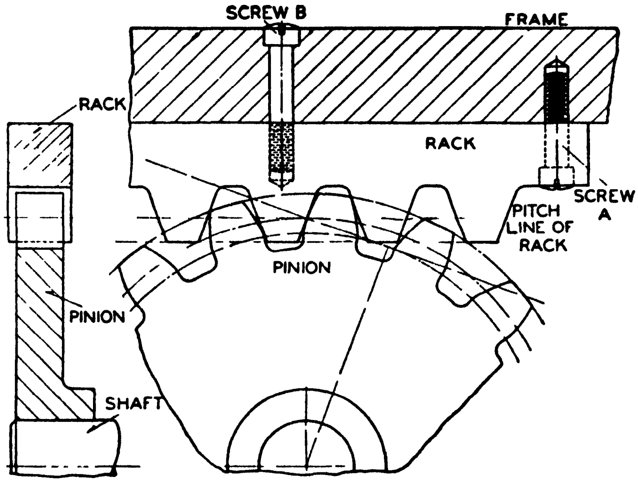


Fig. 15-15. Rack and Pinion.

gears rotate. The nature of the contact is such that with sufficient face width, two or more teeth are in contact and are carrying the load at all times. Helical gears are therefore used for transmission ratios as high as 10:1, and at pitch-line velocities up to 2000 ft. per min. for commercially cut units. Herringbone gear

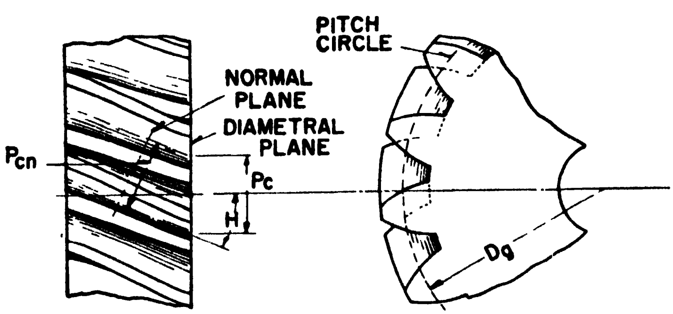


Fig. 15-16. Helical Gear Nomenclature.

sets of special design have been successfully operated at pitch-line speeds of 12,000 ft. per min.

Tooth elements of helical gears are similar to those of spur gears—the most important being illustrated in Fig. 15-16. The helix angle H of the tooth is

measured between the line tangent to the tooth helix at the pitch circle and the shaft axis. In any pair, the gears have teeth with mating right-hand and left-hand helices. The usual method of tooth measurement is by diametral pitch P_d, which corresponds to circular pitch P_c in the diametral plane, perpendicular to the axis of rotation. The pitch diameter of a helical gear can thus be found by Eq. 15-2. By using standard pitches, the pitch diameters (and therefore the center distance of helical gear sets) can be given in commonly used fractions or integers; consequently, a spur gear set of a certain size can be replaced directly by a similar helical gear set. Actual tooth thickness depends upon the pitch and the size of the helix angle. As shown by Fig. 15-16, if the circular pitch P_c be held constant, the actual tooth thickness measured perpendicular to its elements will decrease as the helix angle H is increased. A different cutter is required for every change in helix angle, although the pitch may remain constant. To eliminate an extensive variety of cutters, commercially available helical gears are made in several standard helix angles, among which are 7°30′, 15°, and 23°.

For obtaining continuity of contact or tooth overlap, the minimum theoretical face width should not be less than

$$W_{min} = \pi/(P_d \times \tan H) \qquad (15\text{-}18)$$

For a given rotative force F, the end thrust increases with the helix angle. To minimize the end thrust, a standard helix angle as small as possible should be selected, provided the space available will permit a sufficiently wide tooth face to obtain continuous contact.

By using a standard pitch in a plane normal to the tooth helix, the pitch diameter of a helical gear can be varied to suit a particular center distance by changing the helix angle. In this method of tooth measurement, the normal diametral pitch P_n corresponds to a normal circular pitch P_{cn} in the normal plane. Helical gear teeth designed with normal diametral pitches may be cut with standard spur gear cutters or hobs.

The pitch diameter of a helical gear, based upon normal pitch, is given by

$$D_g = N_g/P_n \cos H \qquad (15\text{-}19)$$

Example 15-3. A gear set for an oil-pump drive consists of two 10-pitch spur gears, with 12 and 35 teeth operating at a center distance of 2.350 in. It is desired to apply this drive to a special pump drive where the center distance must be 2.450 in. Design a replacement of equivalent strength for this center distance.

Solution. The spur gears now employed will not operate at an increased center distance because of incomplete action and excessive backlash. A pair of helical gears, 12 and 35 teeth with a normal pitch of 10 can be substituted, which will have a helix angle that will give the required pitch-diameters and the exact center distance. From Eq. 15-19,

$$\text{Center distance} = (D_p + D_g)/2 = (N_p/2P_n \cos H) + (N_g/2P_n \cos H)$$

Substituting and transposing,

$$\cos H = (12 + 35)/(2 \times 10 \times 2.450)$$

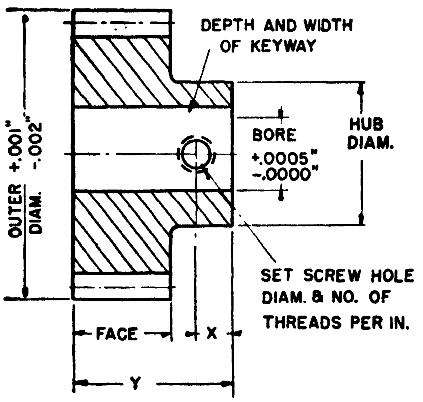
or angle H is equal to 16°26′ for both gear and pinion, one being right-hand and the other left-hand. While these gears cannot be obtained from suppliers' stock, any machine or jobbing shop with a universal milling machine, spiral head, and standard spur gear cutters can cut them to order. Complete data to specify either spur or helical gearing are given in Fig. 15-17.

15-18. Power Transmitting Capacity of Helical Gears. The strength of helical gearing is determined from Eq. 15-15, using the following for the velocity factor K,

$$K = 1200/(1200 + V_m) \tag{15-20}$$

The limiting load for wear may be obtained from

$$F_w = D_p b Q W/\cos^2 H \tag{15-21}$$



General Specifications
- Material
- Type of tooth
- Pressure angle
- Number of teeth

Spur Gear Specifications
- Diametral pitch

Helical Gear Specifications
- Normal Diametral pitch
- Helix angle—degrees and minutes
- Hand—right or left

FIG. 15-17. Spur and Helical Specifications.

where H is the helix angle and the other quantities are analogous to those of Eq. 15-16.

15-19. Herringbone Gearing. End thrust inherent in single helical gears can be eliminated by the use of herringbone gears similar to those shown in Fig. 15-28. These consist virtually of two integral single helical gears of opposite hand, which absorb the axial thrust within the gear. Herringbone gears are extensively used for hoisting and mining machinery, rolling mills, sugar mill and lumber machinery, turbine and compressor drives—in fact for nearly all heavy duty, high transmission ratio applications. If herringbone gear selection is indicated, the manufacturers of such units should be consulted.

GEARING FOR INTERSECTING SHAFT AXES

15-20. Bevel Gearing. Straight-tooth and spiral-tooth bevel gearing are used to transmit motion between shafts whose axes intersect. The operation

of such units is analogous to that of friction cones, which may be considered to represent the pitch cones for the bevel gearing, and which correspond to pitch cylinders for spur gearing. Straight-tooth bevel gears have teeth of involute form, but the straight-line elements converge (if extended) at the intersection of the shaft axes, in contrast to the parallel-tooth elements of spur gears. There are several forms of bevel gearing; in the most important, the gear and pinion

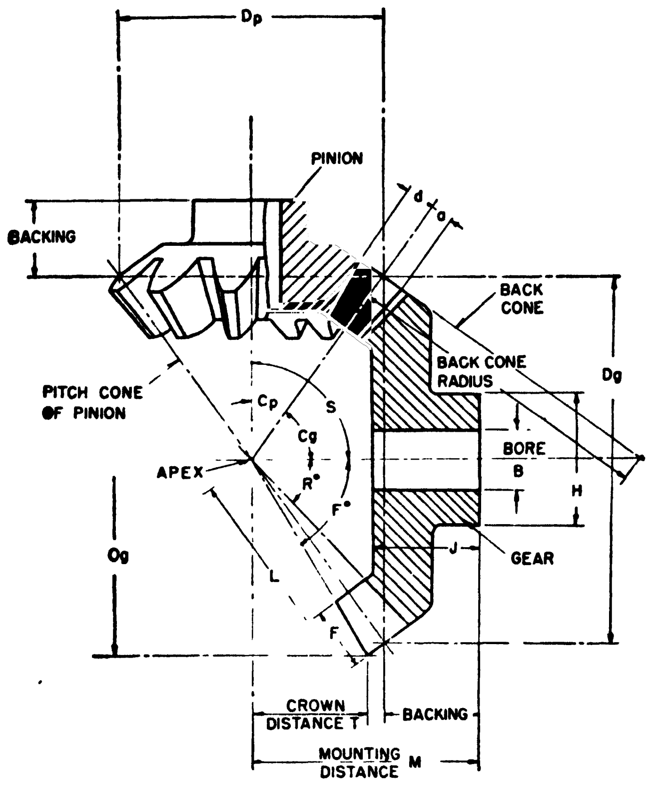


FIG. 15-18. Bevel Gear Nomenclature.

operate at a shaft axes angle of 90°. The unit is termed miter gearing if both gear and pinion have the same number of teeth, and angular gearing if the angle between the shaft axes is less than 90°. Bevel gearing in which the shaft axes angle is greater than 90° is also used to some extent.

Fig. 15-18 shows the important elements of a bevel gear. The pitch cone angles C_p and C_g of the pinion and gear must be complementary for a shaft axes angle of 90°. Table 15-4 gives equations for calculating the important elements of a bevel pinion operating at a shaft axes angle S of 90°. The equa-

tions apply to the gear as well as the pinion by suitable substitution. Other important dimensions necessary for specifications are shown in Fig. 15-18.

The pitch diameter of bevel gearing is measured at the large end of the pitch cones; the addendum and dedendum are not measured in the plane of the pitch circle, as in spur gearing, but are constructed perpendicular to the elements of the pitch cone on the surface of the back cone. The tooth shape is therefore dependent upon the magnitude of the back cone radius, rather than the pitch radius, as in spur gearing.

15-21. Proportions of Bevel Gearing. In contrast to spur gearing, bevel gears of a given pitch are not necessarily interchangeable. In Fig. 15-19, pinions *A* and *C* have the same number of teeth and pitch, but pinion *A* cannot replace *C* in set 2 because of the difference in the pitch cone angles, as illustrated at the right in set 3. It follows that bevel gears should be ordered or specified in sets,

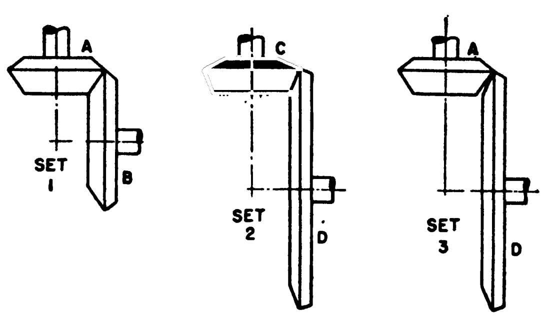


FIG. 15-19. Non-interchangeability of Bevel Gearing.

and if replacement of one element is necessary, it should be ordered with reference to the existing gear of the pair. The inherent non-interchangeability of bevel gear pairs has led to the development of a system of bevel gear teeth in which the pinion, when employed as the driving member, is made with a long addendum, with a correspondingly short addendum of the gear. This system, in which the pressure angle, addendum length, and tooth thickness are varied for differing velocity ratios, has been adopted as a standard by the American Gear Manufacturers Association [4] for drives in which moderate or large amounts of power are to be transmitted. Bevel gearing of this type is in wide use in commercial speed reducers.

Proportions of equal addendum cut-tooth bevel gearing may be obtained from manufacturers' catalogs, and are available in cast iron and steel; cast iron gears with cast teeth can be obtained in the larger tooth sizes.

Mortise gears have cast iron rims with cored slots into which hard maple cogs or teeth are fitted and held in place by wedges at the back of the rim, and are designed to operate with cast iron cast tooth pinions. Cast tooth gearing.

however, is used only where the pitch-line velocity is low and where smooth action is not particularly important.

TABLE 15-4.—BEVEL PINION FORMULAE
(14½° Standard Involute Tooth Form)

No.	Term	Symbol	Formula
1	Pitch diameter	D_p	N_p/P_d
2	Pitch cone angle	C_p	$\text{Tan } C_p = N_p/N_g$
3	Pitch cone radius	L	$\frac{1}{2}\sqrt{D_p^2 + D_g^2}$
4	Addendum	a	$1''/P_d$
5	Dedendum	d	$1.157''/P_d$
6	Face angle	F	$C_p + \text{arc sin } a/L$
7	Root angle	R	$C_p - \text{arc sin } d/L$
8	Outer diameter	O_p	$D_p + 2\,A \cos C_p$
9	Crown distance	T	$O_p\ (\tan F)/2$

15-22. Strength and Wear Characteristics. Like spur gearing, the limiting pitch-line velocity of commercially cut metallic bevel gearing is about 1000 ft. per min., vibration and noise becoming excessive beyond this point. These undesirable characteristics may often be eliminated by using a non-metallic pinion, made of material such as rawhide, for one unit of the set.

The horsepower that commercially obtainable bevel gear sets will transmit, from the standpoint of tooth strength, can be found from the following expressions which are adapted from Eq. 15-15.

For 14½° involute full depth gears:

$$F_s = \frac{2SKYb}{3P_d} \tag{15-22}$$

For 20° involute full depth gears:

$$F_s = \frac{3SKYb}{4P_d} \tag{15-23}$$

where F_s is the allowable load that can be transmitted, S the allowable unit stress from Table 15-1, K a velocity factor obtained from Eq. 15-13 or 15-14, b the face width of the bevel gear, P_d the diametral pitch at the large end of the tooth, and Y the bevel gear tooth-shape factor. The value of Y is affected by the number of teeth in both gears of the set, and may be obtained from Fig. 15-20. The value of Y, for example, for a 20-tooth pinion operating with a 48-tooth

gear, is 0.29. The values of Y given in Fig. 15-20 are based upon a face width equal to one third the pitch cone radius of the gear; the values of Y are in error on the safe side for ratios less than one third, but should not be used for face widths materially in excess of one third the pitch cone radius.

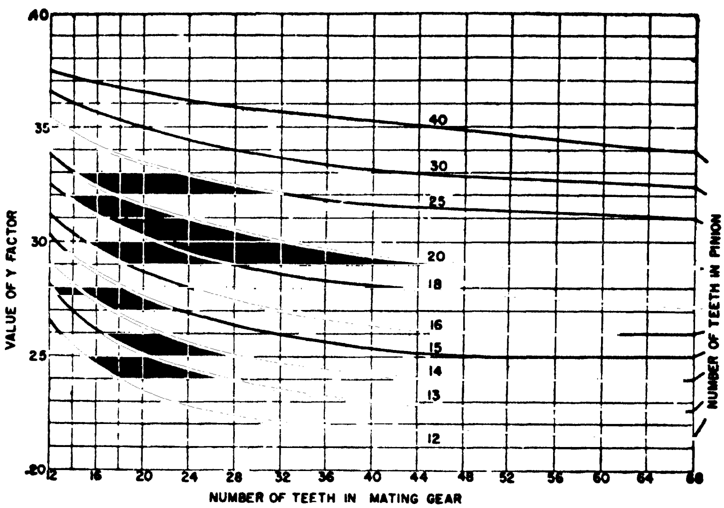


FIG. 15-20. Y Factors—Bevel Gearing.

The limiting load for wear for bevel gearing is found from the following

$$F_w = C\sqrt{D_p} \tag{15-24}$$

where F_w is the limiting load for wear and C is the material and service factor. Values of C are obtained from Table 15-5.

TABLE 15-5.—MATERIAL AND SERVICE FACTORS C FOR BEVEL GEARING

Pinion Material	Gear Material	Type of Service	C
Cast iron or Mach steel	Cast iron	Intermittent Continuous Shock	150 110 60
Hardened steel	Cast iron	Intermittent Continuous Shock	200 150 80
Hardened steel	Mach. steel	Intermittent Continuous Shock	220 180 90
Hardened steel	Hardened steel	Intermittent Continuous Shock	400 300 160

15-23. Spiral Bevel Gears. Spiral bevel gears have teeth cut in the arc of a spiral across the gear face, and bear the same relation to straight bevel gears that helical gears do to spur gears. This construction results in a larger number of teeth in contact than in straight-tooth bevel gearing, and like helical gearing, permits higher pitch-line velocities and greater load-carrying capacities for the same occupied space. Fig. 15-21 illustrates a spiral gear set. They are often used to replace straight-tooth bevel gear sets.

CROSSED-AXIS GEARING

15-24. Worm Gearing. Worm gearing is used to transmit power between shafts with perpendicular, non-intersecting axes. Essential elements are the worm and the worm wheel or gear, shown in Figs. 15-22 and 15-23. The worm is usually of cylindrical form, and resembles a screw; a section through the worm thread shows that the teeth are straight-sided and analogous to those of an involute rack. Worms are cut on a lathe or a thread milling machine and are often ground and polished after cutting and hardening to obtain surface precision and finish.

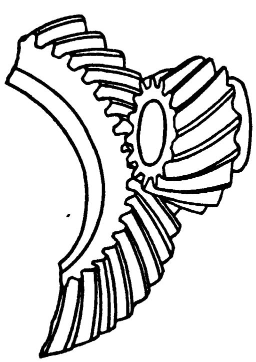


FIG. 15-21. Spiral Bevel Gearing.

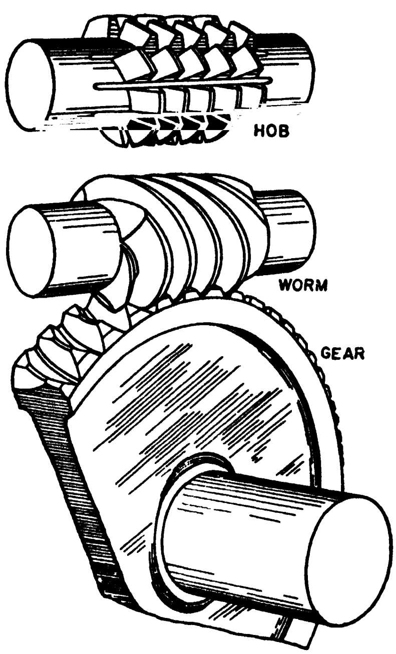


FIG. 15-22. Worm Gearing and Hob.

The worm wheel is essentially a helical gear with a face curved to fit a portion of the worm periphery. The tooth form and shape are obtained by cutting the wheel with a special form cutter known as a hob, shown in Fig. 15-22, which is essentially a replica of the worm, furnished with longitudinal flutes to provide cutting edges. In cutting the worm wheel teeth, the hob and the wheel

blank are rotated at a speed ratio exactly that of the finished set; the hob is properly located with respect to the plane of the wheel and fed in radially until the teeth have been cut to full depth. This cutting action generates worm wheel teeth that are of involute form at the mid-plane of the wheel, and are conjugate to the hob and consequently to the worm. Worm gearing is classified as non-interchangeable, because a worm wheel cut with a hob of one diameter will not operate satisfactorily with a worm of a different diameter, even if the thread pitch is the same.

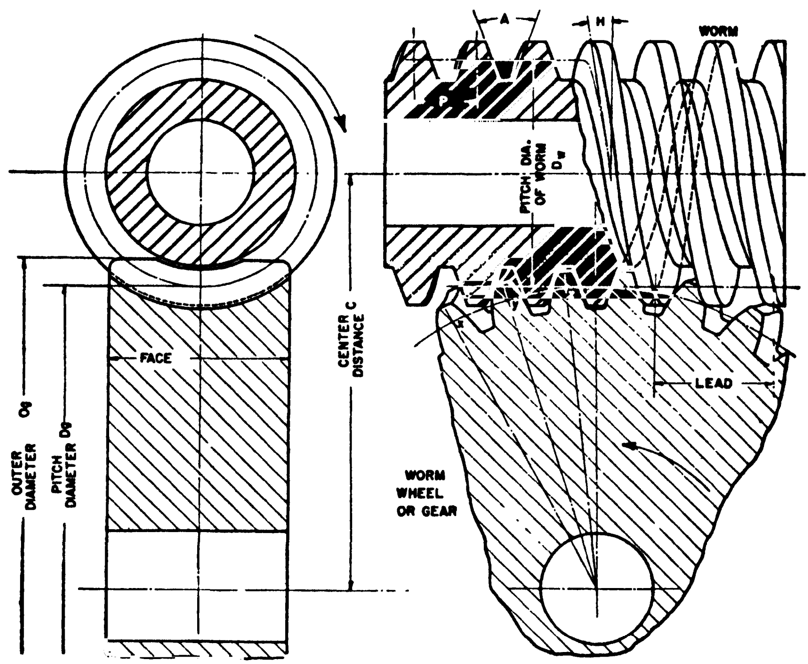


Fig. 15-23. Worm Gearing Nomenclature.

15-25. Worm Gear Nomenclature. Fig. 15-23 shows the principal elements and parts of a worm gear set. Tooth measurement is generally based on circular pitch, although diametral pitch gearing is manufactured and stocked by gear manufacturers. Circular pitch is measured in the diametral plane of the wheel and in a plane passing through the axis of the worm. If D_g represents the pitch diameter of the wheel, P_c the circular pitch, and N_g the number of teeth in the wheel, then

$$D_g = \frac{N_g P_c}{\pi} \tag{15-25}$$

The lead L of the worm is the distance that a thread advances in one turn, or the distance that a point on the pitch circle of the worm wheel will advance

during one revolution of the worm. If N_w represents the number of threads or "starts" in the worm, then:

$$L = N_w P_c \tag{15-26}$$

A triple-threaded worm has a lead equal to three times the pitch; in a single-threaded worm the lead and pitch are alike.

The velocity ratio R of a worm gear set depends upon the lead of the worm and the pitch diameter of the wheel, or,

$$R = \frac{\text{RPM Worm}}{\text{RPM Wheel}} = \frac{N_g}{N_w} \tag{15-27}$$

Unlike most gearing, the velocity ratio is independent of the pitch diameter of one of the elements—the worm. The worm pitch diameter can therefore be selected to suit a particular center distance, or to make use of a stock hob and thereby dispense with the cost of a special cutter.

TABLE 15-6.—PROPORTIONS OF STANDARD WORM GEAR SETS

Symbol	Quantity	Single and Double Thread	Triple and Quadruple Thread
$A/2$	Pressure angle	14½°	20°
	Addendum	$0.318\,P_c$	$0.286\,P_c$
	Whole depth of tooth	$0.686\,P_c$	$0.623P_c$
b	Gear face	$(2.38\,P_c) + (0.25)$	$(2.15\,P_c) + (0.20)$
T_g	Gear throat diameter	$D_g + (0.636\,P_c)$	$D_g + (0.572\,P_c)$
O_g	Gear outer diameter	$D_g + 1.12\,P_c$	$D_g + (0.89\,P_c)$
D_w	Worm pitch diameter	$2.4\,P_c + 1.10$	
L_w	Worm length	$(4.5 + N_g/50)P_c$	

The lead angle H of the worm threads is the angle between a line tangent to the thread helix at the pitch line and a plane perpendicular to the axis of the worm. It is found from the following, where D_w represents the pitch diameter of the worm:

$$\text{Tan } H = \frac{P_c N_w}{\pi D_w} \tag{15-28}$$

The tooth pressure angle is measured in a plane passing through the axis of the worm, and is equal to one-half the thread profile angle A. Pressure angles of 14½° are commonly used for single- and double-threaded worms, and 20° for triple- and for quadruple-threaded worms. However, in many modern worm gear reducer sets, pressure angles as high as 30° are employed. Proportions of worm gear sets are given in Table 15-6. Worm pitch diameters, however, are

often selected to suit standard hobs carried in stock by gear manufacturers. Data regarding the dimensions of these hobs are usually available upon application to the manufacturers. For worm diamete·s not corresponding to the proportions given in Table 15-6, the gear face should have an effective arc length of from 60° to 8C° of the worm pitch-line circumference. The worm length must then be obtained by a scale layout of the unit, and should be sufficiently great to project at least an eighth of an inch past the gear periphery at either end. Other proportions can be obtained from Table 15-6.

15-26. Worm Gear Efficiency. The nature of the tooth engagement in worm gearing causes greater sliding action between the surfaces in contact than in the case of spur gearing. The amount of this sliding action varies with the

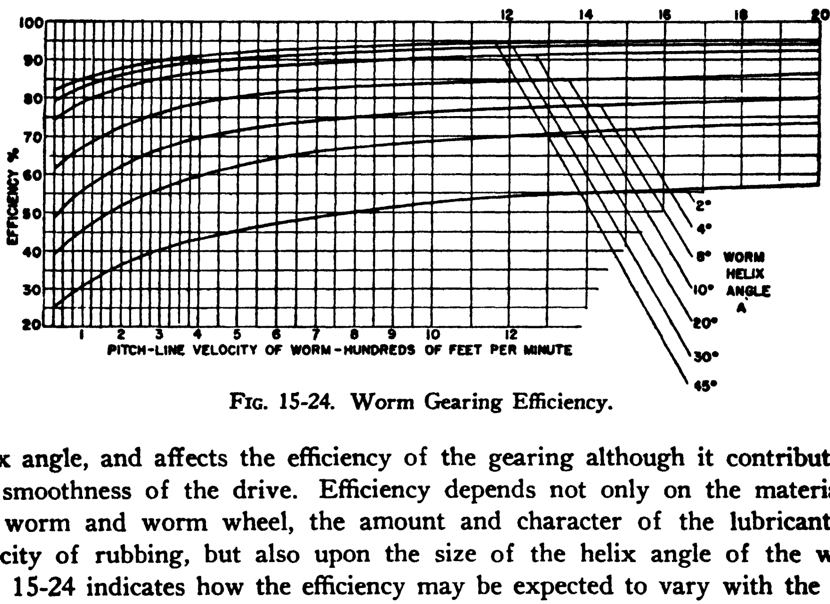


FIG. 15-24. Worm Gearing Efficiency.

helix angle, and affects the efficiency of the gearing although it contributes to the smoothness of the drive. Efficiency depends not only on the material of the worm and worm wheel, the amount and character of the lubricant, the velocity of rubbing, but also upon the size of the helix angle of the worm. Fig. 15-24 indicates how the efficiency may be expected to vary with the helix angles, at various pitch-line velocities V_p of the worm, where

$$V_p = \frac{\pi \times D_w \times \text{RPM}}{12} \tag{15-29}$$

The efficiencies obtained from Fig. 15-24 are based upon well mounted, hardened steel worms and bronze gears, with adequate lubrications supplied by heavy cylinder mineral oil, for operating temperatures not in excess of 160° F. When sets are subjected to indifferent lubrication or attention, the efficiencies obtained from the chart should be modified considerably. The efficiency, based upon the velocity ratio R, can also be estimated very closely by the following:

$$E(\%) = 100 - R \tag{15-30}$$

$$E(\%) = 100 - R/2 \tag{15-31}$$

Eq. 15-30 is applicable to worm gear sets without cases, mounted or installed by the purchaser; Eq. 15-31 applies to commercial worm gear reducers.

Single-threaded worms generally have low helix angles and efficiencies, and are consequently used only where the objective is multiplication of torque or attainment of a large mechanical advantage, as in hoisting machinery. Such worms are also employed where an irreversible or "self-locking" gear set is desirable—one in which the worm wheel cannot drive the worm. In order that a set may be irreversible, the helix angle must generally be less than 5°.

To obtain gear sets whose helix angles are within the maximum range of efficiencies, multiple-threaded worms with two, three, four or even more threads, are often essential. However, it should be remembered that for a given velocity ratio and pitch, a set with a double-threaded worm will necessitate a wheel twice as large as that required in a set using a single-threaded worm.

Fig. 15-24 indicates that the efficiency increases only slightly with any increase in the size of the helix angle beyond 20°, particularly at the higher pitch-line velocities of the worm. Worm helix angles between 30° and 60° have practically the same efficiency as far as tooth action is concerned, but a high helix angle causes a decided end thrust on the worm shaft bearings.

15-27. Power Transmitting Capacity of Worm Gearing. The factors that govern the power-transmitting capacity of a worm gear set are the strength, the ability to resist wear and abrasion, and the heat-radiating capacity. The teeth on the worm gear are weaker than the worm threads, and the design for strength is usually based upon an adaptation of Eq. 15-15, as follows

$$F_s = SbYP_c/\pi \tag{15-32}$$

The allowable unit stress S is taken as one and one half times the values given in Table 15-1 for non-metallic materials, cast iron, bronze, and semi-steel. The values of Y for 14½° and 20° worm gear teeth are computed from Eqs. 15-12 and 15-14; for tooth numbers equal to, or greater than, 40, values of Y equal to 0.314 for 14½° teeth, and 0.392 for 20° teeth, can be used with safety.

The limiting load for wear in a worm gear set is found by

$$F_w = D_g bW \tag{15-33}$$

where W is a constant dependent upon the materials of the worm and gear. Soft worms are likely to abrade readily; for continuous service, the worm should be hardened or casehardened and ground and polished. With hardened worms, the values of W given in Table 15-7 will apply.

In many cases, the design and selection of worm gearing is dictated by the heat-radiating capacity, which depends upon the environment and the type of housing, if any. The heat-radiating capacity for continuous operation estimated from

$$T = 70\,h(100 - E)/D_g^2 \tag{15-34}$$

where h is the input horsepower to the worm, E the efficiency (per cent) and T the temperature rise (° F.) above ambient temperature. The final operating temperatures (T + ambient temperature) should not exceed 200° to 220° F., preferably a maximum of 160° F.

TABLE 15-7.—WEAR CONSTANT W FOR WORM GEARS

Cast iron or semi-steel	$W = 50$
Bronze	$W = 80$
Non-metallic materials	$W = 125$

Worm gear sets should be carefully aligned in the axial plane of the worm, with the shaft axes at 90°. If the set is arranged so that the worm is underneath the wheel, the former may be run in an oil bath to insure adequate lubrication. Installations should preferably be enclosed to retain the lubricant and to prevent the admission of dust or foreign matter.

Example 15-4. Design a worm gear drive for a hoist with a drum diameter of 2 ft., to lift 750 lbs. through a distance of 60 ft. in 15 seconds, driven by an 1175-RPM motor.

Solution. Hoist speed $= \dfrac{\text{Distance lifted per minute}}{\text{Drum circumference}} = \dfrac{60 \times 60/15}{2\pi} = 38.2$ RPM

$$\text{Velocity ratio} = \frac{\text{Motor speed}}{\text{Hoist speed}} = \frac{1175}{38.2} = 30.8, \text{ say } 30.$$

It is possible to use a single-threaded worm with a 30-tooth wheel, a double-threaded worm with a 60-tooth wheel, etc. The single-threaded worm will offer the advantage of a more compact drive, but the multiple-threaded worm will provide a higher efficiency. Assume a single-threaded worm and a 30-tooth bronze gear.

The torque on the hoist is equal to the product of the load and the drum radius, 750×12, or 9000 in.-lbs. The load F that the worm wheel teeth must withstand is equal to the total torque divided by the radius of the worm wheel, or

$$F = 9000/(D_g/2) = 9000/(30\,P_c/2\,\pi) = 600\,\pi/P_c$$

The load F should be at least equal to the safe strength F_s of the worm gear teeth from Eq. 15-32. S may be taken as one and one half times the allowable stress for bronze from Table 15-1, or 18,000 psi. The face width b, from Table 15-6, is given by

$$b = 2.38\,P_c + 0.25$$

and the factor Y, from Eq. 15-12,

$$Y = 0.39 - 2.15/30 = 0.318$$

Equating F_s and F,

$$F_s = \frac{18{,}000(2.38\,P_c + 0.25)(0.318 \times P_c)}{\pi} = 600\,\pi/P_c$$

Solving $$13{,}610\,P_c^3 + 1430\,P_c = 5930$$

A rigorous solution of this expression would result in a fractional, non-standard pitch. To facilitate manufacture, it is advisable to use a pitch for which a hob is available. If a value of ¾ in. for P_c is selected, the expression will read

$$13{,}610 \times 0.75^3 + 1430 \times 0.75 = 6820$$

For this pitch, the pitch diameter of the gear, from Eq. 15-25, is

$$D_g = \frac{30 \times 0.75}{\pi} = 7.16 \text{ in.}$$

From Table 15-6, the face width of the gear is:

$$b = 2.38 \times 0.75 + 0.25 = 2.03, \text{ say 2 in.}$$

Hoist service is usually intermittent, and the question of wear or excessive heating is not always considered. In a design of this character, however, and particularly in view of the comparatively small size of the worm gear, it may be advisable to investigate the possibilities of failure or unsatisfactory service from such causes. If a value of 80 for the wear constant W is selected from Table 15-7, the limiting load for wear from Eq. 15-33 is

$$F_w = 7.16 \times 2 \times 80 = 1150 \text{ lbs.}$$

Comparing this with the actual load carried at the pitch line of the gear

$$F = \frac{\text{Torque}}{D_g/2} = \frac{9000}{3.58} = 2520$$

it is evident that the set is decidedly undersize, even for intermittent service. In view of this it may be advisable to redesign on the basis of a double-threaded worm and a 60-tooth gear set.

Following the procedure outlined in the preceding solution,

$$F = 9000/(D_g/2) = 9000/(60P_c/2\pi) = 300\,\pi/P_c$$

$$F_s = \frac{18{,}000(2.38P_c + 0.25)(0.314 \times P_c)}{\pi} = 300\,\pi/P_c = F$$

Solving $$1348\,P_c^3 + 1410\,P_c = 2965$$

Substituting a standard ⅝-in. pitch

$$3300 + 880 = 4185, \text{ the nearest correct solution.}$$

From Equation 15-25

$$D_g = (60 \times 0.625)/\pi = 11.9 \text{ in.}$$

From Table 15-6

$$b = (2.38 \times 0.625) + 0.25 = 1.75, \text{ say } 1\tfrac{3}{4} \text{ in.}$$

Checking this selection for wear by Eq. 15-33,

$$F_w = 11.9 \times 1.75 \times 80 = 1662 \text{ lbs.}$$

The actual load on the teeth is

$$F = \frac{9000}{11.9/2} = 1510 \text{ lbs.}$$

and it is clear that the actual load is within the limiting load for wear.

An available hob, from a manufacturer's list, has a pitch diameter of 2.118 in. The efficiency of the set is calculated by first finding the lead angle from Eq. 15-29,

$$\text{Tan } H = \frac{0.625 \times 2}{\pi \times 2.118}$$

from which H is equal to 10°38′. From Eq. 15-29 the pitch-line velocity of the worm is

$$V_p = \frac{\pi \times 2.118 \times 1175}{12} = 650 \text{ ft. per min.}$$

and the efficiency, from Fig. 15-24, is about 82%.

A load of 750 lbs. lifted through a distance of 60 ft. in 15 seconds requires a net horsepower of

$$HP = \frac{750 \times 240}{33{,}000} = 5.46$$

The input horsepower is 5.46/0.82, or 6.66. The estimated temperature rise can be obtained from Eq. 15-34,

$$T = (70 \times 6.66)(100 - 82)/11.9^2 = 59° \text{ F.}$$

With a room temperature of 70° or 80°, the actual operating temperature of the unit would probably be between 130° and 140° F., a very satisfactory value.

The necessary manufacturing dimensions and specifications for the worm gear are shown in Fig. 15-25. The critical dimensions are computed from the equations in Table 15-8, as follows:

Tooth depth

$$0.686 \times 0.625 = 0.429 \text{ in.}$$

Throat diameter

$$11.937 + (0.636 \times 0.625) = 12.334 \text{ in.}$$

Outer diameter

$$11.937 + (1.12 \times 0.625) = 12.637 \text{ in.}$$

A tolerance of 0.001 in. over and 0.002 in. under theoretical size for these diameters is usually permissible. The throat radius is computed by subtracting the addendum 0.318 × 0.625, or 0.199, from the pitch radius 2.118/2 of the worm, which gives 1.059 — 0.199, or 0.360. The center distance is of importance for manufacturing and assembly reasons, and is obtained as follows:

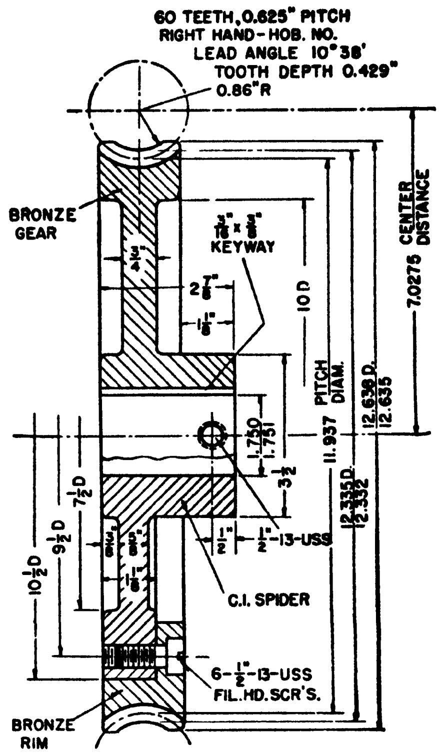


Fig. 15-25. Worm Gear Construction and Delineation.

$$CD = (D_w + D_g)/2 = (2.118 + 11.937)/2 = 7.0275 \text{ in.}$$

The necessary machining dimensions for the worm, in addition to the bore, hub diameters, length, etc., are the pitch and outer diameter, the lead angle, the hand, the pitch, and the number of threads (single, double, triple, etc.), as shown in Fig. 15-25.

Two types of worm gear construction are shown in Fig. 15-25. The gear shown in the upper half of the figure is an integral casting, and must have a hub, web, and rim of the same material as the teeth. The construction shown in the lower half of the figure represents a design in which a bronze rim is mounted on a cast iron spider. This construction necessitates more machining expense than the solid type of wheel, but the additional cost of machining is usually more than compensated for by the saving in material, particularly in large gears.

15-28. Bearing Forces in Worm Gear Drives. The force F found from Eq. 15-9 produces an end thrust on the worm which must be taken care of by a suitable thrust bearing. Similarly, the rotative force Q at the periphery of the worm, which may be found from the following, exerts an end thrust on the gear:

$$Q = \frac{h \times 33{,}000}{V_p} \qquad (15\text{-}35)$$

where h is the input horsepower, and V_p is the pitch-line velocity of the worm. Both F and Q affect the bearing reactions, since they act at the worm and gear peripheries. There is also a separating force between the worm and gear, but its magnitude is such that it is usually disregarded for 14½° and 20° pressure angles.



FIG. 15-26. Spiral Gearing. *Courtesy W. A. Jones Foundry and Machine Co*

15-29. Spiral Gearing. Helical gears for transmitting power between shafts whose axes are neither parallel nor intersecting are illustrated in Fig. 15-26, and are commonly, although incorrectly, called spiral gears. They may be adapted to any shaft axes angles, although they are usually employed for shaft axes at 90° to each other. The tooth elements are similar to those of parallel-shaft helical gears; the usual method of tooth measurement is by normal diametral pitch P_n. In 90° shaft-angle gearing, both pinion and gear have helical teeth of like hand, with complementary helix angles H_p and H_g. The pitch diameters D_p and D_g for the pinion and gear are given by

$$D_p = \frac{N_p}{P_n \cos H_p} \qquad (15\text{-}36)$$

$$D_g = \frac{N_g}{P_n \cos H_g} = \frac{N_g}{P_n \sin H_p} \qquad (15\text{-}37)$$

Consideration of these equations shows that spiral gear pitch diameters, like parallel-shaft helical gear diameters, may be adjusted in design to accommodate a special or unusual center distance. A trial-and-error solution, however, is

necessary; it is evident that slide rule computations are insufficiently accurate for the final stage of the computation. Necessary specifications and dimensions for manufacture or fabrication are similar to those shown for helical gears in Fig. 15-17.

Power transmitting capacity of spiral gearing is limited. Although some operating characteristics are analogous to worm gearing, the contact area between the teeth of spiral gears is theoretically a point, and in actual practice is confined to a very small area. Spiral gears are generally used when motion rather than power is of major importance.

Geared Speed Reducers

15-30. Geared speed reducers afford all the advantages of toothed gearing, and require very little attention or maintenance other than periodic inspec-

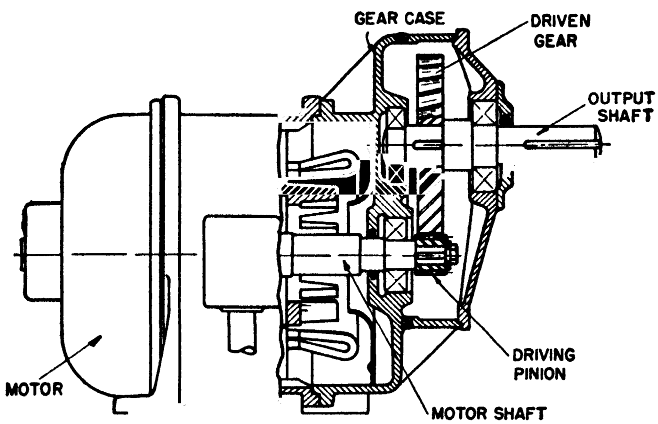


Fig. 15-27. Gearmotor.

tion of the oil supply, if they are properly selected and installed. Practically all types of gearing are employed in their construction; the most important forms of reducers are those with parallel input and output shafting, for which spur, helical, and herringbone gearing are used; and those with perpendicular shaft axes, which employ spiral bevel, hypoid, or worm gearing. One of the simplest forms of speed reducers is the gear motor shown in Fig. 15-27. The unit illustrated in an "offset" reducer, in which a helical pinion on the end of the motor shaft drives a helical gear on the output shaft. The gear is supported by the motor housing, and can be adjusted so that the output shaft is in the same horizontal plane as the motor shaft, or in several intermediate positions, as well as in the position shown, thereby making the unit adaptable to various conditions of installation. Double and triple reduction units are available for high speed

ratios; in some of these the motor is flanged-mounted on the reducer case. Planetary gear motors, with the motor and output shaft axes in alignment, are available in single and double reduction units and for a wide range of ratios and power capacities up to about 75 horsepower.

15-31. Parallel Shaft Gear Reducers. Parallel shaft reducers with a single set of gears are used for reductions up to 10:1, and are known as single-stage reducers; two-stage units, with two sets of gears, are used for reductions from 10:1 to 60:1; and three-stage units for reductions over 60:1. Fig. 15-28 shows a two-stage herringbone gear reducer, with the upper portion of the case removed. The high-speed or input shaft, shown projecting at the right, is mounted in double-row ball bearings; the intermediate shaft and the slow-speed output shaft, which projects at the left and rear, are mounted in Hyatt roller bearings. A three-stage or triple-reduction helical gear unit is shown in Fig. 15-29; the input shaft is directly below the first intermediate shaft and the output shaft is shown at the rear.

15-32. Right-angle Gear Reducers. Right-angle, two-stage speed reducers with a spiral bevel gear set for the high-speed stage, and a helical gear set for the low-speed stage, are common. Single and triple reduction units are also available. Units of this character can also be procured with vertical output shafts, projecting either above or below the gear case, for agitator and mixer drives.

15-33. Worm Gear Reducers. Worm gear reducers are frequently employed for high velocity ratios and heavy power demands. The driving motor is generally connected to the input or worm shaft by means of some form of flexible coupling; the drive from the worm gear shaft, or output shaft, to the driven machine can be effected either through a flexible coupling for direct-connected drives, or by spur gearing, belt or chain drives.

Two types of worm gear reducers are shown in Figs. 15-30 and 15-31. Each type has a high-helix angle worm integral with the input shaft, and is carried in ball bearings. The gear has a bronze rim bolted to a cast iron spider mounted on the slow-speed or output shaft and is carried in roller bearings. Horizontal units are available similar to that of Fig. 15-30, in which the worm is above the gear; there are also double reduction units in which two sets of worm gearing are incorporated in a single case. Since the efficiency of a worm gear reducer can be approximated very closely by Eq. 15-31, it follows that for very high velocity ratios the double-reduction unit, because of the efficiency of the former, is equal to the product of the efficiencies of each worm gear set.

15-34. Worm Gear Reducer Selection. Worm gear reducers should be selected upon consideration of three important factors: the mechanical rating of the unit, which involves the strength and wear load capacities of the gearing; the thermal rating, which takes into account the operating temperature of the unit; and the efficiency. These factors in turn depend upon various conditions



FIG. 15-28 Herringbone Gear Reducer. *Courtesy Foote Bros. Gear and Machine Co.*

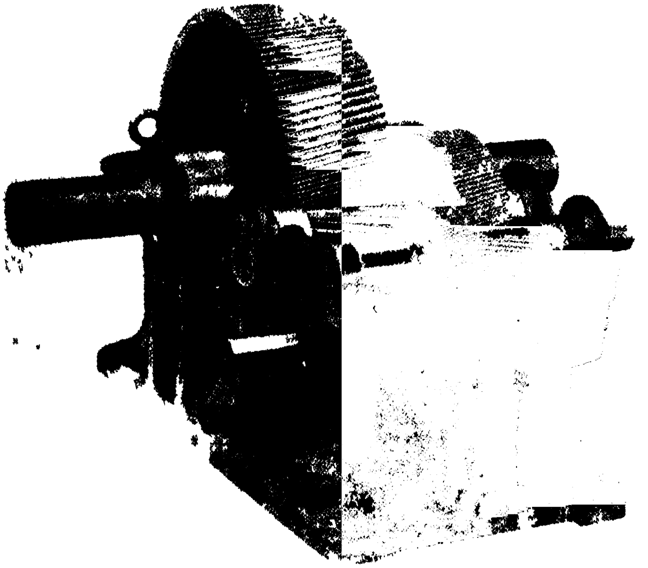


FIG. 15-29. Helical Gear Reducer. *Courtesy Foote Bros Gear and Machine Co.*

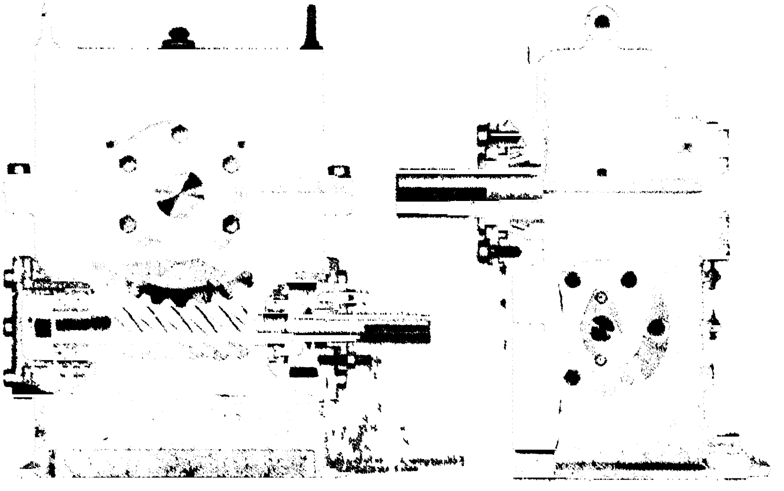


FIG. 15-30. Worm Gear Reducer *Courtesy of Cleveland Worm and Gear Co.*

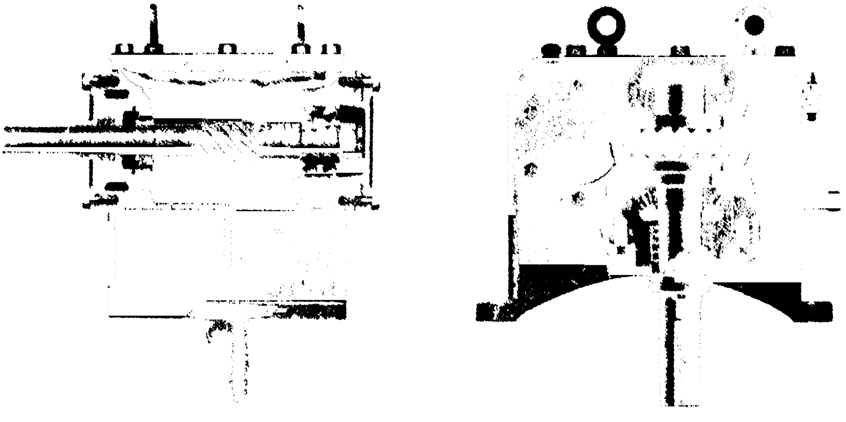


FIG 15-31. Vertical Worm Reducer *Courtesy Cleveland Worm and Gear Co.*

of service, which have been classified as illustrated in Table 15-8 by the American Gear Manufacturers' Association.

TABLE 15-8.—SERVICE CLASSIFICATIONS

(A.G.M.A. Recommended Practice)

Class 1. Normal eight to ten hour service, free from recurrent shock loads (i.e., shock loads that occur at approximately even and frequent intervals). The mechanical ratings listed in Rating Tables found in catalogs of reputable manufacturers are to be applied without a service factor for service in this classification—provided only that the thermal rating of the unit is not exceeded. (For purposes of worm gear selection, any driven machine that imposes load fluctuations at approximately even and frequent intervals, such as reciprocating mechanisms—pumps, compressors, shears—or that has a considerable inertia and is driven at a further reduced speed through spur gearing—dryers, kilns, ball mills—should be regarded as providing a shock load.)
Class 2. Eight to ten hour service where recurrent shock loading is encountered, or twenty-four hour service where no shock loading is experienced. For application to this service classification, the mechanical ratings should be divided by a service factor of 1.2. Thermal rating of the unit must not be exceeded.
Class 3. Twenty-four hour shock load service. For application to this classification, the listed mechanical ratings must be divided by a service factor of 1.33. Thermal rating of the unit must not be exceeded.
Class 4. Intermittent service where the maximum cycle of operation calls for not more than fifteen minutes running in a two hour period. For application to service in this classification, the listed mechanical ratings may be divided by a service factor of 0.7 to obtain the equivalent Class 4 rating. Thermal rating of the unit may be ignored.
Class 5. Low speed service (where the worm speed is less than 100 RPM) will require output torque ratings in inch-pounds, independent of the nature of duration of the load.

Table 15-9 shows a portion of a rating sheet taken from the catalog of a prominent manufacturer which gives the input horsepowers that are permissible under Class 1 ratings for various worm shaft or input speeds. Mechanical ratings of coupled units are obtained directly; for example, a size 200 unit with a speed ratio of 14.5:1 between the input and output shafts, will handle 8.6-HP input load at a worm shaft speed of 720 RPM. When the power take-off from the worm gear shaft to the driven shaft is effected through the medium of spur gearing, belting, or chain drives, an overhung load is induced in the worm gear shaft extension, because of the driving force at the peripheries of these elements. Manufacturers of reducers take this effect into account by specifying the permissible overhung pull, shown in the column headed "Allowable Chain-Pull-Pounds" in Table 15-9, based on the assumption that the load is applied at the pitch-line of a chain sprocket, and at the center of the worm gear shaft key, on a standard shaft extension. The actual pull F that a given amount of transmitted power will induce can be found by

$$F = \frac{126{,}000\, h}{D_s n} \tag{15-38}$$

TABLE 15-9.—WORM GEAR REDUCER RATING TABLES

Ratio	Hand of Thread	Class 1 A.G.M.A. Mechanical Ratings-HP Worm Speed RPM											Thermal Rating HP	Allowable Chain Pull Pounds	Maximum Torque Output in.-lbs.
		100	200	300	580	720	870	1150	1750	2400	3000	3600			
Size 50 Unit														4.000" Centers	
10	*R L*	.63	1.2	1.6	2.5	2.9	3.3	3.8	4.7					900	3780
15½	*R*	.44	.81	1.1	1.8	2.1	2.3	2.7	3.4					1000	4000
19½	*R*	.37	.67	.93	1.5	1.7	1.9	2.3	2.9					1100	4040
40	*R L*	.19	.35	.48	.79	.90	1.0	1.2	1.5					1100	3790
Size 70 Unit														4.750" Centers	
4⅝	*R*	1.7	2.9	3.9	6.0	6.7	7.4	8.5	10.0					900	4870
10	*R*	1.0	1.8	2.5	4.0	4.6	5.0	5.9	7.2					1250	6100
14½	*R*	.77	1.4	1.9	3.0	3.4	3.8	4.5	5.5					1400	6500
30	*R L*	.40	.73	1.0	1.6	1.8	2.0	2.4	3.0					1600	6430
Size 100 Unit														5.500" Centers	
15	*R L*	1.1	2.0	2.7	4.3	4.8	5.4	6.3	7.6	8.7	9.4	9.9	8.6	1700	9800
40	*R*	.46	.82	1.1	1.8	2.0	2.3	2.6	3.2	3.7	4.0	4.2	3.8	1900	9300
45	*R*	.41	.74	1.0	1.6	1.8	2.0	2.4	2.9	3.3	3.6	3.8	3.4	1900	9100
50	*R*	.37	.67	.92	1.4	1.6	1.8	2.1	2.6	3.0	3.2	3.4	3.1	1900	8800

TABLE 15-9.—(*Continued*)

Ratio	Hand of Thread	Class 1 A.G.M.A. Mechanical Ratings-HP Worm Speed RPM 100	200	300	580	720	870	1150	1750	2400	3000	3600	Thermal Rating HP	Allow-able Chain Pull Pounds	Maxi-mum Torque Output in.-lbs.
Size 200 Unit													6.8715" Centers		
14½	*R*	2.1	3.6	5.0	7.6	8.6	9.4	11.0	13.0	14.5	15.5	16.5	13.0	1700	18,000
21½	*R*	1.5	2.7	3.6	5.5	6.2	6.9	7.9	9.6	10.5	11.5	12.0	9.4	1850	18,300
39	*R*	.87	1.5	2.1	3.0	3.6	4.0	4.6	5.5	6.2	6.6	7.0	5.7	1900	17,200
45	*R*	.76	1.3	1.8	2.8	3.1	3.5	4.0	4.8	5.4	5.8	6.1	5.0	1900	16,700
Si														Cei	
15	*R*	3.2	5.6	7.5	11.0	12.5	14.0	16.0	19.0	21.0	22.0	24.0	16.5	2100	28,300
21½	*R L*	2.4	4.2	5.5	8.4	9.2	10.5	12.0	14.0	16.0	17.0	18.0	12.5	2400	28,800
30	*R*	1.8	3.1	4.1	6.2	7.0	7.7	8.8	10.5	11.5	12.5	13.0	9.6	2600	28,200
45	*R*	1.2	2.1	2.8	4.2	4.8	5.3	6.0	7.2	8.0	8.5	9.0	6.7	2600	26,400
Size 600 Unit													13.4365" Centers		
8	*R L*	20.0	33.0	42.5	59.5	66.5	72.0	79.0	90.0	98.5	105	111	60.0	3700	97,500
20	*R*	9.9	16.5	21.5	30.5	34.0	37.0	41.5	47.0	52.0	55.0	58.5	31.0	6500	112,000
31½	*R*	6.6	11.0	14.5	20.5	22.5	24.5	27.5	31.5	35.0	37.0	39.0	21.5	7000	108,000
48	*R*	4.4	7.3	9.6	13.5	15.0	16.5	18.5	21.0	23.5	24.5	26.0	14.5	7000	102,000

where h represents the transmitted horsepower, n the gear shaft RPM, and D_t the pitch diameter of the chain sprocket. To illustrate, consider that the preceding size 200 unit is used to transmit 8 HP with a chain drive having a 13-in. sprocket. The chain pull is

$$F = \frac{126{,}000 \times 8}{13 \times 720/14.5} = 1560 \text{ lbs.}$$

From Table 15-9, the allowable chain pull for the unit in question is 1700 lbs., and the unit would be satisfactory from this point of view. If spur gears and vee- or flat belts are employed as power take-off media, the actual pull on the shaft is somewhat in excess of that exerted by a power chain. To compensate for this increase, the allowable chain pull given in Table 15-9 should be divided by the factors or divisors given in Table 15-10.

TABLE 15-10.—OVERHUNG LOAD FACTORS

Overhung Member	Divisor
Chain sprocket	1.00
Spur gear	1.25
Vee-belt pulley	1.50
Flat belt pulley	2.50

When input shaft speeds are less than 100 RPM, the maximum torque output shown in Table 15-9 governs the reducer selection. These output torque ratings are equivalent to Class 1 mechanical ratings at 100 RPM, and should be used without service factors, in which case they are designated as Class 5 ratings.

15-35. Thermal Rating. Operation of a worm gear set within a housing produces heat because of oil churning and gear and bearing friction. Manufacturers generally consider the maximum allowable operating temperature of the gear unit to be about 200° F. because higher temperatures may introduce difficulties in lubrication. Fig. 15-9 gives the limiting horsepower that may be handled by each unit (referred to as the thermal rating) for a temperature rise of not more than 90° F. above room temperature. Thermal ratings require no service factor, and need not be considered if they are greater than the load transmitted. To illustrate, the size 200 unit previously discussed has been selected on the basis of the mechanical rating, because this value is less than the thermal rating. If the unit is designed to operate at an input speed of 2400 RPM, the 13-HP thermal rating and not the 14.5-HP mechanical rating would govern the selection of the reducer for Class 1 service. Reducer units that operate at input speeds greater than 2000 RPM must in general be water cooled, if the construction is such that cooling coils may be applied; addition of such coils will increase the thermal capacity of an air-cooled unit by about 35%

Example 15-5. Select a worm gear reducer unit for a conveyor headshaft requiring about 3.5 HP at 40 RPM. The unit is driven by a 5-HP 1750-RPM direct-coupled motor, and no shock loads are anticipated.

Solution. The speed ratio of the reducer is equal to the ratio of the motor and headshaft speeds, or 1750/40, or 43.75. The estimated efficiency of the reducer, from Eq. 15-31, is

$$E = 100 - 43.75/2 = 78\%$$

From Table 15-8, the required unit falls into Class 1, and the mechanical rating may be used within the limits of the thermal rating. From Table 15-9, the nearest speed ratio is 45:1 for sizes 100, 200, and 300. For an input speed of 1750 RPM, size 200 has a mechanical rating of 4.8 HP, and is suitable. The thermal rating of 5 HP is greater than the mechanical rating, and need not be considered.

The required input rating is equal to the output horsepower divided by the efficiency, or 3.5/0.78, or 4.5 HP.

Example 15-6. Select a reducer unit or the drive of Example 15-5, using a chain drive between the reducer and the conveyor headshaft, to modify the reduction ratio ol the worm gear unit.

Solution. If a 3:1 reduction ratio for the chain is selected, the required reducer ratio is 43.75/3, or 14.58. The reducer efficiency is

$$E = 100 - 14.58/2 = 92.7\%$$

If an allowance is made for the power loss in the chain drive, the final efficiency may be taken as 90%.

The required input rating is equal to 3.5/0.90, or 3.9 HP. A size 50 unit, Table 15-9, has a Class 1 rating of 3.4 HP for a motor speed of 1750 RPM with a reduction ratio of 15.5:1, and is thus of insufficient capacity; the size 70 unit has a Class 1 rating of 5.5 HP for a motor speed of 1750 RPM, with a reduction ratio of 14.5:1, and has ample capacity. Thermal ratings need not be considered in size 50 and size 70 units since these are in all cases greater than the listed mechanical ratings.

It will be necessary to consider the effect of the load induced by the chain pull on the gear shaft extension. The output speed of the reducer shaft is equal to 1750/14.5, or 121 RPM. The allowable chain pull from Table 15-9 is found to be 1400 lbs. From Eq. 15-38, the minimum diameter of the sprocket is

$$D_s = \frac{126{,}000 \times 3.5}{1400 \times 121} = 2.62 \text{ in.}$$

Any sprocket of larger diameter would be satisfactory, since the overhung load would be less than the allowable 1400 lbs. listed in Table 15-9.

The choice of these solutions for the given problem would depend to a large extent on financial considerations and other factors, such as the relative efficiency of the two methods of driving, the difference in maintenance cost, the amount of space available for installation, the possibility of replacement, etc.

Example 15-7. Select a reducer for Class 1 service, with a reduction ratio of 21.5:1, an input power of 11 HP, and a motor speed of 3000 RPM.

Solution. From Table 15-9, the mechanical rating of a size 200 unit is 11.5 HP and is satisfactory for this purpose, but the thermal rating is only 9.4 HP. It will be necessary to select a size 300 unit of the same ratio, with a thermal rating of 12.5 HP, although its mechanical rating of 17 HP is greatly in excess of that required. This selection would then permit Class 3 use, that is, 24-hour shock load service, with a mechanical rating of 17/1.333, or 12.8 HP, without exceeding the safe limits of its capacity.

Example 15-8. Select a unit to drive a vertical fluid agitator from a 15-HP 1150-RPM motor. The agitator may have a speed varying from 75 to 80 RPM, and will

require approximately 12 HP. The design should be on the basis of continuous 24-hout service.

Solution. The required speed ratio should lie between 1150/75, or 15.3, and 1150/80, or 14.4. The average efficiency of a unit of such ratio will be about 92.5%, which is sufficiently great to permit an output of 12 HP from a reducer driven by a 15-HP motor. The actual required input power that the reducer must take care of will be 12/0.925, or 13 HP.

Driving a fluid agitator would be considered a smooth, shock-free load for a reducer unit, but a Class 2 rating is necessary in view of the 24-hour service required. A size 300 unit is the smallest that can be employed, and has a Class 1 rating of 16 HP at 1150 RPM for a 15:1 ratio. The Class 2 rating would be obtained by dividing this rating by a service factor of 1.2 (from Table 15-8) giving 16/1.2, or 13.3 HP. This rating exceeds the required input horsepower, and since the thermal rating is greater than either, the selection is satisfactory.

15-36. Vertical Geared Reduction Units. Geared reducers with vertical output shafts extending below the case often require special oil seals or stuffing boxes to prevent oil leakage, particularly when driving agitators where the fluid must be kept free from contamination. If the output shaft of the reducer is coupled directly to the agitator, it may be necessary for the lower bearing of the unit to carry part or all of the load induced by the weight of the agitator shaft and component units. In such cases, it is necessary to check the resulting thrust load against the excess thrust capacity of the reducer unit. Thrust capacities and other dimensional data required for design, selection, or installation should be obtained from manufacturers' or suppliers' catalogs.

PROBLEMS—CHAPTER 15

1. Lay out a pair of 20° stub involute form spur gears, one pitch. The driving pinion has 16 teeth, the driven gear 24 teeth. Show four pinion and three gear teeth and determine: (a) the contact ratio, (b) the approaching-receding action ratio, (c) the presence of any interference.

2. A pair of parallel shafts have an 8:1 velocity ratio, the high-speed member rotating at 2750 RPM. The center distance is 9.000 in., and the drive is through the medium of 6-pitch, 2-in. face spur gearing, the pinion being of semi-steel, 12 teeth, and the gear of cast iron. The teeth are cut of 20° stub involute form.

a. How many teeth has the gear?

b. What is the pitch-line velocity?

c. What is your opinion as to this velocity?

d. What HP may be transmitted, strength alone being considered?

e. What HP may be transmitted, with all factors considered?

f. Suggest two alternatives to correct the conditions suggested in *c.* Give full details and data, and suggest any limitations thereof.

3. Find the pitch and face width of a pair of 20°-involute cast iron gears to transmit 20 HP at a gear speed of 250 RPM, if the velocity ratio is 2.2:1 and the center distance is 6 in. The service is intermittent.

4. What is the limiting load for wear in the gear set of Problem 3?

5. Give full design data for a pair of 20°-stub tooth gears to transmit 20 HP at a pinion speed of 1600 RPM and a velocity ratio of 8:1. The center distance should be as small as possible and the drive as inexpensive as possible. (Consider material cost as in direct proportion to strength.)

6. An internal gear drive for intermittent service has a velocity ratio of 29:15, and the driving pinion has 30 teeth, 4-pitch, 3-in. face, and is made of hardened steel. What horse-

power may be transmitted if the pinion rotates at 580 RPM? What should the gear material be?

7. A driving shaft R rotates at 300 RPM and is placed in alignment with a driven shaft N rotating at 25 RPM. A spur gear A on R drives a gear B on a back gear shaft S. Another gear C on S drives a gear D on shaft N. The center distance between shafts S and R is to be less than 8½ in. and more than 6 in. Gears C and D are 4-pitch, A and B are 5-pitch. Find the number of teeth in A, B, C and D if no gear is to have less than 12 or more than 70 teeth.

8. A certain machine has a 7:4 ratio, 8-pitch spur gear drive operating at a center distance of 5.500 in. It is desired to change this ratio to 2:1, maintaining the original center distance, and substituting a drive at least as strong as the original. Give complete data as to the substitute drive.

9. A hoist drum 4 ft. in diameter is to lift 4 tons a distance of 30 ft. in 15 seconds, using 1¼-in. diameter wire rope. The drive is through the medium of worm gearing, using an 870-RPM motor. What horsepower motor must be used for the following reduction media: (a) worm gearing, mounted by owner; (b) commercial worm gear reducer.

10. A worm gear set used for hoisting service has a velocity ratio of 14⅓ to 1, with a ½-in. pitch, hardened steel worm, and a phosphor bronze worm gear. The gear face is 1¼ in. and the center distance is 4½ in. The drive is not enclosed and is grease lubricated. The hoist drum is 8 in. in diameter. The worm is driven by a 600-RPM, 5-HP motor.

a. What is the safe load on the set?

b. What is the limiting load for wear?

c. What is the efficiency of this gear set?

d. What will the probable operating temperature be?

e. What load may be safely lifted by the hoist cable?

f. What is the lifting speed in feet per minute?

11. A shaft rotating at 215 RPM drives a second shaft, whose axis is perpendicular to the first, at 95 RPM. The distance between shaft axes may vary between limits of 8.9475 in.-8.9470 in. The drive is through the medium of a pair of 5-pitch spiral gears. Determine the helix angle and number of teeth of the driving gear and give the actual (theoretical) center distance.

12. A pair of shafts rotating at 1200 and 170 RPM transmit 20 HP through 90° bevel gearings. The pinion is of cast steel, the gear of semi-steel. The gearing is to be as small as possible, consistent with good design.

a. Find the pitch and number of teeth for the gearing, using 14½° involute teeth, for intermittent service.

b. Like part *a*, with 20° teeth, continuous service.

13. A fluid agitator with a vertical shaft rotating at 75 to 80 RPM requires 12 HP and operates 8 hours per day. The agitator is to be driven by a worm gear and a direct-coupled 15-HP, 1150-RPM motor. Select a suitable reducer.

CHAPTER 16

SHAFTING AND BEARINGS

16-1. Shafting and bearings are important elements in all power transmission equipment, and the successful operation of such machinery depends largely upon their correct specification and selection.[18,26,36] A shaft is a rotating member transmitting power or motion. A spindle is a shaft that drives and supports cutting tools or work parts on which machining operations are performed. An axle is a stationary shaft on which pulleys or other members rotate, and is subjected to bending stresses only. In practice there are exceptions to these usages; the full-floating type of automobile rear axle carries practically pure torsional stress and rotates; and the code commonly used for shafting design calls a member a shaft regardless of the type of load or stress.

16-2. Shaft Sizes. Transmission shafting for line shafts, head shafts, and countershafts is usually of uniform cylindrical section, and may be obtained in diameters $\frac{1}{16}$ in. under nominal standard sizes, varying from $\frac{5}{16}$ to 2 $\frac{7}{16}$ in. by quarter-inch increments, and up to $5\frac{7}{16}$ in. by half-inch increments. These sizes were established many years ago when shafting was hot-rolled to a nominal size of 1 or $1\frac{1}{4}$ in. and then turned $\frac{1}{16}$ in. smaller in finishing, and are still maintained for reasons of interchangeability. Machine shafts and spindles, such as motor shafts and lathe and drill press spindles, are generally designed to suit the requirements of the particular installation, and are therefore made to standard nominal sizes, not necessarily of uniform diameter.

16-3. Torque and Torsional Stress. A shaft or rod subjected to a pair of opposite couples in parallel planes at right angles to the shaft length is said to be in torsion between these planes. Fig. 16-1 shows this condition, in which shaft S is in torsion under the action of the applied couple $2yF$ at one end, and $2zF'$ at the other end.

The twisting effect of these couples is known as torque, and may be expressed in either foot-pounds or inch-pounds, in a manner similar to bending moment, with this important distinction—that an external bending moment acts on a section perpendicular to the plane of the forces producing the moment and induces tensile and compressive stresses perpendicular to this section, while torque or twisting moment acts on a section parallel to the plane of the forces and induces shearing stresses in the plane of the section.

In considering the shearing stresses induced in a section, the following basic assumptions hold: the section subject to torque is circular, and neither the proportional nor the elastic limit in shear is exceeded. In Fig. 16-2, r and p represent the outer and inner radii of a section of a shaft or circular form subject to a torque T. In this section, consider an

elemental ring whose mean radius is a and whose wall thickness is da. The area of this ring is

$$dA = 2\pi a da$$

If S represents the unit shearing stress at the outer periphery, at radius r, S/r will represent the unit shearing stress at a unit distance from the center of the section, and Sa/r the unit shearing stress at radius a. The total resisting force of this elemental ring will be

$$SadA/r = (2\pi Sa^2/r)da$$

The resisting moment of this resisting force will be equal to the product of the force and its moment arm a, or

$$(2\pi Sa^3/r)da$$

The summation of all the resisting moments of the elemental rings composing the shaft section must be equal to the external torque, or

$$T = (2\pi S/r)\int_p^r a^3 da = (2\pi S/r)(r^4 - p^4)/4$$

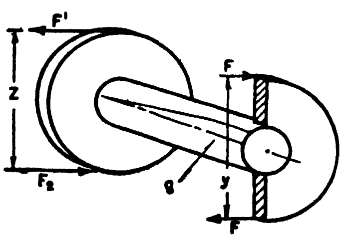


FIG. 16-1. Torsion in a Shaft.

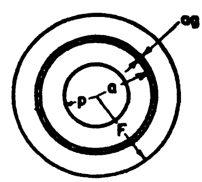


FIG. 16-2. Hollow Shaft Section.

Expressed in terms of the outer diameter D_o and the inner diameter D_i of a hollow shaft:

$$T = (\pi S/16D_o)(D_o^4 - D_i^4) \quad (16\text{-}1)$$

For a solid shaft, where D_i is equal to zero, and D_o is replaced by D:

$$T = \pi SD^3/16 \quad (16\text{-}2)$$

where S is the unit shearing stress, in psi., at the outer periphery of the shaft, and T is the applied torque, in inch-pounds.

16-4. Combined Stresses in Shafting. In addition to torque, most shafts are subjected to flexural stresses because of the beam action resulting from traverse loads applied by gearing, pulleys, or sprockets. The section of the shaft is subjected to a combination of shearing and tensile or compressive forces; since most shafting is made of ductile steels, the resultant shearing stress governs the design.

From Eq. 16-2, the unit shearing stress is equal to $16T/\pi D^3$. The unit flexural stress S_t for a bending moment M in a beam of circular section, from

Eq. 5-13 and Fig. 5-12 is equal to $32M/\pi D^3$. Substituting these values of the shearing and flexural stresses in Eq. 6-3, the resultant shear stress

$$S_m = \sqrt{(16T/\pi D^3)^2 + (32M/\pi D^3)^2/4}$$

or

$$S_m = (16/\pi D^3)\sqrt{T^2 + M^2} \qquad (16\text{-}3)$$

which gives the value of the resultant shearing stress induced by torsional and flexural stresses.

16-5. ASME Code for Shaft Design. Shafting of any character should be designed in accordance with the provisions of the Code for the Design of Power Transmission Shafting of the American Society of Mechanical Engineers, hereinafter referred to as the ASME Shafting Code.[18] Commercial steel shafting used for power transmission is generally hot-rolled from mild steel bars and then cold-rolled or turned in a lathe to the finished size. Since the material is usually ordered or purchased as "shafting," the chemical and physical properties are likely to vary over a rather wide range. On the other hand, some shafting installations (particularly those over 6 in. in diameter) may require a material whose composition and properties can be definitely specified, so that a certain minimum ultimate strength or a definite elastic limit can be used as a basis for design. The ASME Shafting Code recognizes two classes of materials: commercial steel shafting, and steel shafting purchased under definite physical specifications.

For commercial steel shafting, without keyways, the ASME Code recommends a maximum design shearing stress of 8000 psi. For steel shafting purchased under definite physical specifications, without keyways, the recommended design shearing stress may be taken as 30% of the elastic limit of the material in tension, with the further proviso that in no instance shall the design shearing stress exceed 18% of the ultimate strength. For shafting with standard keyways the design stress should be reduced to 75% of the allowable design shearing stress without keyways.

As an example, the allowable design stress S_m to be used in Eq. 16-3 for a 0.20% carbon steel shaft is equal to 30% of 36,000 psi. (from Table 16-1), or 10,800 psi., as this is less than 18% of 65,000 psi. (11,700 psi.). If keyways are to be cut in the shaft, the design stress becomes 75% of 10,800 psi., or 8100 psi. For a shaft made of chrome-vanadium steel, however, 18% of 90,000 psi. equals 16,200 psi., which is less than 30% of 60,000 psi., and thus the lower value based upon the ultimate strength is used as the criterion.

The ASME Shafting Code recognizes that under certain conditions of loading, these stresses must be further reduced. To simplify computation, the torsional and flexural moments are multiplied by numerical combined shock and fatigue factors K and B. Eq. 16-3 thus becomes,

$$S = (16/\pi D^3)\sqrt{(KT)^2 + (BM)^2} \qquad (16\text{-}4)$$

A useful transposition for design purposes is

$$D = \sqrt[3]{\frac{5.09}{S}\sqrt{(KT)^2 + (BM)^2}} \qquad (16\text{-}5)$$

The factor K is 1.0 for a gradually applied or steady load, and varies from 1.5 to 3.0 for suddenly applied loads with major or heavy shocks. The factor B, which is applicable to the flexural moment, is always more than unity for rotating shafts since the tensile or compressive stress at the shaft surface undergoes two stress reversals for every revolution of the shaft. B may be taken as 1.5 for a gradually applied or steady load, may vary from 1.5 to 2.0 for a suddenly applied load with minor shock, and may vary from 2.0 to 3.0 for a suddenly applied load with heavy shock.

TABLE 16-1.—REPRESENTATIVE VALUES FOR SHAFTING STEELS

Material	Approximate SAE Classification	Carbon Content	Ultimate Tensile or Comp. Strength psi.	Elastic Limit in Tension
Forged or hot-rolled steel	1020	.20%	65,000	36,000
Forged or hot-rolled steel	1030	.30%	70,000	40,000
Forged or hot-rolled steel	1040	.40%	75,000	45,000
Forged or hot-rolled steel	1050	.50%	80,000	50,000
3½% Nickel steel		.20%	85,000	55,000
Chrome-vanadium steel		.30%	90,000	60,000

(The ultimate shearing strength of the steels in this table may be taken as 70% of the ultimate tensile or compressive strengths.)

Selection of suitable values for the factors K and B is dependent upon a reasonably accurate knowledge of the operating conditions of the transmission system, and upon experience in design. A condition in which a 50% temporary increase in the normal operating load will occur, even though the load is gradually applied, may require the use of K as 1.5 and B as 2.0. A 100% increase of the normal load for a short time may require values of K of 2.0 and B of 2.5. The latter values would also be used for a suddenly applied load with a temporary increase of 50% in the normal load.

Figs. 16-3 and 16-4 may be used instead of Eq. 16-5 to find the diameter D of a transmission shaft. The curves are based upon an allowable resultant shearing stress of 6000 psi., for values of K and B equal to 1.0. For other values of these factors, find the products KT and BM, and enter the charts on the horizontal and vertical moment-factor scales; the intersection of the ordinates will give the required diameter.

Example 16-1. Fig. 16-5 shows a front view of a commercial steel shaft 2 15/16 in. in diameter. The shaft S rotates at 200 RPM, is supported in bearings L and R, and is driven by a roller chain and sprocket U. The sprocket has 40 teeth, and the chain has a pitch of 1 in. Pulley V is 40 in. in diameter, has a 5-in. face, and drives a machine below

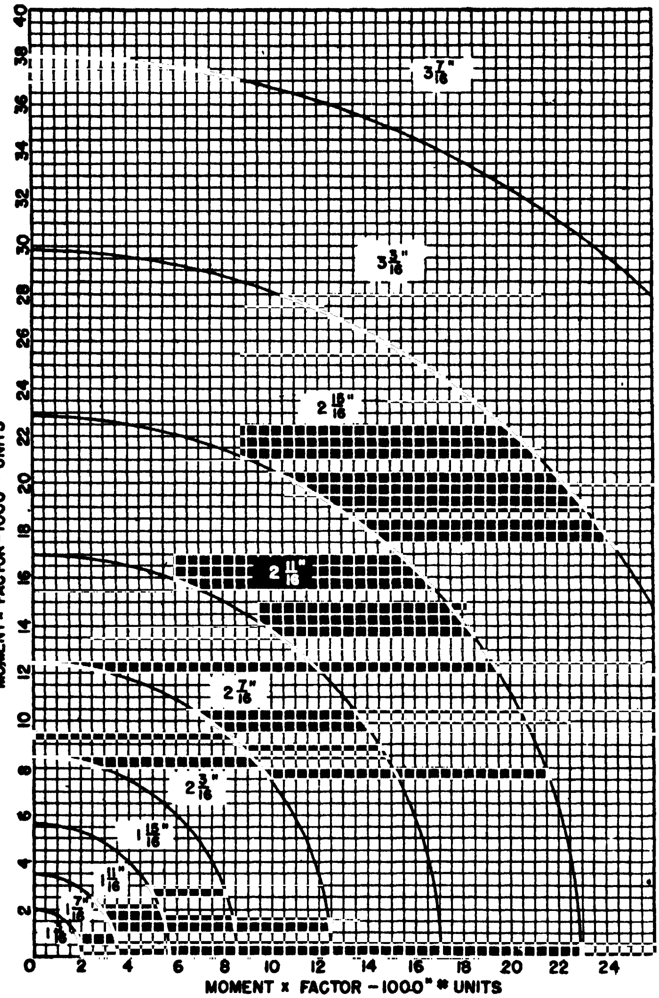


FIG. 16-3. Shaft Diameters Based upon Flexure and Torsion.

shaft S through the medium of a double belt. The belt has an angle of contact on pulley V of 180° and transmits 18 HP. Pulley W is 30 in. in diameter with a 4-in. face, and drives a second machine below shaft S by a double belt, which has an angle of contact of 160° on the pulley, and transmits 10 HP. Both pulleys are made of cast iron; pulley V weighs 100 lbs., and pulley W, 60 lbs. Determine the induced stress in shaft S if the loads are gradually applied with no appreciable overload.

Solution. The first step is to determine the magnitude of the applied torque. From Eq. 14-4, the torque T on pulley W is:

$$T = \frac{63{,}025 \times 10}{200} = 3150 \text{ in.-lbs.}$$

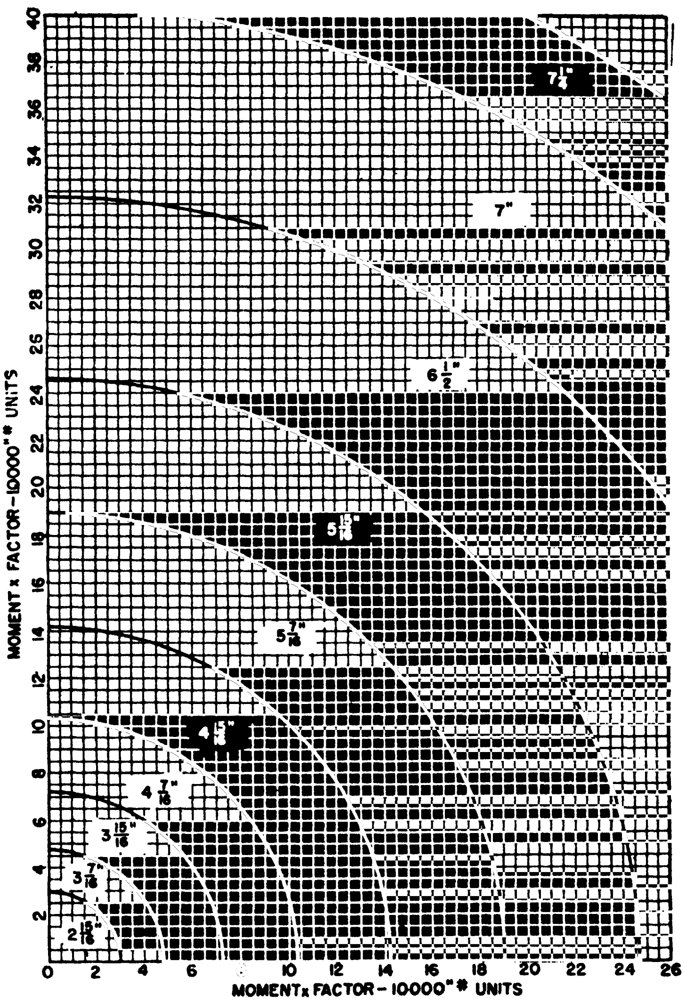


FIG. 16-4. Shaft Diameters Based upon Flexure and Torsion.

The torque on pulley V is

$$T = \frac{63{,}025 \times 18}{200} = 5670 \text{ in.-lbs.}$$

The actual torque between pulley W and pulley V is 3150 in.-lbs. But between pulley V and sprocket U is 3150 + 5670, or 8820 in.-lbs.

The rotative force on the sprocket is

$$E = \frac{2\pi T}{Pn} = \frac{2 \times \pi \times 8820}{1 \times 40} = 1390 \text{ lbs.}$$

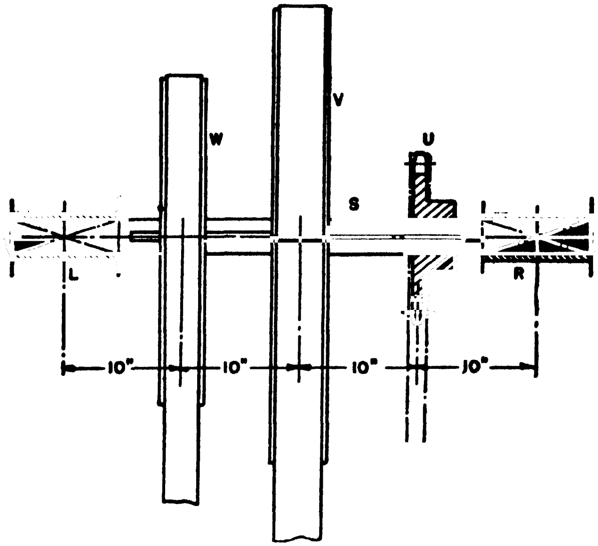


FIG. 16-5. Countershaft Elevation.

and the downward pull exerted by the chain is therefore equal to 1390 lbs., since there is no appreciable tension in the slack side.

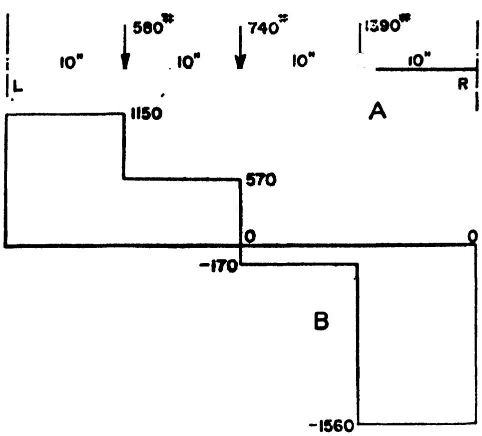


FIG. 16-6. Load and Shear Diagram for Countershaft.

The effective pull E for the belt on pulley W is found by

$$E = \frac{T}{r} = \frac{3150}{15} = 210 \text{ lbs.}$$

The actual force on the shaft is equal to the sum of the tight and loose tensions on the two sides of the belt. The tension ratio p for this belt is found from Fig. 14-6, and has a value of 2.35 for a leather belt on a cast iron pulley with a contact angle of 160°. The force on the shaft from Eq. 14-10,

$$R = \frac{(2.35 + 1)}{(2.35 - 1)} (210) = 520 \text{ lbs.}$$

Similarly, the effective pull E for the belt on pulley V is found to be 5670/20, or 283 lbs.; the tension ratio p for this belt (having a contact angle of 180°) is found from Fig. 14-6 to be 2.60. The force on the shaft is

$$R = \frac{(2.60 + 1)}{(2.60 - 1)} (283) = 640 \text{ lbs.}$$

Fig. 16-6A shows these forces applied to the shaft axis—the load at W is increased by 60 lbs. to allow for the pulley weight, and the load at V by 100 lbs. for the same reason.

The weight of the sprocket is so small that it may be disregarded; further, the shaft weight is approximately 23 lbs. per foot of length and it may also be disregarded.

To find the bearing reactions, moments are taken about the left support L,

$$\Sigma M_L = (L \times 0) + (580 \times 10) + (740 \times 20) + (1390 \times 30) - (R \times 40) = 0$$

or

$$R = \frac{5800 + 14{,}800 + 41{,}700}{40} = 1560 \text{ lbs.}$$

Similarly, for moments about the right bearing,

$$\Sigma M_R = (R \times 0) - (1390 \times 10) - (740 \times 20) - (580 \times 30) + (L \times 30) = 0$$

or

$$L = \frac{13{,}900 + 14{,}800 + 17{,}400}{40} = 1150 \text{ lbs.}$$

Checking these values by a vertical summation:

$$+1150 - 580 - 740 - 1390 + 1560 = 0$$

Fig. 16-6B shows the vertical shear diagram; the position of maximum moment is directly under the centerline of pulley V. The magnitude of the moment at this point is found to be

$$M_V = (+1150 \times 10) + (570 \times 10) = 17{,}200 \text{ in.-lbs.}$$

For a gradually applied load, the service factor values are taken as 1.0 for K and 1.5 for B. Substituting the known values in Eq. 16-4,

$$S = \frac{16}{\pi \times 2.938^3} \sqrt{(1.0 \times 8820)^2 + (1.5 \times 17{,}200)^2} = 5470 \text{ lbs.}$$

This value is less than 6000 psi. and commercial steel shafting is satisfactory for this service.

After the bending and twisting moments have been determined, the charts of Figs. 16-3 and 16-4 are used. The product of the maximum moment and the factor B is $17{,}200 \times 1.5$, or 25,800 in.-lbs.; the product of the twisting moment and the factor K is 8820×1.0, or 8820 in.-lbs. By moving along the 26 and 9 coordinates in Fig. 16-3, their intersection is found to fall within the 2 15/16-in. diameter region, indicating that the shaft size is safe. This method does not, of course, give the actual value of the resultant shearing stress, but it dispenses with the necessity of performing the computation involving Eq. 16-4.

16-6. Loads in Two or More Planes. In many instances, the direction of the forces on the shaft is such that one or more sets of loads lie in different planes. In such cases, all loads can be resolved into horizontal and vertical components, from which the horizontal and vertical components of the reactions may be computed. Horizontal and vertical shear diagrams can be drawn for the load and reaction components, and the maximum flexural moments computed by finding the area of the shear diagram. The maximum bending moment M_m in the shaft is found by combining the horizontal and vertical moments M_h and M_v as follows:

$$M_m = \sqrt{M_h^2 + M_v^2} \tag{16-6}$$

Example 16-2. Find the required diameter of a commercial steel shaft for the conditions given in Example 16-1, with the difference that the loads on pulleys W and V are vertically downward, but the chain drive lies in a horizontal plane.

Solution. Fig. 16-7A represents the load diagram for the belt pulls; by computation, the reactions L and R are found to be 805 and 815 lbs.; Fig. 16-7B shows the shear diagram for this set of forces, which lie in a vertical plane. Fig. 16-7C is a plan or top view of the

shaft with the chain pull of 1390 lbs. in a horizontal plane; the horizontal reactions L and R are found to be 350 and 1040 lbs., respectively. Fig. 16-7D shows the shear diagram for the loading of Fig. 16-7C. In Fig. 16-7, the maximum moment in the vertical plane occurs at the centerline of the pulley V, and the maximum moment in the horizontal plane occurs at the centerline of the sprocket U. In order to determine the maximum resultant moment, it is necessary to compute both horizontal and vertical moments at each of these points, combine each set by Eq. 16-6, and base the shaft design or the shaft stress on the maximum resultant moment.

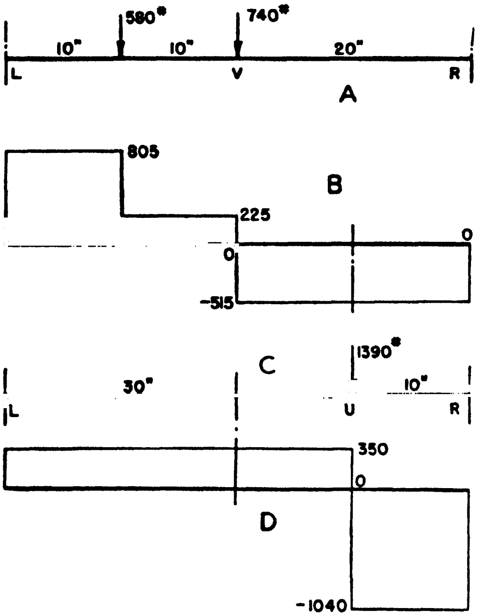


Fig. 16-7. Load and Shear Diagrams for Countershaft.

The moment in the vertical plane at V is equal to 515 × 20, or 10,300 in.-lbs. The moment in the horizontal plane at a corresponding point is equal to 350 × 20, or 7000 in.-lbs. The maximum resultant moment at this point is equal to

$$M_m = \sqrt{7000^2 + 10,300^2} = 12,450 \text{ in.-lbs.}$$

The moment in the horizontal plane at U is equal to 1042 × 10, or 10,420 in.-lbs. The moment in the vertical plane at a corresponding point is equal to 515 × 10, or 5150 in.-lbs. The maximum resultant moment at this point is equal to

$$M_m = \sqrt{10,420^2 + 5150^2} = 11,620 \text{ in.-lbs.}$$

A check value midway between U and V indicates a moment in the vertical plane of 7725 in.-lbs. and a moment in the horizontal plane of 9680 in.-lbs., with a resultant maximum moment at this point of 12,380 in.-lbs. The moment of 12,450 in.-lbs. at point V (obviously the largest) should be used as a basis for stress analysis. If the service factors employed for the first portion of this problem are used, the quantities KT and BM equal 8820 and 18,675 in.-lbs. Reference to Fig. 16-3, shows that the intersection of the 19 and the 9 coordinates is within the $2^{11}/_{16}$-in. diameter range. For this portion of the problem, the next smaller size of shaft would be satisfactory.

Example 16-3. Fig. 16-8A shows the plan view of a shaft W which is driven by a 1200-RPM, 20-HP motor through a gear set Q. The shaft W in turn drives a shaft U at a speed of 100 RPM through a gear set N. The motor pinion has 12 teeth, 4-pitch, and drives a 48-tooth gear on W. The pinion at the right on shaft W has 14 teeth, 3-pitch, and drives a 42-tooth gear on shaft U. All gears have involute teeth with a 20° pressure angle, and gear set Q has a 3-in. face, while gear set N has a 4-in. face. The gears on shaft W are to be placed against the ends of the bearings; the bearing arrangement illustrated must be employed to permit proper clearance between the various elements of the drive. The distance between the centerlines of the faces of the two gear sets must be 50 in. Shaft W is to be made of commercial steel, and the gears are to be keyed in place. The load may be applied suddenly, with slight shock. Find the diameter D of the shaft W and determine the position of the bearings L and R.

Solution. Since the motor V has a speed of 1200 RPM, and the gear set Q a ratio of 48/12, or 4, the speed of shaft W is 300 RPM. From Eq. 14-4 the driving torque on W is

$$T = \frac{63,025 \times 20}{300} = 4200 \text{ in.-lbs.}$$

The gear of set Q has a pitch diameter of 48/4, or 12 in., and the force at the pitch line is equal to 4200/6, or 700 lbs. The pinion of set N has a pitch diameter of 14/3, or 4.667 in., and the force at the pitch line is equal to 4200/2.333, or 1800 lbs. Fig. 16-8B shows the end view of shaft W and the application of these two forces; F is equal to 700 lbs. and acts downward because the gear of set Q is driven by the motor pinion; F' is equal to 1800 lbs., and acts downward because it represents the resistance to rotation induced by the driven gear of set N. (It is of great importance that these force directions, as well as their magnitudes, are correctly determined.)

Weights of the two gears on shaft W can be found in manufacturers' catalogs, or they may be computed with sufficient accuracy by assuming them as solid cylinders with diameters equal to the pitch diameters of the gears. The weight of the gear of set Q is approximately 100 lbs.; that of the pinion of set N about 12 lbs. If the weight of the gear is included and the weight of the pinion disregarded, the forces F and F' are equal to 800 lbs. and 1800 lbs., respectively. With the exception of the shaft weight, these two forces represent the entire

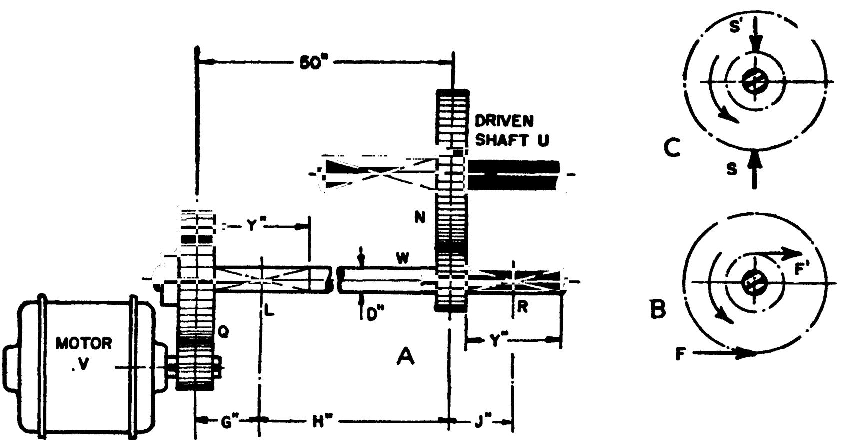


FIG. 16-8. Countershaft and Gear Drive.

vertical loading. The separative forces on the gears are shown in Fig. 16-8C where S represents the action of the separating force of set Q, and S' that of set N, on shaft W; they should be considered in the final analysis, but may be disregarded in this trial computation.

Only the distance between the centerlines of the gear set faces is established and some trial assumptions must be made regarding the bearing lengths. If distances G and J, Fig. 16-8A, are assumed to be 5 in., distance H becomes 45 in., and the load diagram shown in Fig. 16-9A can be set up. The reaction at the bearing R is found by taking moments about the centerline of bearing L:

$$\Sigma M_L = (-800 \times 5) + (1800 \times 45) - (R \times 50) = 0$$

or

$$R = \frac{81{,}000 - 4000}{50} = 1540 \text{ lbs.}$$

(R must induce a counter-clockwise or negative moment, and thus act upward, in order to satisfy the equation.) Moments about R give:

$$\Sigma M_R = (-800 \times 55) + (L \times 50) - (1800 \times 5) = 0$$

or

$$L = \frac{44{,}000 + 9000}{50} = 1060 \text{ lbs.}$$

The vertical summation gives

$$\Sigma V = -800 + 1060 - 1800 + 1540 = 0$$

The vertical shear diagram is shown in Fig. 16-9B and shows that there are two possible positions of maximum moment—one directly under the 1800-lb. load, at point Z, and the other at the left reaction. By inspection, it is clear that the moment at Z is the greater of the two, and has a magnitude of 1540×5, or 7700 in.-lbs.

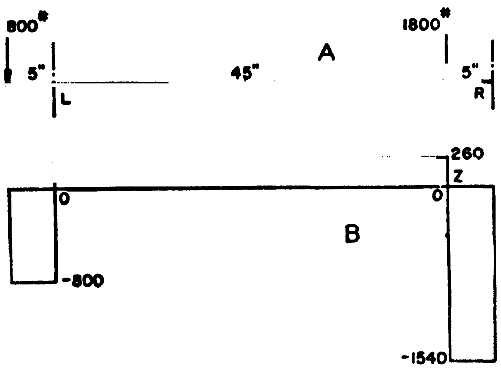


FIG. 16-9. Load and Shear Diagrams for Countershaft.

Since the load is to be applied suddenly, with some possibility of slight shock, it will be advisable to use values of B as 2.0, and K as 1.5 for the service factors. The product of the maximum bending moment and the service factor B is 7700×2.0, or 15,400 in.-lbs., as the design bending moment; the design twisting moment will be equal to 4200×1.5, or 6300 in.-lbs. From Fig. 16-3, the intersection of the coordinates representing these values is seen to fall within the $2\frac{7}{16}$-in. diameter region.

Assuming a $2\frac{7}{16}$-in. diameter shaft, Table 16-5 shows that a bearing suitable for this type of service has a length of $8\frac{3}{4}$ in. The gear set Q has a 3-in. face, and the distance G that will result if the gear is placed against the face of the bearing is $1\frac{1}{2} + 4\frac{3}{8}$, or $5\frac{7}{8}$ in. Similarly, the distance J will be equal to one half the face width of N, or $2 + 4\frac{3}{8}$, or $6\frac{3}{8}$ in. Dimension H will then be equal to $50 - G$, or $44\frac{1}{8}$ in., and the center-to-center distance of bearings L and R will be $H + J$, or $44\frac{1}{8} + 6\frac{3}{8}$, or $50\frac{1}{2}$ in. The revised position of the 1800-lb. load at Z will induce a moment of over 9000 in.-lbs. at that point, which, even without a consideration of the moment induced by the separating forces, will necessitate the next larger size of shaft.

Re-evaluating on the basis of a $2\frac{11}{16}$-in. diameter shaft, a reference to Table 16-5 shows that a bearing $9\frac{3}{4}$ in. long will be required. The distance G will be $1\frac{1}{2} + 4\frac{7}{8}$, or $6\frac{3}{8}$ in.; the distance J will be $2 + 4\frac{7}{8}$, or $6\frac{7}{8}$ in.; H will be $50 - 6\frac{3}{8}$, or $43\frac{5}{8}$ in.; and the center-to-center distance of the bearings will be $H + J$, equal to $43\frac{5}{8} + 6\frac{7}{8}$, or $50\frac{1}{2}$ in. Fig. 16-10A, shows the load diagram for this arrangement. The right reaction is

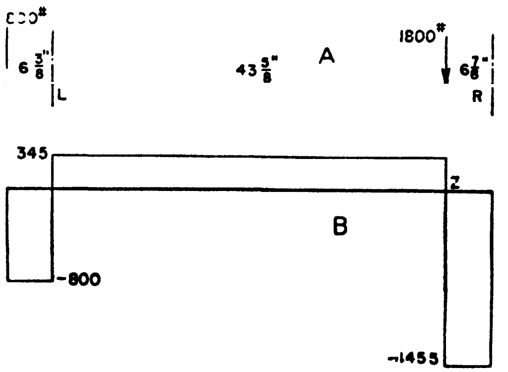


FIG. 16-10. Load and Shear Diagrams for Countershaft.

$$R = \frac{1800 \times 43\frac{5}{8} - (800 \times 6\frac{3}{8})}{50\frac{1}{2}} = 1455 \text{ lbs.}$$

The left reaction is

$$L = \frac{800 \times 56\frac{7}{8} + (1800 \times 6\frac{7}{8})}{50\frac{1}{2}} = 1145 \text{ lbs.}$$

The separating forces between the gears of sets Q and N are found by recalling that he separating force is equal to the product of the driving force at the pitch line and the tangent of the pressure angle of the gearing. The separating force for set Q is then

$$S = F \tan 20° = 700 \tan 20° = 255 \text{ lbs.}$$

and for set N is

$$S' = F' \tan 20° = 1800 \tan 20° = 655 \text{ lbs.}$$

These forces lie in a horizontal plane; their load and shear diagrams are shown in Fig. 16-11. The right reaction is

$$R = \frac{255 \times 6\frac{3}{8} + (655 \times 43\frac{5}{8})}{50\frac{1}{2}} = 600 \text{ lbs.}$$

The left reaction is

$$L = \frac{255 \times 56\frac{7}{8} - (655 \times 6\frac{7}{8})}{50\frac{1}{2}} = 200 \text{ lbs. (toward the front)}$$

The vertical summation gives:

$$+255 - 200 - 655 + 600 = 0$$

From an inspection of the horizontal and vertical shear diagrams of Figs. 16-10 and Fig 16-11, it is clear that the maximum bending moment occurs at the point Z. The magnitude of the bending moment in the vertical plane is $1455 \times 6\frac{7}{8}$, or 10,000 in.-lbs.; the magnitude in the horizontal plane is $600 \times 6\frac{7}{8}$, or 4120 in.-lbs. The combined moment is found by Eq. 16-6, and is equal to $\sqrt{4120^2 + 10{,}000^2}$, or 10,820 in.-lbs.

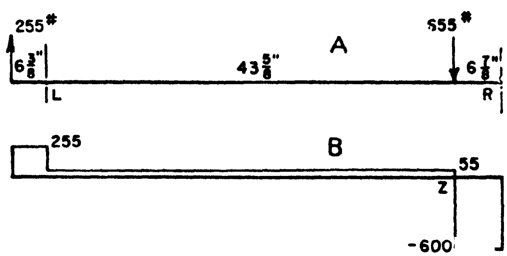


Fig. 16-11. Load and Shear Diagrams for Countershaft.

Using the factors K and B as before, the bending moment on which the design will be based is $10{,}820 \times 2.0$, or 21,460 in.-lbs. The twisting moment is 6300 in.-lbs. From Fig. 16-3 the intersection of the coordinates representing these values falls within the $2\frac{11}{16}$-in. diameter region, so the design may be considered satisfactory.

The actual stress in the shaft is usually of minor importance as long as it is less than 6000 psi., but a check on the results obtained above may be of interest.

Substituting in Eq. 16-4,

$$S = \frac{16}{\pi \times 2.688^3} \sqrt{6300^2 + 21{,}640^2} = 5900 \text{ psi.}$$

16-7. Empirical Design of Shafting. In many cases it is difficult to determine the magnitude of the loads on line shafting, particularly if the member has more than two bearings, which involves continuous beam action and theory. For these reasons, empirical equations to determine the transmitted power are often employed; a representative form is

$$HP = (D^3 \times RPM)/C \tag{16-7}$$

where D is the shaft diameter, and C is a constant with a value of 50 for transmission shafts subjected to torsion only, 80 for line shafting subjected to limited bending loads, and 135 for main or head shafts subjected to heavy bending loads.

The distance between bearings on line shafts and countershafts is limited by excessive shaft bending and bearing wear. An expression which can be used as a guide for such spacing is given by

$$L = D + 5 \tag{16-8}$$

where L is the distance between bearing centers in feet, and D is the shaft diameter in inches. Values of L are satisfactory for average load conditions if the shaft speed does not exceed 400 RPM.

16-8. Critical Speed. Because of lack of homogeneity of a body caused by manufacturing difficulties and variations in material densities, it is impossible to distribute the mass of a rotating body about its geometric center. If the shaft on which the body rotates deflects under load, the center of mass may move from the axis of rotation of the shaft. Shaft rotation will then begin about the geometric axis but at some speed the centrifugal force caused by the displacement of the center of mass will equal the deflecting forces on the shaft. The deflecting force may be opposite to and balance the centrifugal force, or the two may be in phase, causing a periodic variation in the radial force on the shaft, which induces a series of vibrations. Since the magnitude of the centrifugal force depends upon the angular velocity and the mass, the vibrations will attain a maximum value at some speed, which is called the critical speed of the shaft. Above the critical speed a state of equilibrium may again be attained in which the body virtually rotates about its mass center. Second, third, and fourth critical speeds are also possible but the amplitudes of the shaft vibration are progressively less.

This phenomenon occurs in both horizontal and vertical shafts, and is of particular importance in such applications as vertical shaft mixers, in which the shaft is guided by two bearings and has a long extension on which the impeller is mounted. In one application a 24-in. diameter turbine type impeller, weighing 104 lbs. and requiring power input of 1 HP.at 100 RPM, gave the following representative data for varying shaft extensions:

Theoretical Diameter	Actual Diameter Used	Critical Speeds for Various Extensions RPM (Variable liquid level)			
		5 ft.	6 ft	7 ft.	8 ft.
7/8	1 3/4	110	95	80	70
7/8	2	140	125	110	100
7/8	2 1/4	170	145	130	115

From the standpoint of pure torsion a 7/8-in. shaft (based on an allowable unit stress of 5000 psi.) was satisfactory. The smallest shaft that was actually used was 1 3/4 in. in diameter; for an extension of 5 ft. beyond the bearing, the critical speed is 110 RPM, which is sufficiently above the operating speed to permit satisfactory operation. If this shaft, however, is used with an extension of 6 ft. instead of 5 ft., the operating speed would coincide so closely with the actual speed that serious danger of failure would result.

Several other items are of interest in this connection; it has been found that the critical speed is of importance only when the mixing shafts operate with a variable level of fluid. If the impeller is completely immersed in fluid at all times and the shaft is immersed for approximately one fourth of its extension, the fluid has a sufficient dampening effect greatly to increase the critical speed. Unfortunately most problems requiring the use of mixing equipment deal with variable fluid levels. It has been the experience of one of the leading manufacturers of mixing equipment that shaft extensions over 8 ft. long, no matter what the diameter, are impractical with a variable level tank. When extensions exceed this length, the increase in stiffness obtained by increasing the shaft diameter is more than accounted for by the increase in the weight of the shaft.

16-9. Key Application and Selection. Keys are used to prevent relative motion between machine members, and are usually made of steel of rectangular or square section. Several important types of keys are shown in Figs. 16-12 and 16-13. The square or flat key, shown at the left in Fig. 16-12, should be carefully fitted to the shaft keyway. The keyway has a "radius-runout" at one end, which results from stopping the longitudinal feed of the milling cutter. This construction is less expensive than the semi-cylindrical end key and keyway, known as a "drop seat," or "Pratt and Whitney" keyseat. The latter, however, is often preferred to the radius-runout keyway, particularly if the key is close to a bearing, for if any portion of the keyseat extends inside the bearing, it can act as a channel to drain out the lubricant.

TABLE 16-2.—DIMENSIONS OF STANDARD FLAT AND GIBHEAD KEYS (FIG. 16-12) INCHES

Shaft Size	*b*	*t*	*c*	*f*	*e*
15/16 to 1 1/4	1/4	3/16	5/16	1/4	3/16
1 5/16 to 1 3/4	3/8	1/4	7/16	3/8	5/16
1 13/16 to 2 1/4	1/2	3/8	5/8	1/2	7/16
2 5/16 to 2 3/4	5/8	7/16	3/4	5/8	1/2
2 7/8 to 3 1/4	3/4	1/2	7/8	3/4	5/8
3 3/8 to 3 3/4	7/8	5/8	1 1/16	7/8	3/4

Gibhead keys, Fig. 16-12, right, have a tapered upper surface which induces a large frictional force between the shaft and the hub when the key is driven into place. The contact prevents axial motion and assists materially in the transmission of power. Gibhead keys are provided with heads to enable them to be easily removed if one end of the keyway is inaccessible. The projecting

head may be a source of danger, and gibhead keys should not be used for moderate or high speed power transmission unless a suitable guard is provided.

Key design should be based upon the assumption that the key is as strong as the shaft. Frequently, however, the key is intentionally made weaker than the shaft and other parts, so that if failure occurs, the comparatively inexpensive key will be the only replacement required. By the latter principle, the allowable

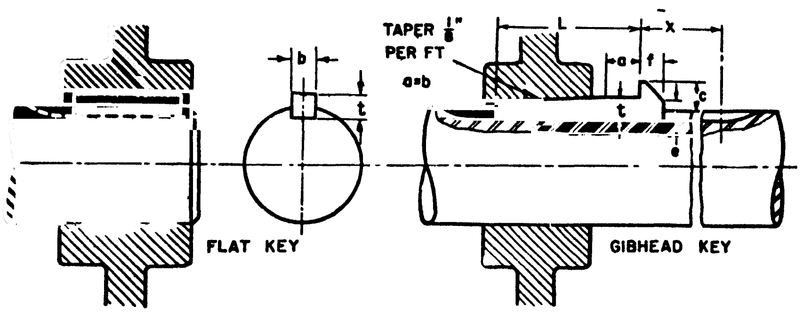


FIG. 16-12. Flat and Gibhead Keys.

shearing stress S_s in the key should be based upon the ultimate shearing strength of the key material divided by the shock and fatigue factor K, and is equal to $10{,}000/K$ for mild steel, and $12{,}000/K$ for machinery steel equivalent to SAE 1025. Because the effective portion of the key is completely enclosed by the shaft and hub, the allowable compressive or bearing stresses S_b may be quite high, and are usually taken as $23{,}000/K$ for mild steel, and $28{,}000/K$ for machinery steel.

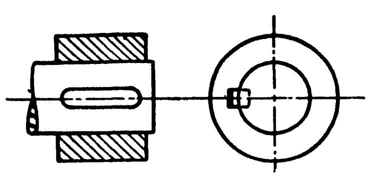


FIG. 16-13. Drop-seat Key.

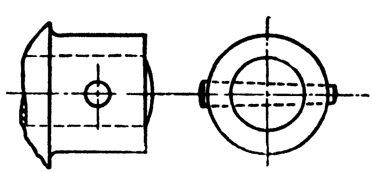


FIG. 16-14. Taper Pin Key.

The allowable load F that can be transmitted by a key of rectangular or square section is

$$F = bLS_s \qquad (16\text{-}9)$$

or

$$F = tLS_b/2 \qquad (16\text{-}10)$$

where b is the width, t the height, and L the effective length of the key.

A feather key is one which permits axial motion of the hub along the shaft, while preventing relative rotation of the two elements. Feather keys are frequently employed for gear transmissions in variable speed drives, and for jaw

or friction clutches. Feather keys are attached to either the hub or shaft; integral feather keys are termed splines or splined fittings. Multiple-splined fittings are preferred to a single feather key, for the axial force necessary to move a member along a shaft with two or more equally spaced keys is only one half that required to move the hub if a single key is present.

Taper pins are sometimes used as transverse keys for light drives, as illustrated in Fig. 16-14. The pin serves to prevent axial, as well as radial, motion. The transmission capacity of the pin is found from

$$T = 0.785 S_s D d^2 \qquad (16\text{-}11)$$

where T is the shaft torque and d is the mean diameter of the pin. Computation for the compressive strength of the pin is not necessary. Allowable values of the unit shearing stress may be the same as those for keys. Straight dowels, forced into a reamed hole in the hub and shaft, are also used as transverse keys.

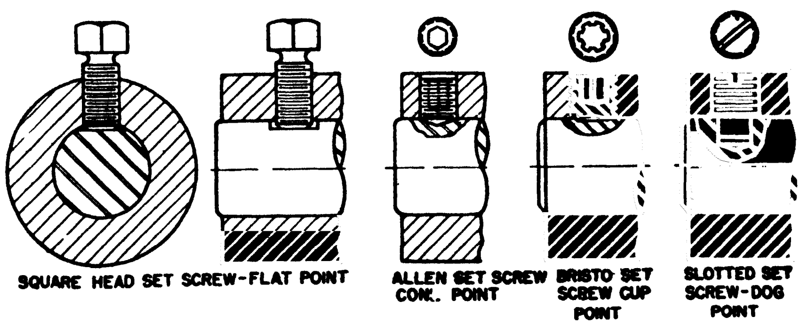


FIG. 16-15. Set Screw Types and Applications.

Transverse keys should not be used for heavy service, or for reversing drives. In the latter instance, there is danger of loosening the key by the reversal load

For some types of drives, where sudden shock or overload may occur, a shear pin may be used. A shear pin is one so designed that failure will occur by shearing or breaking the pin before any serious overload can be transmitted to the rest of the mechanism. One form is used in pulley drives; the pulley is driven by a shear pin connected to a disk keyed to the shaft. The shear pin is recessed or "necked" so that failure will occur at a load slightly greater than normal. Shear pins are usually designed so that they may be removed and replaced in a few minutes. In some cases, keys are deliberately made weaker than the shaft or other members, so that failure will occur in the comparatively inexpensive element.

16-10. Set Screw Application and Selection. Set screws prevent relative motion by pressure exerted on their points. Several varieties are shown in Fig. 16-15. Square head set screws have heads in which the distance across

the flats is equal to the diameter of the screw, with a height equal to three fourths the screw diameter. Safety or headless set screws may be set so that they do not project above the surfaces of the hub; Allen and Bristo screws require special wrenches, but can be seated more firmly than headless set screws with screwdriver slots. Any of the varieties of screws shown in Fig. 16-15 are obtainable commercially with all types of points shown. Cone and cup point set screws raise burrs on the shaft, and create difficulties in disassembly; shafts are usually made with a "flat," or "spotted" with the point of a drill, to provide a secure seat for the screw and to eliminate burring.

An empirical equation for the size of a set screw is

$$d = 0.25 + (D/8) \tag{16-12}$$

where d is the set screw diameter and D the shaft diameter. The transmission capacity of cup point set screws is given by

$$F = 2500\, d^{2.3} \tag{16-13}$$

where F is the resisting force at the surface of the shaft.

16-11. Torsional Rigidity. Shafts of small diameter and considerable length may not possess sufficient torsional or flexural rigidity to transmit power with a uniform steady motion, or to eliminate excessive deflection between bearings. The latter may be checked by considering the shaft as a beam supported at the bearings, and computing the deflection. Line shafting should be limited to a transverse deflection of 0.01 in. per foot of span. In other cases, notably in the design of shaft for electric motors and generators, the deflection rather than the flexural or torsional moments may control the selection of the shaft diameter; design of this character, however, is beyond the scope of this text.

A problem frequently encountered is one in which a shaft is used as a control rod for valve operation or indication. In such cases, the torsional rigidity is extremely important. For a solid rod of circular cross section, the angular deflection A, in degrees, is given by

$$A = TL/20{,}500D^4 \tag{16-14}$$

where T is the twisting moment, in inch-pounds, and L and D are the length and shaft diameter in inches.

16-12. Couplings and Clutches. Couplings are generally used for connection between collinear shafts, where disconnection is required only in case of repairs or other special considerations. Couplings are of two types—rigid and flexible. The sleeve coupling, shown in Fig. 14-2, is one of the simplest and least expensive of all rigid couplings; there are several available commercial varieties, but sleeve couplings are often fabricated for light drives. For such applications, the outer diameter of the coupling should be equal to about twice the shaft diameter, with a length equal to three times the shaft diameter. For transmitting moderate or large amounts of power, sleeve couplings should be

furnished with both keys and set screws; the latter alone, however, will suffice for light drives. Another form of rigid coupling is the so-called compression coupling, which is made in semi-cylindrical halves bolted together, and may be removed without displacing the shafts. In some instances, rigid couplings are fitted with a shear pin (section 16-9) to eliminate the possibility of overload or shock damage to the machine elements.

Perfect alignment of two theoretically collinear shafts is practically impossible to attain, and still more difficult to maintain because of bearing wear and other causes. Slightly misaligned shafts will undergo continuous stress reversal and cause excessive bearing wear. For these reasons, some form of flexible coupling is usually employed for moderate or heavy-duty transmission service, where installation is affected by millwrights or maintenance men. Such couplings prevent the transmission of shock from one shaft to the other and eliminate stress reversals when either shaft is subjected to deflection at or near the coupling.

There are numerous forms of flexible couplings; one type is illustrated in Fig. 16-16 and consists of a pair of cast iron flanges with integral lugs, which engage openings in a flexible intermediate member, made of fiber, leather, or rubber. This member permits individual deflection of the flanges (which are rigidly fastened to the shafts), acts as an insulator, and provides noiseless operation. Table 16-3 gives necessary dimensions, and the power capacities at 100 RPM; capacities at other speeds are in direct proportion.

TABLE 16-3.—CAPACITIES AND DIMENSIONS OF FLEXIBLE COUPLINGS (FIG. 16-16) INCHES

No.	HP at 100 RPM	Max. Bore	*A*	*B*	*C*
4	1	1	4	4¼	1 13/16
5	1½	1¼	5	5	2 3/16
6	3	1⅝	6	5⅝	2½
7	5	1¾	7	5⅞	2 9/16
8	8	2	8	6½	2⅞
9	13	2¼	9	7¼	3 3/16
10	18	2½	10	7⅞	3½
12	35	3	12	9⅝	4 1/16

The capacities listed are suitable for uniform load, with electric motor drive. for such service as drives for liquid agitators, centrifugal blowers, brew kettles, light and normal duty line shafting, and centrifugal pumps. For moderate shock loads with electric motor drive, such as beaters, centrifugal and

reciprocating compressors, grinders, rotary kilns, heavy service line shafting, and ball, pebble, and tube mills, a coupling with a 50% excess capacity should be selected. For heavy shock loads, such as single or two-cylinder compressors, reciprocating conveyors and feeders, and hammer mills, a coupling with an excess capacity of 100% is desirable. The load to be transmitted should be

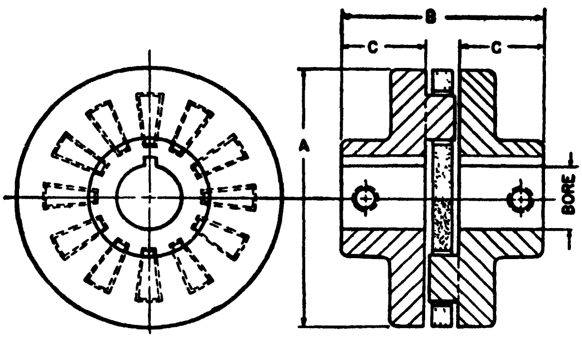


FIG. 16-16. Flexible Coupling.

increased by about 25% if a steam or gasoline engine is the driving medium, and by about 50% if the drive is subjected to 24-hour service.

Universal joints are rigid couplings that connect shaft whose axes will intersect if prolonged. The coupling shown in Fig. 16-17 consists of two forks *F* which fit on the shafts *S*, and to which they are held by set screws *W* and Pratt & Whitney keys *K*. The connection between the forks is by pins *P*

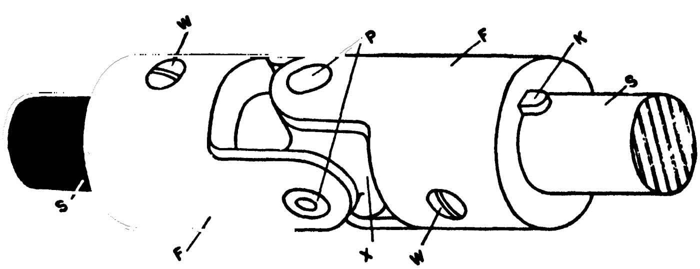


FIG. 16-17. Universal Joint.

fitting in a cross *X*. The angle between the shaft axes may vary slightly during operation, but universal joints should not be used to compensate for excessive misalignment.

The Oldhams, or cross-keyed, coupling is a rigid coupling for connecting shafts whose axes are parallel and a short distance apart, and consists of two coupling halves which are fastened to the shafts and a central member with perpendicular tongues that engage slots in the halves. In the position shown in

Fig. 16-18, the left half of the coupling can move from front to back, and the right half up or down. This combined action takes care of the non-coincident alignment of the shafts. Cross-keyed couplings are sometimes used as flexible couplings; in such cases the central member is made of fiber, or has leather-faced contact surfaces.



FIG. 16-18. Oldhams Coupling.

16-13. Clutch Application. Clutches are connection media used where frequent starting and stopping of the driven shaft is required. Two types are commonly employed; positive clutches and friction clutches. Jaw clutches, shown in Fig. 16-19, are slow-speed service positive connectors; square-tooth clutches can transmit motion in either direction; the other types can transmit motion only as indicated. The square-tooth clutch is difficult to engage or disengage under load, while the two-tooth clutch permits ready engagement, but all jaw clutches transmit considerable shock if engaged under load, particularly at high velocities. Jaw clutches for standard transmission shafting, with shifting apparatus, are listed in catalogs of power transmission equipment manufacturers.

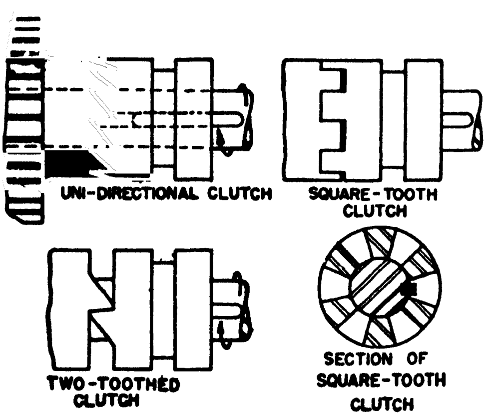


FIG. 16-19. Jaw Clutches.

Friction clutches similar to those of Fig. 14-3, and single- and multiple-disk clutches are extensively employed for both high- and low-speed service; although they are non-positive, they will permit engagement without excessive shock or noise between stationary and rotating members, or between members rotating at different speeds. Numerous forms of friction clutches for a variety of service can be found in manufacturers' catalogs; the problem of clutch selection is essentially similar to that of coupling selection, being dependent upon the power, speed, and type of service.

SLIDING BEARINGS

16-14. Bearing Theory. Bearings are used to support and guide rotating, oscillating, or reciprocating elements, and are classified as sliding or rolling

bearings. Sliding bearings for rotating elements are called journal bearings, and are composed of two essential parts: the journal, which is the inner cylindrical or conical part and which usually rotates, and the surrounding shell or bearing which is usually stationary.

The sliding action between the outer surface of the journal and the inner surface of the bearing results in friction, and is modified by the presence of a film of lubricating oil. Under the proper conditions of oil viscosity, pressure, and surface speed, the oil is forced between the contacting surfaces to build up a fluid pressure and a continuous oil film under the load; the frictional force that is present is due to the force necessary to shear the oil film and does not depend, theoretically, upon the materials of the journal and bearing. This condition is referred to as fluid-film lubrication. Continuous oil films are difficult to maintain in slow-speed, heavily loaded rotating bearings, and in oscillating or reciprocating bearings. Such elements are said to be imperfectly lubricated and more or less frequent metal-to-metal contact may be anticipated; the type and character of the metal surfaces is therefore of importance. Incidentally, the surface materials of fluid-film lubricated bearings are important in design because every journal must start and stop at some time in the operation of the bearing, and fluid-film lubrication is impossible of attainment until the moving surfaces have attained certain relative speeds.

In general, unlike materials, such as cast iron and hardened steel, Babbitt metal and heat-treated steel, or bronze and hardened steel, operate best as bearing and journal materials. Lubricated cast iron surfaces used for both elements are an exception to this rule, for they operate very satisfactorily after a suitable running-in period, particularly in reciprocating bearings. The member that is most easily replaced is usually made of the softer material.

16-15. Types and Methods of Lubricating Plain Bearings. Intermittent lubrication may be provided by using grease or oil; the lubricant is usually applied by the operator through an oil hole, oil cup, or grease cup. Limited lubrication insures a continuous supply of a limited quantity of the lubricant, and can be effected by a drop feed oil cup which permits a constant supply of oil through an adjustable needle valve, or by a pad or wick which presses against the journal as it rotates, thereby permitting the oil to flow to the contact surfaces by capillary action. Continuous lubrication insures an adequate supply of oil to the bearing surfaces. Ring and chain oiled bearings have a loose ring or chain resting on the journal and bringing oil from an oil reservoir in the bearing housing to the top of the journal as it rotates. In bath lubrication, the journal is partly or wholly submerged in a pool of oil. Splash lubrication is used on reciprocating mechanisms, as in internal combustion engines where the shaft is enclosed and the reciprocating member can dip into a reservoir of oil at each stroke. Pressure lubrication requires a circulating system where the oil is pumped from a reservoir to the bearing and returns by gravity to the reservoir.

Fig. 16-20 shows a gibbed pillow block for transmission shafting; the unit consists of a base *F* and a cap *C*, which is held to the base by four studs *S* and nuts *N*; alignment of the two is furnished by the gib *G* on the cap, which fits into a carefully machined groove or slot in the base. The bearing surfaces of the base and cap are lined with Babbitt metal, which is cast and locked in place

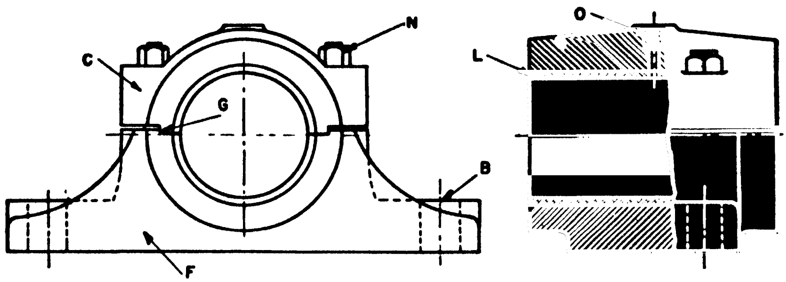


Fig. 16-20. Babbitt-lined Rigid Split Pillow Block.

by recesses or anchors in these members. The bearing is lubricated by a drop feed oil cup or a grease cup screwed into the threaded hole *O*. Bearings of this character are also made in solid form, with an integral cap; these are less expensive than split bearings, but require removal of pulleys or other members if the shaft is to be removed from the bearings. The entire shaft assembly can usually be removed from split bearings by taking off the cap. Pillow blocks are fastened in position by cap screws or foundation bolts in the cored bolt holes *B*; the hole is sufficiently larger than the bolt to permit shaft and bearing alignment.

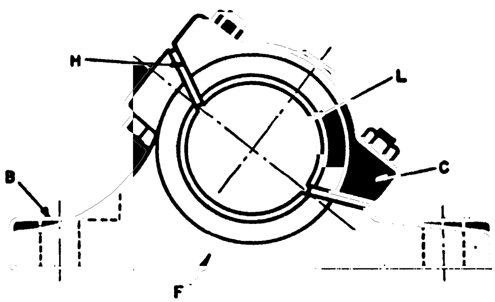


Fig. 16-21. Angular Cap Split Pillow Black.

The pillow block shown in Fig. 16-20 is suitable for loads whose resultant direction is vertical or inclined at an angle not exceeding 60° with the vertical; a load direction parallel to the base of the bearing will cause the greatest film pressure concentration at the juncture of the cap and base, which tends to break the oil film at that point. For such a load, the angle pillow block shown in Fig. 16-21 may be more satisfactory; this type of bearing can also be mounted on vertical as well as horizontal supports. The bearing of Fig. 16-21 differs to some extent from that of Fig. 16-20 in that the base and cap are not gibbed; the unit is provided with a shim *H*, and may be easily increased or decreased in size to vary the bearing clearance or to compensate approximately for the

effects of wear. Important selection and installation dimensions and data for these types of bearings are given in Table 16-4. It should be noted that bearing dimensions are not standardized, and are not alike for all manufacturers; the designer should refer to manufacturers' or suppliers' catalogs for up-to-the-minute dimensional data.

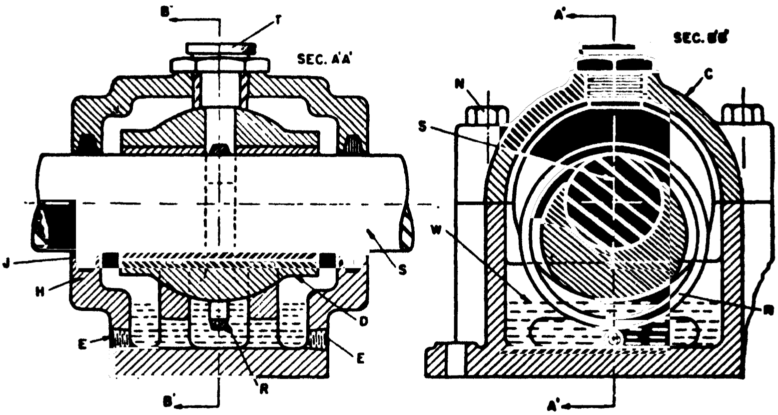


FIG. 16-22. Ring-oiling Pillow Block.

Fig. 16-22 shows a heavy-duty, Babbitt-lined, ring-oiled bearing in which the shaft *S* rotates in a sleeve *D* whose spherical exterior is seated in a corresponding recess in the housing base *H*, and held in place by the hollow screw *T* which also serves as a passage for replenishing the lubricant. The function of the spherical seat is to permit the bearing to accommodate itself to the deflection of the shaft, as shown at *A*, Fig. 16-23. Self-aligning bearings are usually more satisfactory than rigid bearings, but should not be considered a panacea for incorrectly installed or misaligned shafting. A self-aligning bearing may, however, be of no appreciable value in an overhung load condition, such as shown at C, Fig. 16-23.

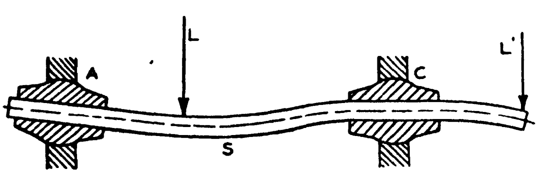


FIG. 16-23. Effect of Shaft Deflection on Self-aligning Bearings.

The bearings shown in Figs. 16-20 and 16-21 have the disadvantage that the lubricant may flow out at the ends. If regular oil replacement is not provided, the bearing may generate excessive heat and seize or score. In the bearing of Fig. 16-22, any lubricant that escapes at the ends runs into the reservoir *W* in the base of the housing *H*, and is recirculated through the bearing by the oil ring *R* bringing oil to the upper surface of the shaft which

TABLE 16-4.—RIGID SPLIT PILLOW BLOCKS
INCHES

Shaft Diam.	Overall Length	Bearing Height	Number of Bolts	Bolt Diam.	Length of Base	Width of Base	Transverse Distance Between Bolts	Axial Distance between Bolts
15/16	2	1 1/8	2	3/8	1 3/8	5 1/2	4 1/4	
1 3/16	2 1/2	1 3/8	2	3/8	1 5/8	6 1/4	5	
1 7/16	3	1 1/2	2	1/2	2	7 1/2	5 3/4	
1 11/16	3 1/2	1 3/4	2	1/2	2 1/4	8 1/4	6 1/2	
1 15/16	4	2	2	5/8	2 3/4	9	7	
2 3/16	4 1/2	2 1/4	2	5/8	3	9 1/2	7 1/2	
2 7/16	5	2 1/2	2	3/4	3 1/4	10 1/4	8	
2 11/16	6	2 3/4	2	7/8	4	12 1/4	9 1/2	
2 15/16	6	2 3/4	2	7/8	4	12 1/4	9 1/2	
3 7/16	7	3 1/4	4	3/4	5	13	10 1/2	2 3/4
3 15/16	8	3 1/2	4	3/4	5 1/2	14 3/4	12	3
4 7/16	9	4 1/8	4	7/8	6 1/4	16 1/2	13 1/2	3 1/2

disperses it radially and axially by its wiping action. The housing H is supplied with two threaded holes E; one holds a sight gage which determines the oil level; the other is fitted with a pipe plug for draining the housing. Each end of the cap and base is provided with a circumferential groove that holds a felt washer J, which serves the dual function of retaining the oil within the housing and keeping dirt or foreign matter out of the bearing. Representative dimensions for the bearing of Fig. 16-22 are given in Table 16-5.

Fig. 16-24 shows a Babbitt-lined bearing for overhead transmission shafting. The bearing is split so that it may be disassembled readily, and is lubricated by two oil rings. This bearing is shown supported by two positioning screws in

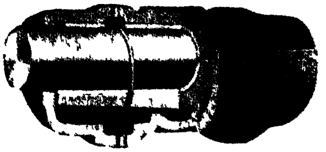


FIG. 16-24. Ring-oiled Hanger Bearing.

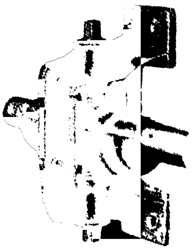


FIG. 16-26. Wall or Post Hanger.

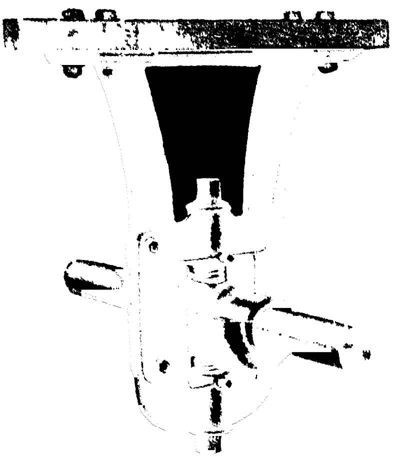


FIG. 16-25. Drop Hanger with Ring-oiled Bearing.

the drop hanger of Fig. 16-25; the screws are used to secure bearing and shafting alignment, and have cupped ends which fit spherical seats on the bearing housing, so that the bearings are permitted some self-alignment for shaft deflection. Hangers of cast iron, as illustrated, or of pressed steel are commercially obtainable; hangers may be attached to wooden ceiling joists by through bolts, as illustrated, or by lag screws or hanger bolts. Steel girder clamps for attaching hangers to the lower flanges of I beams and channels are also obtainable. Fig. 16-26 shows a wall or post hanger, for attachment to vertical surfaces, such as columns or walls. Drop hangers are available in a shaft size range from $1\frac{5}{16}$ to $4\frac{15}{16}$ in.; the "drop," or distance from the shaft axis to the hanger footing, may vary from 8 to 36 in., depending upon the size and type. Dimensions of hangers and hanger bearings can be obtained from power transmission equipment catalogs.

TABLE 16-5.—RING-OILING PILLOW BLOCKS

INCHES

Shaft	Overall Length	Bearing Length	Bearing Height	Number of Bolts	Bolt Diam.	Length of Base	Width of Base	Transverse Distance Between Bolts	Axial Distance Between Bolts
$1\frac{7}{16}$	$5\frac{1}{4}$	$3\frac{1}{2}$	2	2	$\frac{1}{2}$	$3\frac{1}{2}$	7	5	0
$1\frac{11}{16}$	$6\frac{1}{4}$	$4\frac{1}{4}$	$2\frac{1}{4}$	2	$\frac{5}{8}$	4	8	6	0
$1\frac{15}{16}$	7	$4\frac{3}{4}$	$2\frac{9}{16}$	2	$\frac{5}{8}$	5	9	$6\frac{1}{2}$	0
$2\frac{3}{16}$	8	$5\frac{1}{2}$	$2\frac{7}{8}$	2	$\frac{3}{4}$	$5\frac{1}{2}$	10	7	0
$2\frac{7}{16}$	$8\frac{3}{4}$	6	$3\frac{3}{8}$	2	$\frac{7}{8}$	6	11	8	0
$2\frac{11}{16}$	$9\frac{3}{4}$	$6\frac{1}{2}$	$3\frac{7}{16}$	2	$\frac{7}{8}$	$6\frac{1}{2}$	12	$8\frac{1}{2}$	0
$2\frac{15}{16}$	$10\frac{1}{2}$	7	$3\frac{15}{16}$	4	$\frac{5}{8}$	7	$12\frac{1}{2}$	9	$3\frac{1}{2}$
$3\frac{7}{16}$	$12\frac{1}{2}$	$8\frac{1}{2}$	$4\frac{3}{8}$	4	$\frac{5}{8}$	8	15	$10\frac{1}{2}$	4
$3\frac{15}{16}$	$14\frac{1}{4}$	$9\frac{1}{2}$	5	4	$\frac{3}{4}$	10	17	$12\frac{1}{2}$	5
$4\frac{7}{16}$	16	11	$5\frac{1}{2}$	4	$\frac{7}{8}$	11	$18\frac{1}{2}$	$13\frac{1}{2}$	$5\frac{1}{2}$

16-16. Plain Bearing Selection and Design. Many bearings now in use, particularly line shaft, countershaft, and head shaft bearings, in which the entire drive is assembled and installed by the purchaser, should be considered in the incomplete-film or limited lubrication classification. Since there is very little theoretical basis for the design of such equipment, the selection of a suitable bearing is generally dependent upon past experience and experimental data.[46] Such selection is usually based upon three important factors: the load-carrying capacity, the frictional effects, and the ultimate cost.

The load-carrying capacity of a journal bearing depends upon the unit bearing pressure and the rubbing velocity between the bearing surfaces. Unit bearing pressure is determined by dividing the radial load on the bearing by the projected area of the bearing, or the product of the length and the diameter, and is given as pounds per square inch of projected area (psipa.). A 3-in. diameter, 10-in. long bearing, for example, subjected to a load of 1500 lbs., has a unit bearing pressure of 50 psipa. Allowable unit pressures for countershaft and head shaft bearings vary from 15 to 25 psipa. for cast iron bearings; from 50 to 200 psipa. for Babbitted bearings; and up to 400 psipa. for bronze bearings. Values of 100 psipa. for Babbitted, and of 150 psipa. for bronze, bearings for transmission shafting may be used if the lubrication is good; manually or intermittently lubricated bearings should be designed on the basis of lower values.

The minimum journal diameter is usually determined by the shaft strength or stiffness; the bearing length by selecting such bearings as are available commercially. The length-to-diameter ratio of countershaft and head shaft bearing generally varies between 2 and 3. In bearings whose length is much more than three times the diameter a uniform load distribution is sometimes difficult to effect, particularly if the shaft is subject to appreciable deflection. Bearings with a length-to-diameter ratio of less than 2 are likely to suffer from leakage of the lubricant at the ends, making it difficult to establish even a partial oil film. In some instances it may be advisable to reduce the unit bearing pressure by increasing the shaft diameter, even though consideration of strength or stiffness do not necessitate the change.

The load-carrying capacity of a bearing is also dependent upon the velocity of rubbing, which is found by the following equation

$$V = 0.262\,DN \qquad (16\text{-}15)$$

where V is the sliding velocity in feet per minute, D is the diameter of the journal in inches, and N is the number of revolutions per minute. Medium and moderately high rubbing velocities assist in the film formation, but excessively high velocities may cause an excessive increase in the frictional work. As previously stated, it is difficult to obtain any appreciable film formation with very slow rubbing speeds.

In some installations the bearing selection is based upon a constant C, which is equal to the product of the rubbing velocity V and the unit pressure P.

Values for this constant vary from 1400 for low-speed machine tool bearings up to 25,000 for countershaft and head shaft bearings. Use of the latter constant is probably on the safe side for Babbitted or bronze bearings.

TABLE 16-6.—VALUES OF THE BEARING FACTOR K

Bearing Type and Operation	K
A—Oil bath or flooded lubrication Hand-scraped, self-aligning bearings Very careful maintenance and attendance Clean and protected location	0.006
B—Ring-oiled or wick-oiled lubrication Hand-scraped, self-aligning bearings Usual weekly or semi-weekly maintenance Ordinary location	0.009
C—Ring or wick-oiled lubrication Bored or reamed rigid bearings, careful alignment Usual maintenance Ordinary location	0.012
D—Constant drop-feed lubrication (sight-feed oil cup) Hand-scraped, self-aligning bearings Usual maintenance Ordinary location	0.012
E—Constant drop-feed lubrication Bored or reamed rigid bearings, ordinary alignment Usual maintenance Ordinary location	0.015
F—Oil-cup or grease lubricated bearing Bored or reamed rigid bearings, ordinary alignment Usual maintenance Ordinary location	0.021
G—Oil-cup or grease lubrication Bored or reamed rigid bearings, ordinary alignment Intermittent maintenance Ordinary location	0.024
H—Oil-cup or grease lubrication Unbored bearings, ordinary alignment Intermittent maintenance Unfavorable location	0.030
J—Oil-cup or grease lubrication Unbored bearings, ordinary or indifferent alignment Intermittent maintenance Unfavorable location	Up to 0.045

Frictional effects are the principal sources of bearing power losses. Any frictional force engendered by the bearings may be dissipated as heat, and an excessive bearing temperature can cause bearing failure by reducing the oil viscosity to such a point that the lubricant no longer fulfills its primary purpose. With a constant diameter, load, and rubbing speed, the frictional effects

are directly dependent upon the magnitude of the coefficient of friction. The probable coefficient of friction of an incomplete-film bearing can be obtained from the equation

$$f = K \sqrt[4]{P/V} \qquad (16\text{-}16)$$

where K is a constant from Table 16-6, P is the average unit bearing pressure per square inch of projected area, and V is the velocity of rubbing, from Eq. 16-15. Values of the bearing factor K depend upon the kind and type of bearing, the method of lubrication, the surface characteristics of the bearing surfaces, the care taken in the installation of the bearings and the alignment of the shafting, and the degree of attention and maintenance which the bearing will probably receive. Several representative values of the factor K are given in Table 16-6.

The surface characteristics of a bearing depend upon the method of manufacture and the care that is taken in finishing the bearing bore and the surface of the shaft. Inexpensive one-piece, Babbitted bearing surfaces are produced by pouring molten Babbitt into the space between the shell and a smooth, highly polished mandrel whose diameter is equal to the size of the shaft. After the Babbitt cools the mandrel is removed and the bearing is put into service without machining the bore. Although the bearing surface is quite smooth, the actual area available for contact with the journal is limited and this type of surface is usually used for cold-rolled steel shafting only, and for comparative low loads and slow speeds. The bearing shown in Fig. 16-20 has a Babbitted surface that is cast with an undersize bore so that the Babbitt may be hammered or peened in place to produce a considerably greater density. The bearing bore is then reamed or bored to suit the shaft. This type of bearing surface is extensively used for most power transmission and hoisting machinery bearings. For very high pressures and speeds, bored bearings are usually hand scraped to fit turned and ground shaft surfaces, which insures adequate bearing surfaces. In bearings made for mass production machinery the bearing bore and the journal surface are often Superfinished (a process analogous to slow-speed honing) to obtain a high degree of polish on the surface.

The maximum operating temperature of standard transmission bearings is about 180° F. when the usual room temperature in the vicinity of the bearings is between 70° and 80°; the heat-dissipating capacity of hanger bearings, and those similar to Figs. 16-20 and 16-21, is about 300 ft-lbs. per min. per sq. in. of projected bearing area; self-aligning bearings similar to Fig. 16-22 have heat-dissipating characteristics of about twice this value. If the frictional work is within these limits, the bearing will operate satisfactorily. It is often of interest to determine the actual operating temperature, and this can be obtained from the following:

$$\Delta t = \sqrt{CH} - 30^\circ \qquad (16\text{-}17)$$

where Δt is the difference between the bearing and air temperature, C is a constant whose value is 55 for rigid and 30 for self-aligning bearings; and H is the heat (ft.-lbs.) that must be dissipated per minute per square inch of projected area. The resultant value of Δt should be added to the room temperature to determine the final operating temperature of the bearing.

The frictional energy that must be dissipated in the form of heat is given by

$$F = fPV \tag{16-18}$$

where F is the frictional energy, in foot-pounds per square inch of projected area, f is the coefficient of friction, V is the unit bearing pressure, in pounds per square inch of projected area, and V is the sliding velocity of the bearing, in feet per minute, from Eq. 16-15.

Axial thrust in transmission bearings is usually small, and can be taken care of by cylindrical collars held to the shaft by set screws. Two-piece split collars, which are clamped on the shaft by two screws, can be used, although they are more expensive than one-piece or solid collars. For estimating and design purposes, the outer diameter of one-piece collars is taken as $1.5D$, and the thickness as $0.75D$, where D is the shaft diameter.

Example 16-4. Investigate the bearing selected for the transmission shaft of Example 16-3.

Solution. The bearing is shown in Fig. 16-22; from Table 16-5, for a diameter of $2\frac{11}{16}$ in., the unit has a bearing length of $6\frac{1}{2}$ in. The reactions at the right bearing are a maximum, and the resultant reaction is found from Eq. 16-6 to be

$$R_m = \sqrt{R_h^2 + R_v^2} = \sqrt{600^2 + 1455^2} = 1572 \text{ lbs.}$$

The unit bearing pressure is $1572/(2.688 \times 6.5)$, or 90 psipa. This value is within reasonable limits for a self-aligning ring-oiled bearing. The sliding velocity V, from Eq. 16-15, is

$$V = 0.262 \times 2.688 \times 300 = 211 \text{ ft. per min.}$$

The PV constant C is 90×211, or 18,990, which is materially less than the maximum recommended value of 25,000 given in a preceding section.

The bearing surface is usually reamed, and considerable care is taken to align the shaft and the bearings. For the normal maintenance, therefore, the bearing characteristics are analogous to Classification C, Table 16-6, and the value of the bearing factor K may be taken as 0.012.

The probable coefficient of friction, from Eq. 16-16, is

$$f = 0.012\sqrt[4]{90/211} = 0.0097$$

The frictional energy to be dissipated as heat, from Eq. 16-18, is

$$F = 0.0097 \times 90 \times 211 = 184 \text{ ft.-lbs.}$$

The maximum heat-dissipating capacity of this bearing at the maximum operating temperature is 600 ft.-lbs., which is far greater than the actual frictional energy. From Eq. 16-17, the temperature difference between the bearing and the atmosphere will be

$$\Delta t = \sqrt{30 \times 184} - 30° = 44°$$

and the estimated operating temperature will be about $44° + 70°$, or 114°, for a room temperature of 70°. This bearing is considered very satisfactory for its intended purpose.

Anti-friction Bearings

16-17. Ball and roller bearings are known as anti-friction bearings, and have certain advantages over journal bearings. The actual bearing friction is less than in sliding bearings and, since it is principally rolling friction, results in little danger of abrasion to machines that are frequently started and stopped under load. Rolling bearings will maintain relatively accurate alignment of parts over long periods of time, can carry heavy momentary overloads without failure, and are easily lubricated.

16-18. Radial Ball Bearings. Fig. 16-27 illustrates the important elements of a single-row radial ball bearing. The bearing has four elements: the outer race *O* which fits in the housing or machine frame; the inner race *J*

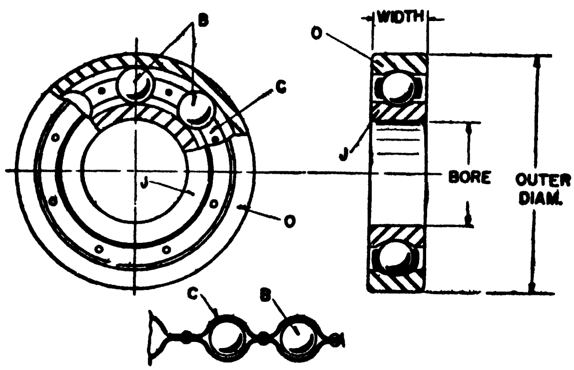


Fig. 16-27. Ball Bearing Elements.

which fits on the shaft; the balls *B*; and the cage or retainer *C* which separates the balls and keeps them properly spaced about the periphery of the unit. Theoretically there is no reason why the balls could not roll on the shaft and in the housing, but races are used to maintain the proper fit and to provide satisfactory rolling surfaces of the proper degree of hardness.

The ball bearing shown in Fig. 16-27 is a single-row bearing primarily intended for radial loads, although it has a thrust capacity equivalent to 75% of its rated radial load. Bearings of this type are also available with deeper and more closely fitting grooves with a rated thrust capacity as high as 200% of the rated radial load. Double-row ball bearings, Fig. 16-30, have two independent rows of balls in single inner and outer races, and have approximately twice the load capacity of a single-row bearing. They are useful for two-directional thrust loads, particularly where an increase in bearing capacity is desired at the expense of axial rather than radial space. Self-aligning ball bearings, also illustrated in Fig. 16-30, have an outer race whose inner surface is of spherical form, permitting some degree of deflection of the inner race, for shaft deflection

under load, or to compensate for a small degree of misalignment in erection and installation.

Preloaded ball bearings are those placed under an initial load that is independent of the working load. Preloading tends to reduce the axial deflection under working loads, thereby maintaining accurate alignment of the shaft. The use of preloaded bearings is usually confined to machine tool spindles and to other precision equipment.

16-19. Radial Ball Bearing Nomenclature. Ball bearing bores, widths, and outer diameters are usually given in millimeters since the bearings were originally used in quantity in Germany, but bearings in standard inch sizes are also available at the present time. Radial ball bearings are made in three series—light, medium, and heavy—and are numbered as follows: 205, 305, 405, respectively. Bearings having the same terminal numeral, such as 205, 305, and 405, have the same bore but the medium and heavy series bearings, which are used for greater loads than the 205 bearing, have larger outer diameters and greater widths. The bore, in millimeters, between sizes 04 and 13 is five times the terminal digits. The bore of a 205 bearing, for example, is 25 millimeters (mm.), while a 307 bearing has a bore of 35 mm.

Light series bearings are used when shaft sizes are comparatively large, and the loads are moderate. They permit the smallest bearing width and outer diameter for a given bore size, and are preferred for limited space. Medium series bearings have a load-carrying capacity from 30 to 40% greater than light series bearings of the same bore, but occupy more axial and radial space on the shaft and in the housing. Heavy series bearings have a load-carrying capacity of from 20 to 30% greater than medium series bearings. Since medium series bearings usually have about as much capacity as the ordinary shaft requires, the heavy series bearing is usually limited in application to special shafts, and is not as frequently employed as the first two types.

Fig. 16-28 shows the application of a single-row ball bearing to the end of a machine tool shaft. The outer race is carried in a cartridge, or intermediate housing, which is pressed into the machine frame; the inner race is a light press fit on the shaft, and is locked in place by the nut shown. A felt-packed groove at the right and the cap at the left assist in sealing the lubricant in the bearing, and aid in keeping out foreign matter. Fig. 16-29 shows a pair of single-row ball bearings applied to a loose pulley; in this installation the outer races rotate, and their load capacity is somewhat less than for the conventional or rotating inner race application.

16-20. Ball Bearings for Transmission Shafting. Ball bearings cannot be conveniently applied to transmission shafting by the method of mounting shown in Fig. 16-28, and some form of adapter-type bearing must be used. In Fig. 16-30, the inner race of the bearing has a tapered bore which fits over a split sleeve so that, as the bearing is forced along the sleeve, the latter is clamped to the shaft and thereby locates the bearing in place. In Fig. 16-31,

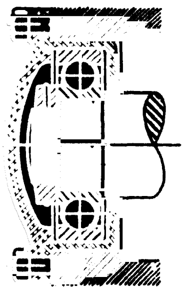


FIG. 16-28. Cartridge-type Mounted Ball Bearing.

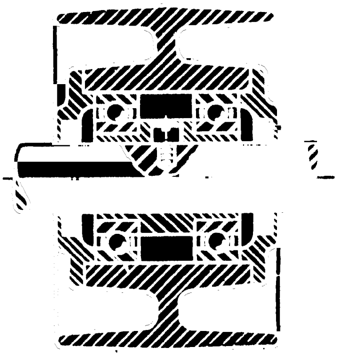


FIG. 16-29. Idler or Loose Pulley Ball Bearing Application.

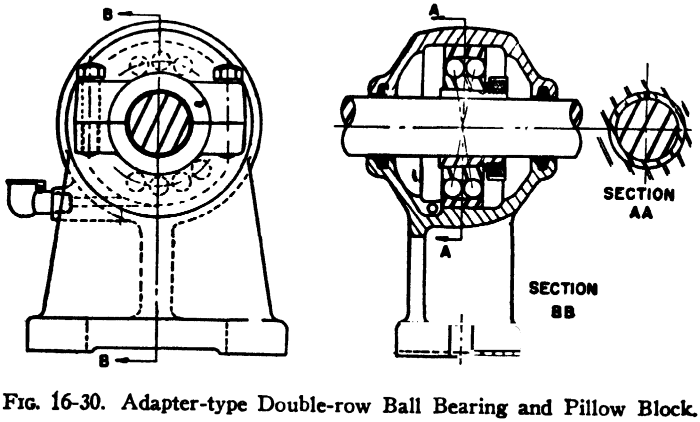


FIG. 16-30. Adapter-type Double-row Ball Bearing and Pillow Block.

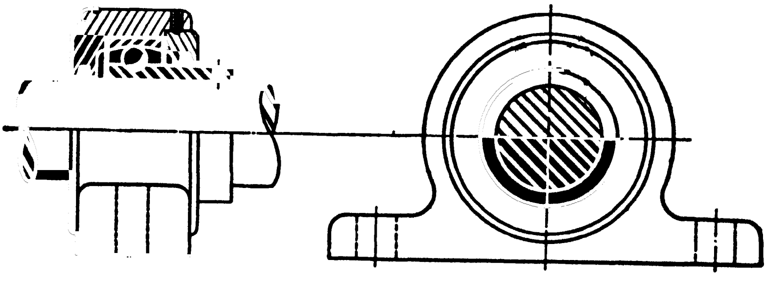


FIG. 16-31. Adapter-type Single-row Ball Bearing and Pillow Block.

the inner race has an extension which is held to the shaft by two set screws General dimensions of this type of bearing are given in Table 16-7.

TABLE 16-7.—SELF ALIGNING BALL BEARING PILLOW BLOCKS (FIG. 16-31) INCHES

Shaft Diam.	Overall Length	Bearing Height	Bolt Diam.	Length of Base	Width of Base	Axial Distance Between Bolts	Distance from Bolt Center to End of Sleeve
1 5/16	2 5/16	1 7/16	3/8	11 1/16	5 3/8	4 1/8	1 11/32
1 3/16	2 7/16	1 11/16	1/2	13 1/4	6	4 1/2	1 7/16
1 7/16	2 9/16	1 7/8	1/2	11 3/16	6 9/16	5	1 9/16
1 11/16	2 13/16	2 1/8	1/2	11 5/16	7 1/8	5 1/2	1 23/32
1 15/16	2 15/16	2 1/4	1/2	2	7 7/8	6 1/4	1 7/8
2 3/16	3	2 1/2	5/8	2 1/16	8 1/2	6 5/8	1 7/8

16-21. Roller Bearing Types. Roller bearings have a greater load-carrying capacity but develop more friction than ball bearings of similar size. Cylindrical roller bearings are made in three series, similar to ball bearings, and have rollers whose diameters are approximately equal to their length. Needle bearings have cylindrical rollers of small diameter and considerable length, and operate without a cage or retainer. They occupy very little radial or diametral space in relation to their load-carrying capacity, and are therefore coming into extensive use for gear mountings in transmission units, and for piston pin bearings in large internal combustion engines. Hyatt roller bearings have hollow cylindrical rollers that are made by winding strip steel into helical form. The hollow construction permits greater deflection under load. The bearing is made with inner and outer races but has been successfully applied to transmission shafting where the rollers bear directly on the surface of the shaft.

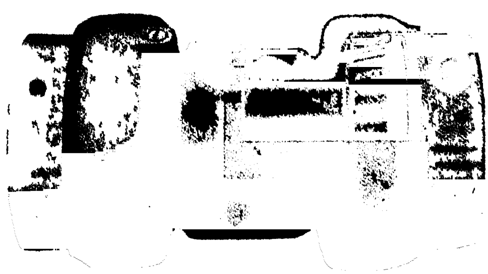


FIG. 16-32. Tapered-roller Hanger Bearing.

Bearings with tapered rollers are extensively used for machine tool and automotive applications, and can handle heavy uni-directional thrust loads as well as large radial loads. They are used in pairs for two-directional thrust Fig. 16-32 shows this type of bearing for use in the hanger of Fig. 16-25, to

replace the ring-oiled bearing shown in that figure. The bearing proper is mounted on a split sleeve which is clamped to the transmission shaft by the clamp nuts shown at the ends of the bearing. Roller bearings of semi-spherical or barrel form, and "hour-glass" type roller bearings, such as shown in Fig. 16-33, are also in wide use. Dimensional data on the latter are given in Table 16-8.

TABLE 16-8.—ROLLER BEARING PILLOW BLOCKS
(FIG. 16-33) INCHES

Shaft Diam.	Overall Length	Bearing Height	Number of Bolts	Bolt Diam.	Length of Base	Width of Base	Transverse Distance Between Bolts	Axial Distance Between Bolts
1 7/16	3 7/8	2 1/8	2	1/2	2 1/2	7 7/8	5 7/8	0
1 11/16	4 7/16	2 5/16	2	1/2	2 13/16	8 9/16	6 1/2	0
1 15/16	4 1/4	2 1/2	4	1/2	3 13/16	9	7	2 1/4
2 3/16	4 5/8	2 3/4	4	5/8	4	9 15/16	7 1/2	2 1/4
2 7/16	4 7/8	3	4	5/8	4 3/8	10 5/8	8 1/4	2 5/8
2 11/16	5 5/8	3 1/2	4	3/4	4 3/4	12 5/8	9 3/4	2 3/4
2 15/16	5 5/8	3 1/2	4	3/4	4 3/4	12 5/8	9 3/4	2 3/4
3 3/16	5 7/8	4	4	3/4	5	14 1/4	11 1/2	3
3 7/16	5 7/8	4	4	3/4	5	14 1/4	11 1/2	3
3 15/16	6 1/2	4 7/16	4	7/8	5 5/8	15 5/8	12 1/4	3 3/8
4 7/16	7 5/8	5 3/4	4	7/8	6 1/2	18 1/8	14 3/4	4

Ball or roller bearings for thrust loads only are used to some extent in machine tools in combination with radial sliding bearings. Fig. 16-34 is a

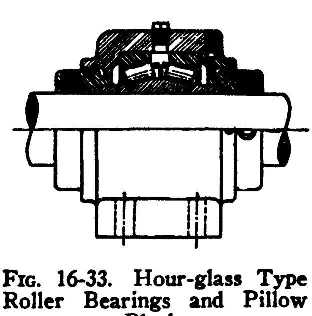


FIG. 16-33. Hour-glass Type Roller Bearings and Pillow Block.

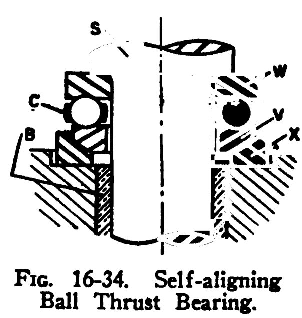


FIG. 16-34. Self-aligning Ball Thrust Bearing.

representative example of a self-aligning, spherical-seated ball thrust bearing: this unit is used to some extent for jib crane columns, and in crane hook swivels, as well as for the usual thrust applications.

16-22. Anti-friction Bearing Selection. The selection of a ball or roller bearing unit for a given installation is dependent upon five primary factors which are: the load-carrying capacity; the speed, in revolutions per minute; the type of service; the anticipated life of the bearing; and the proportion of thrust to radial load. Fig. 16-35 shows the variation in radial load capacity with speed for the ball bearing unit of Fig. 16-31 and Table 16-7, and the roller bearing

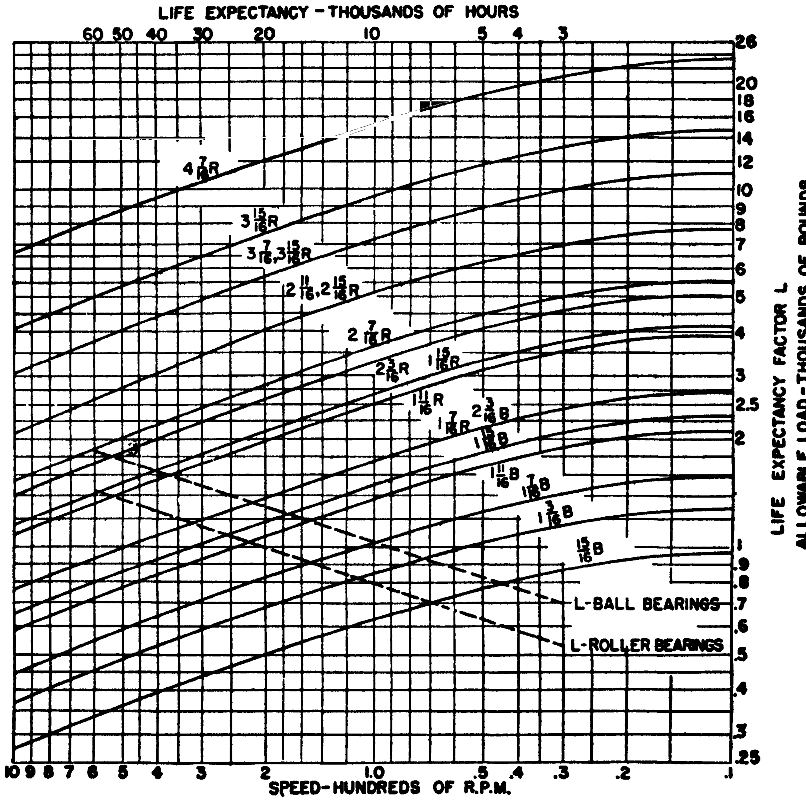


Fig. 16-35. Load Capacity and Life Expectancy of Ball and Roller Bearings.

unit of Fig. 16-33 and Table 16-8. In this chart the shaft diameter followed by letters B or R, which designate ball or roller bearing units respectively, is shown on the curves; the allowable radial load, in thousands of pounds, is given on the vertical scale at the left, and the speed, in hundreds of revolutions per minute, is given on the horizontal scale at the bottom. It may be noted that there is a sharp decrease in capacity as the speed increases. The ratings as given in Fig. 16-35 are based upon a light shock load, and should be divided by an operational factor Q for other classes of service.

For seasonal or intermittent operation, or intermittent peak loads, Q is taken as 0.75; for general machine service which moderate shock loads, Q is taken as 1.35; and for heavy-duty machinery operating under severe shock loads or extreme vibration, Q should have a value of 1.75. In some cases it may be necessary to use intermediate values of these factors, which the judgment and past experience of the designer will control.

If a ball bearing operates continuously, its life expectancy measured in hours, days, or years, will obviously be shorter than if operated intermittently. The relative rapidity of fatigue failure in ball bearings depends not only upon the material characteristics of the contact surfaces and the magnitudes of the stresses involved, but also upon the frequency of the stress repetitions per unit of time. The greater the number of stress cycles, the more rapidly will fatigue failure occur. Lightly loaded contact surfaces will withstand many more stress repetitions without fatigue failure than those in which the load is heavy. Practically all published load ratings are based upon a uniform life expectancy, but unfortunately not all suppliers and manufacturers are entirely in agreement as to the basic figure. The bearing ratings in Fig. 16-35 are based upon a life expectancy of 9000 hours for ball bearing and 20,000 hours for roller bearing units. If bearings are selected from Fig. 16-35 on a life expectancy basis other than these values, the bearing capacity should be divided by the life expectancy factor L taken from the curves near the bottom of the figure. To illustrate, a roller bearing unit for a 2 $^{11}/_{16}$-in. diameter shaft has an allowable radial capacity of 4000 lbs. at a speed of 200 RPM. Since these curves are based upon a light shock load, the same bearing would carry a load of 4000/0.75, or 5333 lbs., if the service is intermittent, and if the estimated life of the bearing is 20,000 hours. If the bearing life is to be 50,000 hours, the life expectancy factor L will be 1.36, and the bearing capacity will be 4000/1.36, or 2940 lbs., for light shock load service. For intermittent service with a bearing life of 50,000 hours, the capacity is 5333/1.36, or 3920 lbs.

Bearings or units which serve as supports for transmission shafting driven by belts, chains, or spur gears are usually subjected to radial loads with little or no thrust load. For combined thrust and radial loads on the ball bearing unit of Table 16-7, the allowable thrust load for continuous operation will be one third the unused radial capacity. To illustrate, from Fig. 16-35 the 1$^{3}/_{16}$-in. diameter ball bearing unit has a radial capacity, for 9000 hours of service and for a light shock load, of 450 lbs. at a speed of 600 RPM. If the actual radial load on this bearing is 270 lbs., the unused radial capacity will be 450 − 270, or 180 lbs., and the bearing will therefore carry a continuous thrust load of one-third this value, or 60 lbs., in addition to the radial load.

For the roller bearing units of Table 16-8, any combination of thrust and radial loads should be considered under two classifications; one in which the thrust load is less than 30% of the radial load, and the other in which the thrust

load predominates. In the first classification, the required radial capacity must be equal to the following

$$C = [R + (10T/3)]LQ \qquad (16\text{-}19)$$

where C is the capacity of the unit at the operating speed, R and T are the actual radial and thrust loads, and L and Q are the life expectancy and service factors. To illustrate, consider a $3^{15}/_{16}$-in. diameter roller bearing unit subjected to a radial load of 3000 lbs. and a thrust load of 700 lbs. at a speed of 129 RPM, with a life of 40,000 hours, and a moderate shock load. The ratio between the thrust and radial loads is 700/3000, or 0.233, which is less than 0.30 or 30%. The factor L, from Fig. 16-35, for roller bearing units is 1.3, the service factor Q is 1.35, and the required capacity, from Eq. 16-19, is

$$C = [3000 + (10 \times 700)/3]1.2 \times 1.35 = 8630$$

From Fig. 16-36, it is seen that this bearing has a capacity of 9000 lbs. at a speed of 120 RPM, which is satisfactory.

For thrust loads greater than 30% of the radial load, the required radial capacity C is given by

$$C = (1.4R + 2T)LQ \qquad (16\text{-}20)$$

If the bearing of the preceding illustration had been subjected to radial and thrust loads of 1000 and 2000 lbs., respectively. the required capacity would be

$$C = [(1.4 \times 1000) + (2 \times 2000)]1.2 \times 1.35 = 8750 \text{ lbs.}$$

for which the rated radial capacity is still satisfactory.

16-23. Bearing Baseplates. Pillow blocks and anti-friction bearing units may be mounted directly on wooden or structural steel beams and joists, or on concrete or masonry bases. For the latter type of foundation, some form of baseplate is preferred, so that adjustment for shaft height and alignment can be made readily. Data on types and specifications of such baseplates may be obtained from manufacturers' and suppliers' catalogs.

PROBLEMS—CHAPTER 16

1. A rotating shaft transmits a torque of 43,000 in.-lbs. and is subjected to a flexural moment of 70,000 in.-lbs.

a. What is the diameter of the shaft if made of commercial steel? (Assume a steady load, with keyways.)

b. What material should be used if the shaft diameter is $3^{7}/_{16}$ in., with a light shock load and no keyways?

c. What is the diameter of the shaft of part *a* if chrome-vanadium steel is used?

2. An eccentric is to be fastened to a 2-in. shaft. The center of the eccentric is $^{7}/_{8}$ in. from the shaft axis, and a force of 600 lbs. is applied through the connecting rod to this center.

a. Determine the size of a set screw for holding the eccentric to the shaft.

b. Like *a*, for a transverse taper pin, made of mild steel.

3. A commercial steel shaft, with keyways, is subjected to pure torsional, suddenly applied loads with heavy shock and transmits 150 HP at 200 RPM.

a. What is the theoretical diameter of the shaft?

b. If the shaft of part *a* is to be replaced by a hollow shaft of the same outer diameter, but made of 3½% nickel steel, what should be the inside diameter?

c. What is the saving in shaft weight?

4. A flanged coupling for a 2¹⁵⁄₁₆-in. diameter shaft has a flange diameter of 10 in., a flange thickness of 1 in., a bolt circle diameter of 8⅛ in., a ¾-in. square key, a hub length and diameter of 3¾ and 5 in., and is fitted with six ¾-in. bolts. The flanges are made of gray cast iron, the bolts, key, and shaft of commercial steel. Determine the unit stresses in the coupling elements if the coupling is rated for 75 HP at 180 RPM.

5. A shaft for the remote control of a valve is subjected to a torsional moment of 150 in.-lbs. and has a length of 18 ft. Determine the diameter of the shaft if the angular deflection is limited to about 1° and select a suitable set screw for attaching the control crank to the shaft.

6. An overhead 300-RPM countershaft for a machine tool carries a driven pulley *A*, 10 in. diameter, and three driving pulleys *B*, *C*, and *D*, 20 in., 12 in., and 14 in. diameter. The distance between the bearing centerlines is 40 in. The centerline of pulley *A* is located 6 in. to the left of the centerline of the left bearing and is driven from a lineshaft in the same horizontal plane. The centerline of pulley *B* is 16 in. to the right of the left bearing and the belt is vertically upward. The centerlines of pulleys *C* and *D* are 15 in. and 6 in. to the left of the centerline of the right bearing, and the belt is vertically downward. Pulleys *B*, *C*, and *D* transmit 2, 1, and 1½ HP, respectively, and the belt tension ratio is about 3. The countershaft is made of CRS, the pulleys are keyed to the shaft, and the load is steady.

a. Find the bearing reactions in the horizontal and vertical planes, disregarding the weight of the pulleys.

b. Draw shear diagrams for the horizontal and vertical plane force systems.

c. Determine the magnitude and position of the maximum moment.

d. Find the theoretical shaft diameter and select a suitable standard shaft.

e. Check the load-carrying capacity and the frictional and heat losses of the most heavily loaded bearing.

f. Select suitable anti-friction bearings for this countershaft.

7. The headshaft for a conveyor is driven by an 870-RPM, 25-HP motor through the medium of a roller chain and rotates at 145 RPM. The headshaft carries a belt conveyor pulley 22 in. in diameter and 32 in. wide, placed midway between the bearings. The conveyor belt is of rubberized fabric with 180° of contact. The distance between bearing centers is 40 in.; the centerline of the chain is about 10 in. from the centerline of one of the bearings. The direction of the belt pull is in the same plane but opposite to that of the chain.

a. Select a suitable chain for this drive.

b. Find the belt and chain pulls and draw a shear diagram of the forces on the shaft.

c. Determine the diameter of a commercial steel shaft, allowing 10% for bearing friction.

d. Select suitable plain bearings for this drive and check the frictional and heat losses.

e. Select suitable anti-friction bearings for this shaft.

8. The driving bevel friction wheel of Problem 2, Chapter 14, is mounted on a CRS shaft. The centerlines of the bearings are symmetrical with respect to the axis of the slow-speed shaft and are 20 in. apart.

a. Find the diameter of the shaft.

b. Select suitable anti-friction bearings for the shaft.

9. Prove that the axial force required to move a member along a shaft having one key is twice that required to move the same member if two or more keys, equally spaced, are employed.

CHAPTER 17

HANDLING EQUIPMENT AND MECHANICAL FRAMES

17-1. Material handling equipment[82] is of major importance in the chemical and mechanical processing industries. Handling systems can be classified as manual and mechanical. The most useful manual systems are skids or skid-mounted boxes or racks, and caster trucks or live skids. The principal mechanical systems are conveyors, which move material continuously over fixed lines of travel between fixed points; industrial trucks, which move material intermittently over any desired lines of travel throughout any area having suitable running surfaces and clearances; and cranes and hoists, which move material intermittently over any desired lines of travel within a restricted area.

17-2. Chain Conveyors. Conveyors are used primarily for horizontal transportation, but certain varieties may be employed for vertical movement, either independently of or in combination with horizontal movement. There are three general classes of horizontal conveyors: chain, belt, and screw conveyors. Chain conveyors make use of either the detachable link type or the pintle type of chain shown in Fig. 14-12. The former is widely used for moderate loads, and is standardized in a wide range of sizes by most of the well known manufacturers of material handling equipment.[85] Detachable link chain pitches and permissible loads vary from 1 in. and 120 lbs. to 4½ in. and 1650 lbs. Special links with integral lugs, plates or pins for attaching buckets, flights, or scrapers are obtainable (Fig. 14-12).

Pintle chain is used for heavy loads, in pitches from 1⅜ to 9 in., and can be obtained with a variety of attachments. Fixed or swinging buckets in a wide range of types and sizes are available for conveying coal, ashes, grain, and other granular materials. Bucket elevators are vertical conveyors in which buckets are attached to a chain or belt, with head and tail pulleys at the top and bottom of the elevator. The buckets pick up the material to be lifted as they turn around the tail pulley, or they may be filled on the rising side by a chute. The material is discharged as the buckets turn around the head pulley. Portable conveyors with buckets are used for loading cars and trucks; they are usually inclined at an angle of about 15°, loaded at the lower end and discharged at the upper end.

Pintle chain conveyors with flat plates or flights that overlap or abut each other are used as moving platforms carrying boxes and racks. Flights can also be mounted at right angles to the direction of movement of the chain, and slide in a trough, thus serving as scrapers to move material. Scraper conveyors are useful for horizontal or slightly inclined movement of non-abrasive materials;

the material may be fed and discharged at any desired points in the length of the conveyor.

17-3. Belt Conveyors. Belt conveyors usually employ a heavy rubber-faced cotton duck belt, and are available in standard widths from 1 to 5 ft., for lengths up to 2000 ft. Belt conveyors are usually used for bulk material but can be used for carrying individual pieces, such as boxes or cartons. The belt is carried on sets of small rollers spaced about 5 ft. apart, arranged in such a manner that the upper or load-carrying surface of the belt is troughed or dished upward at the edges to prevent spilling. Belt conveyors are used for solid or loose material that will not stick to the belt, and where single pieces are not too large or too heavy. Belt conveyors are used for moderate inclines, usually limited to about 20°; the magnitude of the angle depends to a considerable extent upon the angle of repose of the material carried. Belt conveyors usually have a larger capacity and require less power to operate than chain conveyors, and may be used for either exterior or interior service. Belt conveyors are not suitable for moving heavy single pieces or fluid materials.

17-4. Screw Conveyors. Screw conveyors consist of a helicoidal flight attached to a shaft which revolves inside a trough without motion in an axial direction. Material lies below the shaft and is moved along the trough by the helicoidal surfaces of the flight. Screw conveyors are available in a wide range of sizes for the transportation of light, non-abrasive bulk material. They may be fed at one end or anywhere along their length, and may discharge at one or more points by means of sliding gates in the bottom of the trough. In some process industries, screw conveyors are equipped with flights that will act so as to mix materials while in transit. Troughs may be steam-jecketed for heating, crystallizing, or drying, or they may be perforated to allow moisture to drain out as the material moves along.

17-5. Roller Conveyors. Roller conveyors consist of a framework supporting a series of anti-friction rollers in a horizontal plane and are used for moving boxes or work parts by hand. In some cases the plane of the rollers has a slight incline so that the work will move slowly under the influence of gravity. Roller conveyors are made in sections consisting of straight runs and curves; switching units, consisting of balls instead of rollers, are obtainable for moving work parts between roller conveyors operating at right angled directions.

17-6. Industrial Trucks. Industrial trucks are classified as haulage or as handling devices. The conventional tractor-trailer arrangement, which consists of a series of trailer trucks drawn by an electrically operated, manually controlled tractor, falls into the haulage classification since, inherently, the system does not contain a means of loading and unloading the material transported. The outstanding example of a handling system is the lift truck, which may be either manually or electrically operated. Power lift trucks have a low projecting platform which is capable of an independent vertical movement, usually limited to two or three inches but in some cases to as much as 6 ft. The work or material

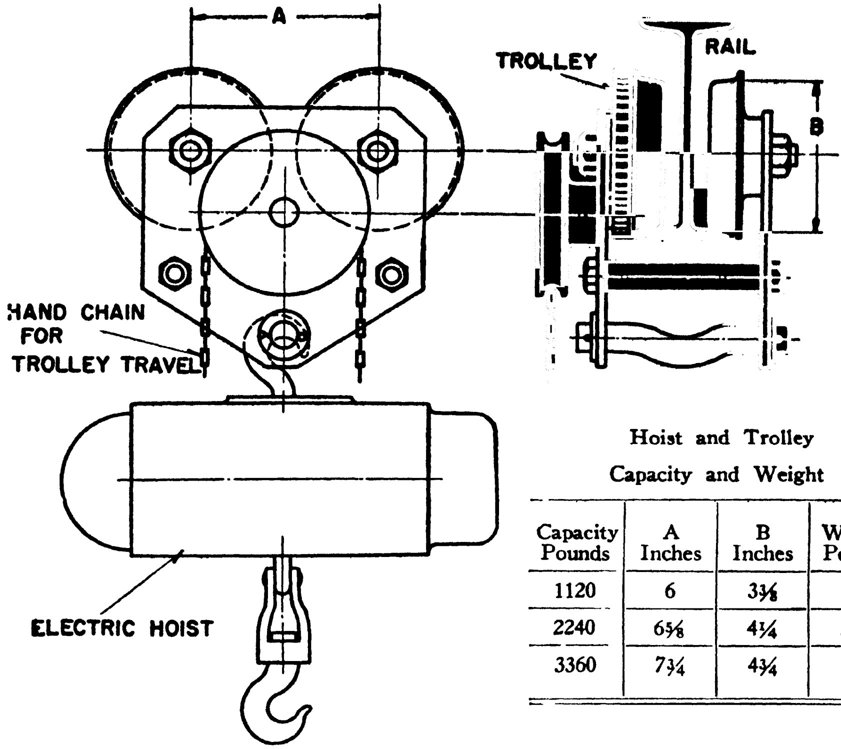
to be transported is carried in boxes or special containers mounted on legs slightly longer than the height of the truck platform at its lowest platform position. The truck is brought up to the container, with the platform underneath; the platform lifts the container so that it clears the floor, and carries it to the desired location. The truck platform is then lowered and moved away from the container. High lift trucks with special platforms or attachments for handling barrels, pipe, or other parts, are available; trucks with winch or derrick attachments are also used to a considerable extent.

17-7. Hoists and Winches. Hoists may be manually, pneumatically, or electrically operated. The chain hoist is used for temporary or intermittent service where the load and the height of lift are moderate, and where the work is expected to hang in position for some time.[56] For frequent operation, pneumatically operated hoists powered by compressed air are used to a considerable extent. For heavier loads and higher lifts, electric drum hoists with wire cables or hoisting chain are used. Hoists of this character are obtainable in a wide range of capacities and speeds, varying from a capacity of 500 lbs. at a lifting speed of 25 ft. per min., to hoists having capacities of several hundred tons. The smaller sizes of hoists are often hung from a bracket or beam for operation at a fixed position. For movable operation, the supporting hook on the hoist may be attached to a movable trolley, shown in Fig. 17-1, and moved along the hoist beam by operating the hand chain. Hoists for larger sizes of cranes are constructed with integral trolleys that move along the crane beam by power.

Winches are usually hand operated, and consist of a drum rotated by a worm gear, which is driven in turn by a worm and hand crank, as shown in Fig. 17-2. Worm gear or screw winches are self-locking; winches actuated by spur gearing are usually provided with a suitable ratchet. Power-actuated winches are available for stationary hoist service, or for use as car pullers for "spotting" or locating freight cars on a railway siding.

17-8. Cranes and Tramways. Cranes may be classified as traveling cranes and jib cranes. The traveling crane consists of a moving bridge carried on trucks running on rails mounted at the sides of a building. A suitable hoist and trolley moves lengthwise on the bridge. The crane and hoist are controlled from an operator's cage on the lower side of the bridge, or by electrical floor controls. The traveling crane permits transportation over the entire floor area of the building. Gantry cranes are traveling cranes used in yards and at docks; the bridge is supported by frames which move on rails laid on the ground.

17-9. Jib Cranes. Jib cranes are used for local service, and usually consist of a vertical mast supported at the top and bottom, and reasonably free to move about its axis. An arm or beam is attached to the mast and carries a trolley and hoist. Jib crane service is limited to a sector whose radius is equal to the effective radius of the arm, and may be as great as 25 or 30 ft. A representative example of a wall-supported crane is shown in Fig. 17-3.



Hoist and Trolley
Capacity and Weight

Capacity Pounds	A Inches	B Inches	Weight Pounds
1120	6	3⅛	220
2240	6⅝	4¼	250
3360	7¼	4¾	340

FIG. 17-1. Hoist and Trolley.

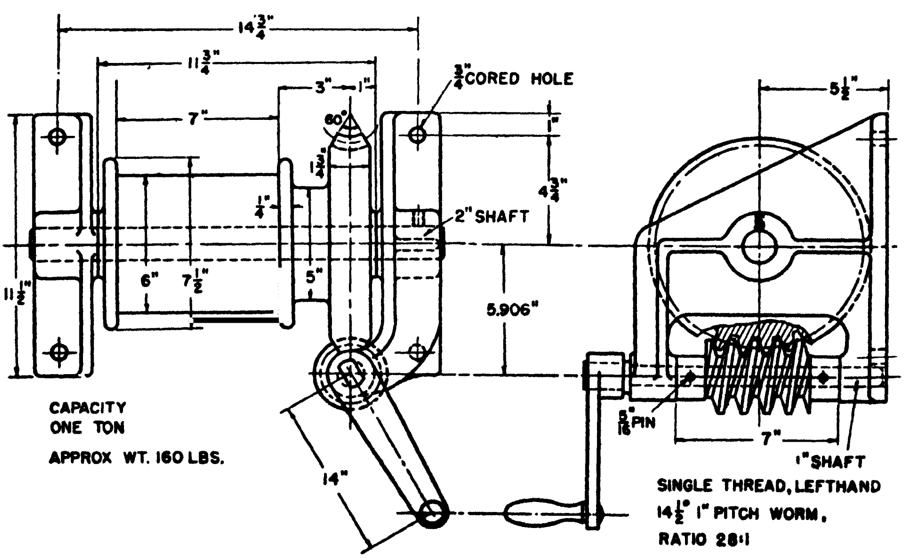


FIG. 17-2. Worm Gear Winch.

Overhead tramways are widely used in industrial plants, and consist of a system of tracks, usually a standard I beam, suspended from ceiling girders or the lower chords of roof trusses. The lower flange of the I beam supports a trolley to which an electric hoist may be attached. Overhead tramways are very flexible, and offer no interference with floor layouts or machinery. They may be used for inter-building as well as inter-departmental work, and are usually superior to traveling cranes for loads up to five tons.

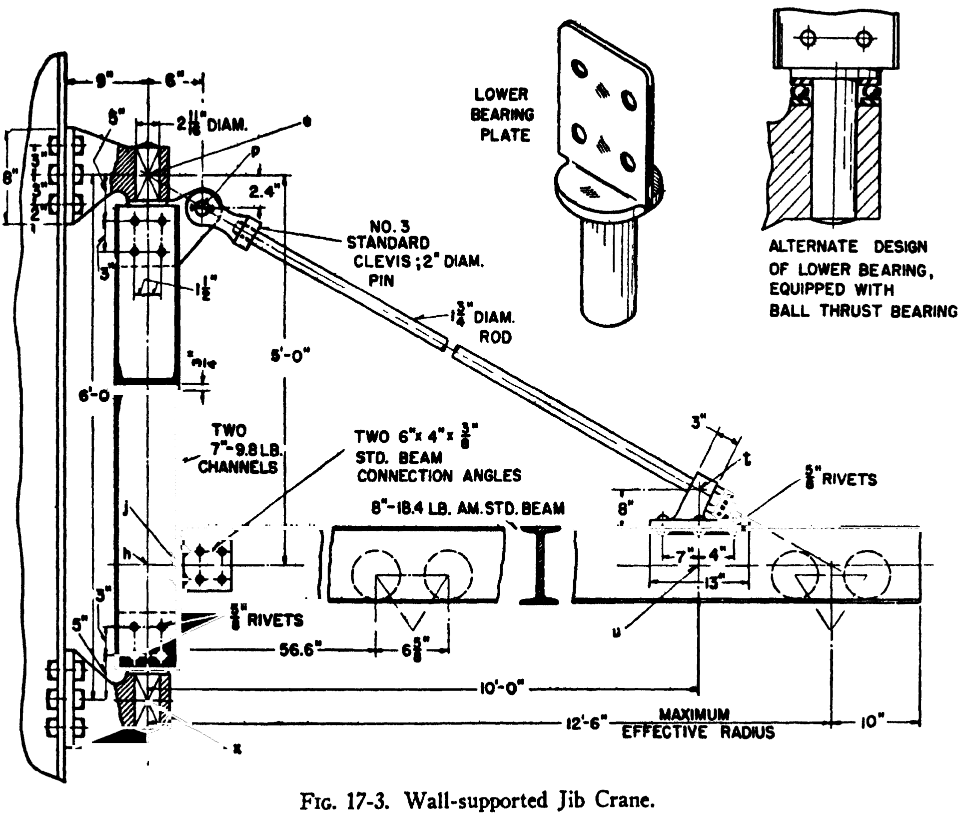


FIG. 17-3. Wall-supported Jib Crane.

17-10. Attachments for Overhead Rails. Several types of hangers and attachments for overhead tramway rails are shown in Figs. 17-4 and 17-5. For attachment to wooden ceiling beams or girders, the detail at A, Fig. 17-4, is preferred to that of B. In the latter the body of the screw is subjected to tension, while the threads cut in the beam by the lag screws are subject to shear. The construction shown at A permits the screw body to be subjected to direct shear; through bolts may be substituted if the beam breadth is not excessive. The hanger shown at C can be used for attaching tramway beams to the lower chord of a roof truss. The filler plate is used between hangers so that they may be

adapted to differing tramway beam widths. Two views of two methods of attaching tramway beams to I beam or WF ceiling girders are shown in Fig. 17-5. The detail at A necessitates a pair of clamps for each side of the flange; that at B requires drilled holes in the lower flange of the ceiling beam. For the latter construction, a wedge-shaped washer W should be used in conjunction with the sloping flange beam.

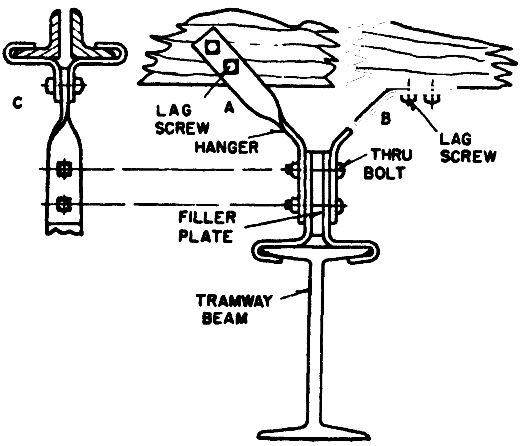


Fig. 17-4. Tramway Hangers.

Hangers and attachments can be purchased from suppliers, but are very easily fabricated from strap iron or steel. The design of such attachments offers no particular difficulties; they should never be made of strap material smaller than $1\frac{1}{2} \times \frac{1}{4}$ in., and $\frac{1}{2}$-in. bolts or lag screws should be a minimum requirement.

17-11. The design and selection of most forms of material handling equipment is a highly specialized activity, and it is imperative that chemical, industrial, and mechanical engineers concerned with the process industries should consult manufacturers, suppliers, and specialists for detailed information. Several excellent texts on the subject of material handling are available; catalog and other descriptive data can be obtained from manufacturers. For these reasons, further treatment of conveyors and truck systems and a detailed presentation of power crane design are not included in this text. Comparatively simple structures, such as jib cranes or overhead rails for material transportation, however, are often fabricated and installed by mechanics associated with a manufacturing organization, and the process engineer may be called upon to design such equipment, or to investigate the capacity or reliability of existing structures.

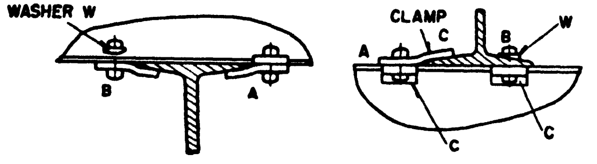


Fig. 17-5. Tramway Beam Attachments.

17-12. Frames. Structural and mechanical frames are arrangements of members or component parts that act as brackets or beams. Frames differ from

trusses in that the members composing the former are usually subjected to flexural or eccentric loads, in addition to the simple and direct stresses induced in truss members.

Brackets for supporting dead or stationary loads, such as pipe lines, bases for motors, pumps, are representative examples of structural frames. Mechanical frames actually constitute a part of a machine; they are often subjected to moving or variable and live loads, as in jib cranes and the like.

The wall bracket shown in Fig. 9-48 is an example of a structural frame, and consists of a wall plate *P*, and pairs of horizontal beams *B* and supporting braces *C*. If the pipe weight were concentrated at the point of intersection of *B* and *C*, and if the bracket were supported at the juncture of *B* and *P*, the stresses in members *B* and *C* would be direct tension and compression. With the load in the position shown, however, the beam *B* is subjected to a flexural as well as a direct tensile stress, and the plate *P* is subjected to both compressive and eccentric loads. Since the clamping arrangement for the supporting rollers for the pipe may be shifted at will, the magnitude and character of the stresses may be affected materially by a change in the lateral position of the pipe.

The jib crane shown in Fig. 17-3 is an example of a mechanical frame. The maximum flexural stress in the crane beam will occur when the hoist is approximately midway between the points *j* and *u*. The hoist weight and load induce a direct tensile stress in the rod, which in turn causes a compressive stress in the crane beam. The maximum tensile stress in the rod, and the maximum load on the bracket bolts will be induced when the hoist is at its maximum effective radius. The rivet groups at the upper and lower bearing plates, and those at the connection angles and the rod bracket, are subjected to eccentrically applied forces, since (in contrast to the usual roof truss construction) the design is such that the force lines do not pass through their centroids.

17-13. Stresses in Mechanical Frames. Allowable unit stresses in the members of structural frames are taken from Table 7-1, but a carefully detailed analysis is essential so that the likelihood of high stress concentration due to eccentricity may be properly evaluated.

Allowable unit stresses for mechanical frames, as in hoisting and conveying equipment, are usually taken as one half to three fourths the allowable stresses for structural design because of the greater likelihood of suddenly applied or shock loads. An average working stress would therefore be two thirds the values given in Table 7-1 for tension, flexural, and shearing stresses. A prominent manufacturer of cranes recommends allowable tensile stresses of 12,000 psi. and allowable compressive stresses of 9000 psi. in crane girders and booms, with a maximum ratio of span length to girder flange width of 60 for hand operated and 50 for electric cranes. Column design in cranes is based upon Eqs. 5-15 and 5-16, using a design stress of 50% of a yield point of 30,000 psi. (approximately the yield point of structural steel).

To guard against any possibility of buckling or lateral failure, crane beam sections with unsupported flanges, selected on the basis of flexural stress, should be checked for localized load by the following:

$$S = \frac{20{,}000}{1 + \dfrac{L^2}{2000b^2}} \tag{17-1}$$

where S is the maximum allowable stress, psi., L is the beam length and b the width of the compression flange, in inches.

In the application of trolleys to crane and tramway beams, the total load on the beam (including the load lifted and the trolley weight) is carried by four wheels, and is considered as acting at two points on the beam. It is evident that the maximum moment will occur under one set of wheels, as the vertical shear must be zero at this point. If L represents the span of a simply supported beam, A the fixed distance between the loads, and X the distance from the reaction or point of support, the magnitude of the right reaction is found by taking moments about the left end of the beam, and the magnitude of the reaction is

$$R = \frac{F(L-X) + F(L-X-A)}{L} = \frac{2FL - 2FX - FA}{L}$$

The moment directly under the left wheel is

$$M = R(A+X) - FA = RA + RX - FA$$

Substituting the value of R,

$$M = \frac{2FLA - 2FXA - FA^2}{L} + \frac{2FLX - 2FX^2 - FAX}{L} - FA$$

$$= \frac{2FLA - FA^2}{L} + \frac{2FLX - 2FX^2 - 3FAX}{L}$$

If the moment is differentiated and set equal to zero, the value of X thus found will determine the position of the left wheel for the maximum bending moment on the beam. Differentiating M,

$$dM = 0 + \frac{2FL - 4FX - 3FA}{L} = 0$$

and

$$X = \frac{2L - 3A}{4} = \frac{L}{2} - \frac{3A}{4}$$

or, the maximum moment in the beam will occur when one of the wheels is positioned at a distance equal to one fourth the wheel span from the center of the beam span.

The methods of stress analysis for structural and mechanical frames parallels those for truss analysis. The usual procedure is to determine the external supporting forces or reactions by treating the entire frame as a free body. The

effect of these external forces on each member is then handled by considering every member as a free body, and evaluating direct and secondary stresses for each. Probably no general form for such analyses can be developed, since frame design and construction differ in type and degree, but the following example illustrates one analysis.

Example 17-1. The electroplating division of a manufacturing plant requires a jib crane for handling large quantities of work parts. A second-hand jib crane, shown in Fig. 17-3, can be obtained for this purpose. This crane can be fastened to a column near the plating tank by bolting the crane brackets to the column flanges, using six ¾-in. bolts in each bracket. The maximum effective radius of the crane beam is 12 ft. 6 in., and is ample for the purpose at hand. The maximum working load on the crane beam will be 1 ton. Select a suitable trolley and hoist, and determine the suitability and capacity of the crane for this purpose.

Solution. From Fig. 17-1, a 2240-lb. capacity trolley and hoist can be purchased. The combined weight of the trolley and hoist, from the manufacturers' catalog, is 250 lbs. The design load for the jib crane, exclusive of the weight of the crane itself, is $2000 + 250$, or 2250 lbs.

The weight of the beam is equal to the product of the length and the weight per unit length. The length is 12 ft. 6 in. $+ 10 - 3.5$, or 156.5 in.; the beam weight is $156.5 \times 18.4/12$, or 240 lbs. The length of the mast is $72 - (2 \times 2.4)$, or 67.2 in., and its weight is $67.2 \times 2 \times 9.8/12$, or 110 lbs.

There are several possibilities which should be considered in attempting to determine the maximum stresses in the component members of this crane. If the hoist is placed at the maximum effective radius of the beam, the reacting forces at e and x will have their greatest value, and the direct tensile stress in the tie rod, and the direct compressive stress in the beam will have maximum values. The greatest flexural stress in the beam, however, will occur when one of the hoist wheels is placed at a distance equal to one fourth the wheel span from the center of the beam span. This position will reduce the compressive stress in the beam, and may result in a lower or higher stress than is the case with the load at the maximum effective radius.

Fig. 17-6 shows a skeleton layout of the crane, with the hoist at the maximum effective radius of 150 in., and the mast and beam weight concentrated at their centroids. The lower bearing resists the entire vertical load, and the resisting force at this point is

$$F_3 = 110 + 240 + 2250 = 2600 \text{ lbs.}$$

The reaction F_1 at the upper bearing is horizontal; its magnitude is obtained by taking moments about the lower bearing, and is

$$F_1 = \frac{240 \times 81.75 + (2250 \times 150)}{72} = 4960 \text{ lbs.}$$

By horizontal equilibrium, F_2 is equal to F_1.

Fig. 17-7 shows a skeleton layout of the crane beam ju with the hoist load of 2250 lbs. acting at the maximum effective radius. The forces F_4 and F_5 are the vertical and horizontal components of the supporting force in the tie rod tp. The slope of the axis of the tie rod tp is 60/150, or 0.4, and the magnitude of component F_4 is equal to 0.4 F_5. Taking moments about point j,

$$M = -12\,F_5 - 116.5\,F_4 + (78.25 \times 240) + (146.5 \times 2250) = 0$$

Substituting 0.4 F_5 for F_4 and solving

$$F_5 = \frac{18{,}780 + 32{,}925}{58.6} = 5945 \text{ lbs.}$$

$$F_4 = 0.4 \times 5945 = 2378 \text{ lbs.}$$

By a vertical summation

$$F_7 = 2250 + 240 - 2378 = 112 \text{ lbs.}$$

By a horizontal summation

$$F_6 = F_5 = 5945 \text{ lbs.}$$

The action of the external forces F_4 and F_5 on the tie rod *tp* are shown in Fig. 17-9.

Fig. 17-8 shows a skeleton layout of the crane mast *xe,* with the various forces acting on it; forces F_6 and F_7 induced by the beam, F_4 and F_5 induced by the tension rod, 110 lbs. due to the mast weight, and the external forces F_1, F_2, and F_3.

Checking the force system by taking moments about point *x,*

$$M = -72\,F_1 + 69.6\,F_5 + 6\,F_4 + 3.5\,F_7 - 12\,F_6 = 0$$

or $M = (-72 \times 4960) + (69.6 \times 5945) + (6 \times 2378) + (3.5 \times 112) - (12 \times 5945) = 0$

Fig. 17-10 shows a skeleton layout of the crane, with the hoist at the position which develops the maximum bending stress in the beam. The maximum moment will occur when one of the wheels is positioned at a distance equal to one fourth of the wheel span from the center of the beam span. From Fig. 17-1, the wheel span is 6⅝ in.; one fourth of this distance is 1.656 in. The beam span, from Fig. 17-3, is 116.5 in.; one half of this distance is 58.25 in.; and the distance from the axis of the mast to the center of the span is 58.25 + 3.50, or 61.75 in. The distance from the axis of the mast to the centroid of the hoist is 61.75 + 1.656, or 63.406 in., say 63.4 in.

The magnitude of the upper bearing reaction F_1 is found by

$$F_1 = \frac{240 \times 81.75 + (2250 \times 63.4)}{72} = 2254 \text{ lbs.}$$

By horizontal equilibrium, F_2 is equal to F_1. The vertical resisting force F_3 at the lower bearing is 2600 lbs.

Fig. 17-11 shows a skeleton layout of the crane beam *ju* with the hoist load of 2250 lbs. acting at the position of maximum moment in the beam. Considering F_4 equal to $0.4\,F_5$, the moment about point *j* is

$$M = (-12 \times F_5) - (0.4 \times 116.5 \times F_5) + (78.25 \times 240) + (59.9 \times 2250) = 0$$

from which

$$F_5 = \frac{18{,}780 + 134{,}775}{58.6} = 2620 \text{ lbs.}$$

and

$$F_4 = 0.4 \times 2620 = 848 \text{ lbs.}$$

By a horizontal summation

$$F_5 = F_6 = 2620 \text{ lbs.}$$

By a vertical summation

$$F_7 = 2250 + 240 - 848 = 1642 \text{ lbs.}$$

Checking the force system for the mast, shown in Fig. 17-8, by taking moments about point *x,*

$$M = -72F_1 + 69.6F_5 + 6F_4 + 3.5F_7 - 12F_6$$

$$= (-72 \times 2254) + (69.6 \times 2620) + (6 \times 848) + (3.5 \times 1642) - (12 \times 2620) = 0$$

Fig. 17-12 shows a diagram of the vertical forces on the crane beam, for the condition in which the hoist is at its maximum effective radius. The weight of the members, 240 lbs. (represented as a concentrated load in Figs. 17-6 and 17-7), is shown distributed across the

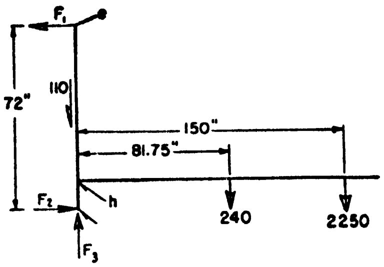


FIG. 17-6. Skeleton Layout of Crane with Hoist Load at Maximum Effective Radius.

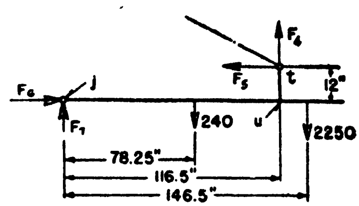


FIG. 17-7. Forces Acting on Crane Beam.

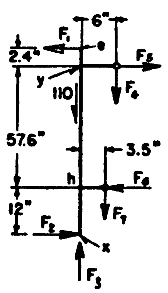


FIG. 17-8. Forces Acting on Crane Mast.

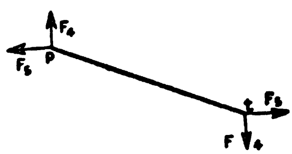


FIG. 17-9. Forces in Tension Rod.

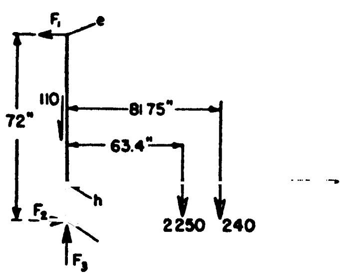


FIG. 17-10. Skeleton Layout of Crane with Hoist Load Positioned for Maximum Flexural Moment.

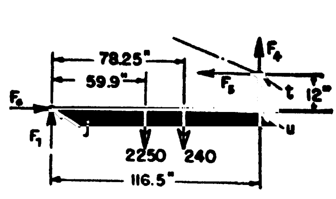


FIG. 17-11. Forces Acting on Crane Beam.

length of the beam. For the vertical force action, the maximum moment occurs at point u and, from a summation of the area of the diagram to the right of that point, its value is

$$M_u = 30(2311 + 22{,}625)/2 + (10 \times 15/2) = 68{,}715 \text{ in.-lbs.}$$

This quantity represents the moment caused by the vertical forces only, and does not take into account the effect of the couple F_6F_6. (An inspection of Fig. 17-12 will show that the areas to the right and left of point u are not equivalent.) The force F_6 induces a moment of $12F_6$, or 12×5945, or 71,340 in.-lbs. at the point u. Since the sign of this moment is opposite to that of the moment of 68,715, the net moment at this point, considering the effect of the horizontal force F_6, is 68,715 — 71,340, or —2625 in.-lbs. The apparent discrepancy introduced at point u may be visualized readily by considering that the moment at an infinitesimal distance to the right of point u is +68,715 in.-lbs., and the moment at an infinitesimal distance to the left of point u is —2625 in.-lbs. The section modulus of the crane beam section is 14.2 (Table 7-2), and the unit tensile stress, from Eq. 5-9, is

$$S = M/Z = 68{,}715/14.2 = 4840 \text{ psi.}$$

From Table 7-1, the allowable tensile stress in flexure for structural sections is 20,000 psi., and an allowable stress of two thirds of this value, or 13,400 psi., is well above that just calculated and the design is safe.

In addition to the flexural stress in the crane beam, the section is subjected to a compressive force of 5945 lbs. which induces a columnar or buckling effect. Bending will occur in a vertical plane, but the least radius of gyration of the beam section is a minimum about the vertical axis, and the beam tends to buckle in a horizontal plane. For the most conservative analysis, therefore maximum stresses may be assumed to occur at the upper right-hand or left-hand corners of the beam section.

For the analysis of buckling action, Eq. 5-15 or 5-16 may be employed. The minimum radius of gyration is 0.84 about the vertical axis; the area of the section is 5.34 sq. in.; the effective length of the column, from Fig. 17-3, is 116.5 in.; the L/k ratio is 116.5/0.84, or 139, and since this value is greater than 120, Eq. 5-16 will be used

It is necessary to determine the value of the unit load P/A in Eq. 5-15 for an L/k ratio of 120, in order to find the value of the constant f in Eq. 5-16. Using a yield point stress S_y of 20,000 psi., and equivalent stress S of one third this value, or 10,000 psi., and an end condition factor C of 2 (corresponding to Case C, Fig. 5-29), the allowable unit load, from Eq. 5-15, is

$$P/A = 10{,}000\left(1 - \frac{30{,}000 \times 120^2}{4 \times 2 \times \pi^2 \times 29 \times 10^6}\right) = 8100 \text{ psi.}$$

Equating this value to Eq. 5-16,

$$8100 = \frac{2 \times \pi^2 \times 29 \times 10^6 \times f}{120^2}$$

and

$$f = 0.204$$

The allowable unit load is equal to

$$P/A = \frac{2 \times \pi^2 \times 29 \times 10^6 \times 0.204}{139^2} = 6050 \text{ psi.}$$

The actual unit load in the beam is

$$\frac{P}{A} = \frac{5945}{5.34} = 1110 \text{ psi.}$$

That portion of the crane beam between the point u and the application of the hoist load is not subjected to columnar action. To determine the effect of a combination of flexural and compressive stresses, the maximum flexural stress may be obtained by finding the maximum flexural moment for that portion of the crane beam between points j and u. This

moment may be obtained from Fig. 17-12 by taking the area of the vertical load diagram to the left of the point of zero load, and is equal to $73 \times 112/2$, or 4188 in.-lbs. The unit tensile stress, from Eq. 5-9, is

$$S = \frac{4188}{14.2} = 295 \text{ psi.}$$

Members subjected to flexural and buckling stresses should satisfy the stress condition indicated in Eq. 7-10, as follows:

$$\frac{1110}{6050} + \frac{295}{13{,}400} = 0.205$$

which is considerably less than unity, and the total stress on the beam is therefore well within the allowable limits.

Fig. 17-13 shows a diagram of the vertical forces on the crane beam for the condition in which the hoist load exerts the maximum flexural stress. For the vertical force action,

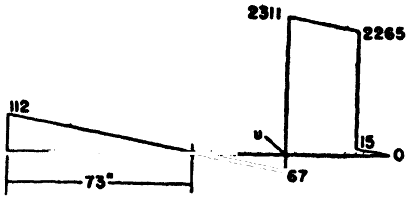


FIG. 17-12. Diagram of Vertical Forces on Crane Beam with Hoist at Maximum Effective Radius.

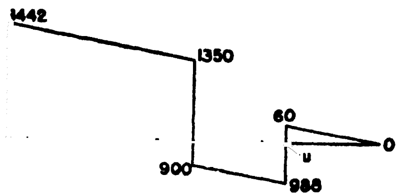


FIG. 17-13. Diagram of Vertical Forces in Crane Beam with Hoist Positioned for Maximum Flexural Moment.

the maximum moment occurs at the hoist position and is equal to $59.9(1642 + 1550)/2$, or 95,500 in.-lbs. The flexural stress in the crane beam at this point, from Eq. 5-13, is

$$S = \frac{95{,}500}{14.2} = 6725 \text{ psi.}$$

The unit compressive load is

$$\frac{P}{A} = \frac{2620}{5.34} = 490 \text{ psi.}$$

The stress condition of Eq. 7-10 is given by

$$\frac{490}{6050} + \frac{6725}{13{,}400} = 0.583$$

indicating that the crane beam section is satisfactory for this condition. In this discussion the increase in the moment arm of force F_5, due to the deflection of the crane beam, has been neglected since the change in stress is comparatively small.

The crane beam should be checked for the maximum permissible stress based upon lateral failure. From Eq. 17-1, with a length L of 116.5 in., and a flange width b of 4 in. (from Table 7-2), the permissible stress is

$$S = \frac{20{,}000}{1 + \dfrac{116.5^2}{2000 \times 4^2}} = 14{,}700 \text{ psi.}$$

This value is appreciably greater than the flexural stress, and there is very little danger of localized flange failure.

The stresses in the mast will attain their maximum values when the hoist is at the maximum effective radius. An examination of Fig. 17-8 will show that the maximum moment may occur at points y or h. The moment at y is

$$M = -2.4F_1 + 0F_5 + 6F_4 = (-2.4 \times 4960) + (6 \times 2378) = 2400 \text{ in.-lbs.}$$

The moment at h is

$$M = -12F_1 - 0F_6 + 3.5F_7 = (-12 \times 4960) + (3.5 \times 112) = -59{,}130 \text{ in.-lbs.}$$

Flexure is induced and will occur in the plane of the crane members; the section modulus of a single 7-in., 9.8-lb. channel about the axis perpendicular to this plane, from Table 7-4, is 6.0 in.^{3}, and the unit tensile stress, from Eq. 5-13, is

$$S = 59{,}130/12 = 4930 \text{ psi.}$$

The mast is subjected to a buckling load induced by force F_3 (2600 lbs.). The mast may buckle about an axis perpendicular to the plane of the crane members, for which the radius of gyration is 2.72 in., or it may tend to fail about an axis in the plane, for which the radius of gyration may be found by an application of Eqs. 5-7 and 5-8. The distance from the centroid of the mast section to the centroid of the channel is 0.375 + 0.55, or 0.925 in.; the area of the section is 2.85 sq.in.; the moment of inertia about the axis parallel to the web is 0.98 in.^{4} The moment of inertia of one channel with respect to the axis of the mast section, from Eq. 5-6, is $0.98 + 2.85(0.925)^2$, or 3.42 in.^{4} From Eq. 5-8, the radius of gyration is $\sqrt{3.42/2.85}$, or 1.095 in.

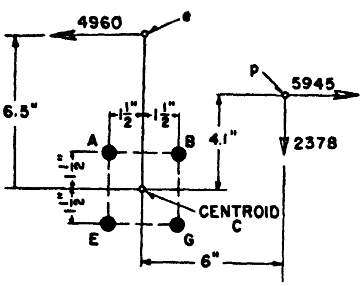


FIG. 17-14. Layout of Upper Joint of Crane.

The L/k ratio for this condition is 72/1.095, or 65.7, and the bearings afford sufficient rigidity to permit using an end condition constant C equal to 2 If the same significant and yield point stresses that are used for the beam are employed for the mast, the allowable unit load is given by

$$P/A = 10{,}000\left(1 - \frac{30{,}000 \times 65.7^2}{4 \times 2 \times \pi^2 \times 29 \times 10^6}\right) = 9436 \text{ psi.}$$

The actual unit load is $2600/(2 \times 2.85)$, or 456 psi. The stress condition indicated in Eq. 7-10 is given by

$$\frac{456}{9436} + \frac{4930}{13{,}400} = 0.416$$

which shows that the actual load is appreciably less than the permissible load.

Because of the eccentricity of load, a detailed analysis of the joints at the upper and lower bearing plates is advisable. A scale layout of the upper joint is shown in Fig. 17-14 for the condition in which the hoist is at its maximum effective radius. The moment about the centroid of the rivet group is

$$M = -(6.5 \times 4960) + (4.1 \times 5945) + (6 \times 2378) = 6500 \text{ in.-lbs.}$$

The total horizontal force on the rivet group is equal to 5945 − 4960, or 985 lbs.; the total vertical force is 2378 lbs. The resisting forces induced by the horizontal and vertical loading are represented in Fig. 17-15, and are equal to 985/4 and 2378/4, or 246 and 595 lbs., respectively. These forces act upward and toward the left to equilibrate the external forces acting downward and to the right. The moment exerted by the external forces on the rivet group is resisted by a horizontal and a vertical force component at each rivet, as shown in Fig. 17-16. The distance from the centroid of the group to the force line is 1.5 in.; since

there are four horizontal and four vertical components, the magnitude of either component F is obtained from $M/(8 \times 1.5)$, or $6500/(8 \times 1.5)$, which gives 542 lbs. These components have a counterclockwise rotational tendency about the centroid C, to equilibrate the clockwise effect of the external forces.

Fig. 17-17 shows the combination of direct and secondary forces on the rivets, and represents the algebraic summation of the forces of Figs. 17-15 and 17-16. The total resisting force will be a maximum for rivet B, and is equal to $\sqrt{1137^2 + 788^2}$, or 1383 lbs.

From Eq. 7-1, with a design stress of 15,000 psi., the shearing strength of a ⅝-in. rivet for structural design in double shear is equal to $15{,}000\pi \times 0.625^2/4$, or 9210 lbs.

From Eq. 7-2, with a bearing stress of 30,000 psi., the bearing strength of a ⅝-in. rivet for structural design in double shear is $30{,}000 \times 0.625 \times 2 \times 0.25$, or 9380 lbs. (The thickness t is based upon the thickness of the webs of both channels, because this value is less than the thickness of the bearing plate.) Even though two thirds of these values be considered a maximum for hoist service, the actual strength of the rivet in either shear or bearing is considerably greater than the actual load.

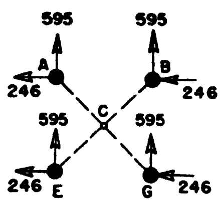


FIG. 17-15. Horizontal and Vertical Resisting Forces on Upper Joint Rivets.

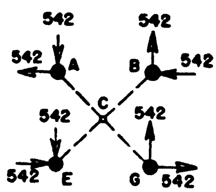


FIG. 17-16. Torsional Force Components on Upper Group of Rivets.

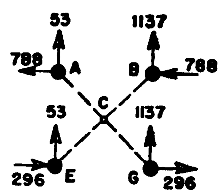


FIG. 17-17. Summation of Resisting Forces on Upper Joint Rivets.

The lower joint is subjected to a moment of 6.5×4960, or 32,200 in.-lbs., a vertical load of 2600 lbs., and a horizontal force of 4960 lbs. The resisting force of the lower rivets of the group, induced by the moment, is $32{,}200/(8 \times 1.5)$, or 2683 lbs. The horizontal and vertical resisting forces are 4960/4 and 2600/4, or 1240 and 650 lbs., respectively. By combining these forces in a manner analogous to the analysis of the upper joint, the horizontal and vertical resisting forces on the lower right rivet are $(1240 + 2683)$ and $(2683 + 650)$, or 3923 and 3333, which combine into a maximum of $\sqrt{3923^2 + 3333^2}$, or 5150 lbs. This value is less than either the allowable shearing or bearing values for hoist design, based upon two thirds of the allowable values given for structural design. Should the actual load on the rivets have been greater than the allowable, it would still have been possible to use the crane by removing the ⅝-in. rivets, re-reaming the holes, and substituting ¾-in. turned bolts, which have allowable shearing and bearing strengths of 13,260 and 11,260 lbs., respectively.

The upper and lower bearing plates of the crane are forged from cylindrical steel bar stock, and the ends of the channels composing the mast are milled so that they are properly seated against the circular flange. This construction will be of material assistance to the rivet groups in resisting primary and secondary shear, but since the actual proportion of such resistance afforded by the flange is difficult to evaluate, it is usually disregarded in stress computations.

The forces and stresses in the connection angles at j, and in the rod bracket at t, require investigation. The rod bracket is a steel casting attached to the upper flange of the crane beam by four ⅝-in. rivets, and has a thickness of 4 in. in a plane perpendicular to the plane of the crane at the region of attachment. The maximum force is induced in the rod when the hoist is at its maximum effective radius, and is $\sqrt{F_4^2 + F_5^2}$, giving $\sqrt{2378^2 + 5945^2}$, or 6400 lbs. This force has a moment arm of approximately 8 in., and the flexural moment is 6400×8, or 51,200 in.-lbs. The section modulus Z of the bracket arm is $(4 \times 3^2)/6$, or 6 in.³, and the unit flexural stress is 51,200/6, or 8533 psi., which is acceptable for cast steel.

For a stress analysis of the rivets, the bracket may be assumed to rotate about the left edge. If the unit force at a unit distance from this axis is taken as F_u, the resisting moment of the rivets is given by

$$2F_u + (2 \times 11 \times 11 \times F_u) = 51{,}200 \text{ in.-lbs.}$$

or

$$F_u = 212 \text{ lbs.}$$

The tensile force on the rivets at the right is 212×11, or 2332 lbs. In addition, the four rivets are subjected to a direct tensile force F_4, or 2378 lbs., or 595 lbs. each. The rivets are also subjected to a direct shearing force F_5, or 5945 lbs., or 1486 lbs. each. The resultant force is a maximum shear, and is equal to $\sqrt{1486^2 + 2927^2/4}$, or 2090 lbs. The unit shearing stress in the rivets at the right is $2090/(\pi \times 0.625^2/4)$, or 6800 psi., which is an acceptable value even if two thirds of the allowable stress for structural rivets are considered.

The analysis of the beam connection at the point j is similar to that developed in Chap. 7; a computation will show that the connection is amply safe.

The stress in the tie rod will be a maximum at the thread root; from Table 6-1, the root area is 1.746 sq. in., and the unit tensile stress is obviously far below the permissible. Similarly, the induced shearing and bearing stresses on the clevis pin at joint p are well within permissible limits.

It will also be necessary to check the size of the bolts for fastening the bearing brackets to the column. The lower bracket bolts are subjected to a horizontal reaction F_2 of 4960 lbs., and a vertical reaction F_3 of 2600 lbs. The horizontal reaction acts towards the column face, and does not affect the bolts. The moment of the vertical reaction is 2600×9, or 23,400 in.-lbs. Assuming the unit force in a bolt at a unit distance as F_u, the total resisting moment of the bolts is

$$(2 \times 2 \times 2 \times F_u) + (2 \times 5 \times 5 \times F_u) + (2 \times 8 \times 8 \times F_u) = 23{,}400 \text{ in.-lbs.}$$

or

$$F_u = 23{,}400/186 = 126 \text{ lbs.}$$

The tensile force on each of the upper bolts is $8F_u$, or 8×126, or 1008 lbs. The shearing force on each bolt is 2600/6, or 333 lbs., and the maximum resultant shearing force is $\sqrt{333^2 + 1008^2/4}$, or 604 lbs.

The upper bracket bolts are subjected to a horizontal reaction of 4960 lbs., with a consequent moment about the lower edge of the bracket of 24,800 in.-lbs. The unit force F_u at a unit distance is equal to 24,800/186, or 133 lbs., and the tensile force on each of the upper bolts is 8×133, or 1064 lbs. The direct stress is 4960/6, or 827 lbs. per bolt. The maximum load on the upper bolts is, therefore, $1064 + 827$, or 1891 lbs.

The permissible unit shearing and tensile stresses for unfinished bolts, from Table 7-1, are 10,000 and 13,000 psi.; allowable stresses for hoisting equipment may be taken as two thirds of these, resulting in values of 6667 psi. for maximum shear, and 8667 psi. for tension at the thread root. The shearing strength of a ¾-in. bolt is equal to $6667 \times \pi \times 0.750^2/4$, or 2940 lbs. The tensile strength is equal to $8667 \times \pi \times 0.750^2/4$, or 3830 lbs. A comparison of the actual and allowable loads shows that the ¾-in. bolts are satisfactory.

Several other connection details should be checked, such as the angle connection between the mast and beam, and the rivets attaching the tie rod bracket to the crane beam, but since the stress analysis is essentially the same as those given in this chapter and in Chap. 7, they are omitted from this discussion.

When the crane is installed it may be advisable to use a ball thrust bearing, as shown in the alternate design in Fig. 17-3, to make for easier handling and operation. A suitable bearing may be obtained by reference to the catalog of any bearing manufacturer.

17-14. Many items of material handling equipment can be fabricated in the plant in which they are to be used by employing standard or stock parts available from suppliers and manufacturers. If the plant has a capable machinist and welder, and is equipped with a small shop furnished with a lathe, drill press, forge, and welding equipment, a variety of plant-fabricated apparatus can be

made up quickly and inexpensively. The processing engineer may be called upon to design bases, brackets, and other equipment for which no similar commercial products are available, and should be reasonably familiar with the possibilities and limitations of manufacturing and fabricating processes. This information can be acquired by experience, or by a study of reference works on engineering tools and processes.[28]

17-15. Arc-welded Steel Replacements. From the standpoint of plant fabrication, castings of steel or cast iron are not desirable, because of the expense and time required to make patterns. Similarly, die castings and forgings

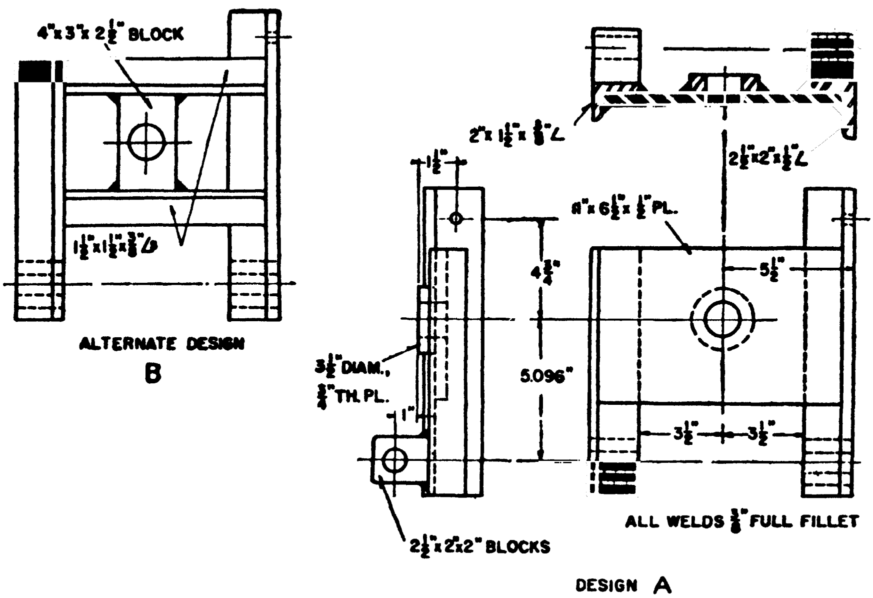


FIG. 17-18. Arc-welded End Brackets for Winch.

more complex than the bearing plates shown in Fig. 17-3 are not usually available. Arc-welded frames composed of structural members and plates are easily fabricated, however, and may be used to replace castings or forgings. The winch shown in Fig. 17-2 is an example of a commercial article which is manufactured in sufficiently great quantity to permit the use of three castings, the two end brackets and the worm wheel-hoist drum unit. A plant-fabricated unit can be constructed by purchasing a suitable worm gear set and attaching the worm wheel to a drum turned from cast iron or steel bar stock. The end brackets can be made of arc-welded structural steel members, as design A or B shown in Fig. 17-18, and either design is satisfactory; alternate B is used when plate material is not readily available. The crank can be fabricated by welding a cylindrical section to one end of the crank to serve as a hub, and forcing a commercial handle, or a plain cylindrical pin, into a drilled hole at the other

end of the crank. The worm wheel and worm shafts can be made of cold-rolled steel bar stock, without further machining other than drilling and reaming for the taper pins.

Fig. 17-19 illustrates several other examples in which arc-welded steel parts are employed to replace riveted joints in structural connections and castings in machine construction. The arc-welded column and beam connections may be

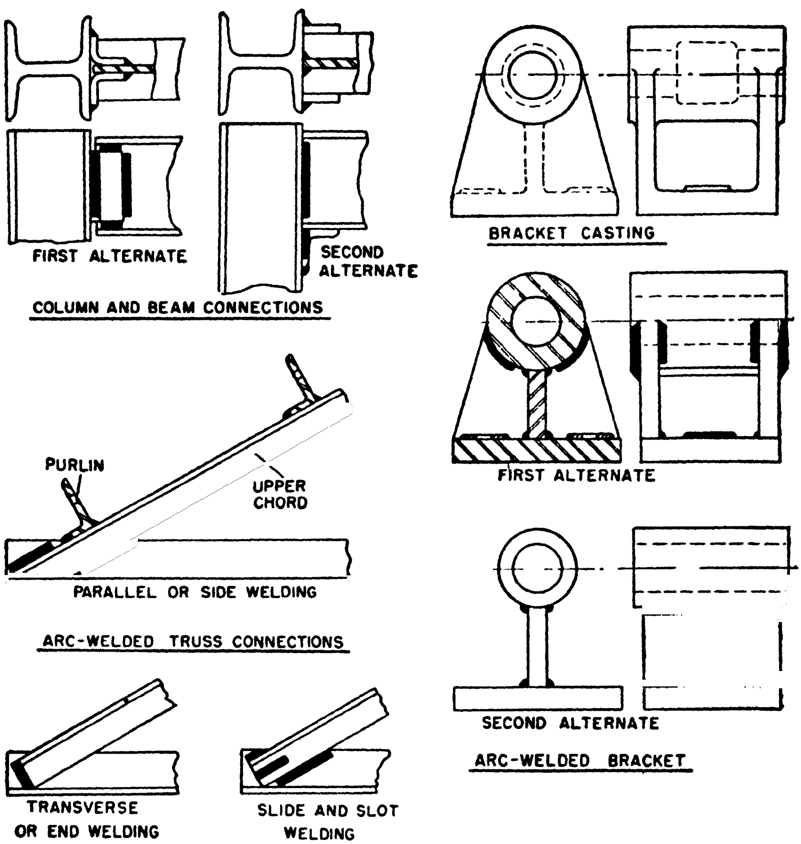


Fig. 17-19. Redesign for Arc Welding.

used to replace similar riveted beam connection angles. The first alternate is similar in design to a riveted connection, and makes use of connection angles to join the I beam to the H column; the second alternate differs from the riveted construction, but is less expensive to erect and weld than the first design. In many instances, the seat angle has two bolt holes in the horizontal flange, so that it may be welded to the column and used as a bracket to support the beam while the latter is properly set and aligned. After alignment, the beam is bolted to the seat angle and welded as indicated.

Three methods of welding truss joints are shown, any of which may be used to replace the riveted construction shown in Fig. 8-8. Transverse or end welding is preferred to parallel or side welding, but the latter is often used when the joint length is limited. In some cases the upper chord angle is slotted so that additional weld length can be obtained. In other cases gusset plates are employed if the angles composing the truss have insufficient leg lengths to permit adequate weld lengths.

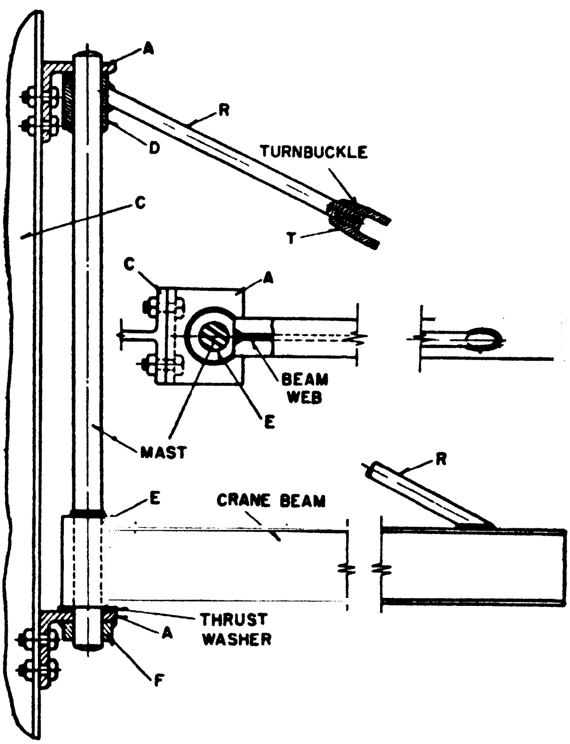


Fig. 17-20. Wall-supported Arc Welded Jib Crane.

Fig. 17-20 shows an arc-welded jib crane of the same capacity and size as the commercial product represented in Fig. 17-3. The mast is made of cold-rolled steel bar stock, with hollow CRS cylinders *D* and *E* welded in place. These cylinders have a flat face to which the crane beam and the tie rod *R* are welded. The lower end of the tie rod is fillet welded to the upper flange of the beam. The bearing brackets *A* are sections of structural angles, with drilled holes to serve as bearings for the mast. A collar *F* may be welded to the brackets if additional bearing area is required. The brackets *A* can be welded to the column if desired. Fig. 17-21 shows alternative designs for the joints of the crane of Fig. 17-20. It should be noted that when iron castings are replaced by arc-

welded steel frames, there is a justifiable tendency on the part of the designer to reduce the size and weight of the parts in direct proportion to the greater strength of the steel. This practice may result in a loss of rigidity, for steel is far more elastic in proportion to its strength than is cast iron. If rigidity and stiffness are important factors in the function of the part, due consideration should be given to a possible increase in deflection if the part is redesigned for arc-welded construction on an equal strength basis.

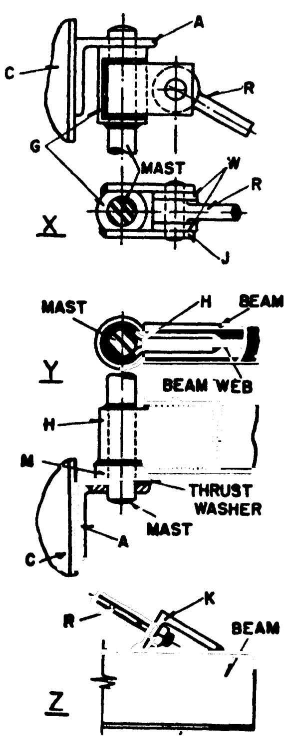


Fig. 17-21. Alternate Construction for Arc Welded Jib Crane.

17-16. Hoisting Chain and Rope. Several forms of coil chain (described in Chap. 14) are used for hoisting purposes. Vegetable fiber rope, made from manila or cotton fibers, is used extensively for temporary hoists and slings and for service of intermittent character. The ultimate strength S of manila fiber rope is given by

$$S = 7000\,d^2 \qquad (17\text{-}2)$$

where d is the rope diameter. The ultimate strength of cotton rope is approximately 70% of this value. The working load for manila and cotton rope should be from one thirtieth to one thirty-fifth this value. The size of the sheave or drum over which the rope is run has an important effect on the life of the rope, since the bending action induces internal friction and external chafing of the rope fibers. For slow-speed haulage, not exceeding 50 ft. per min., the minimum sheave diameter may be as low as 8 to 10 times the diameter of the rope; high speeds, up to 600 ft. per min., may necessitate sheave diameters up to 40 or 50 times the rope diameter to insure a reasonable rope life.

17-17. Wire Rope. Wire rope is in wide use for hoisting and haulage, and for static loads, such as guy and supporting wires for stacks, masts, and towers.[8] Wire rope consists of cold-drawn steel wires wrapped into strands and twisted around a hemp center or core saturated with lubricant. Commercial wire rope is obtainable in several grades of steel, referred to as cast steel, plow steel, and extra-high-strength steel. Rope made of aluminum, bronze, and stainless steel wire is also available. Rope with wires fabricated from cast steel is used for ordinary service; plow steel, high-strength or improved plow steel, or extra-high-strength steel ropes are used for high degrees of security and severe service conditions. In preformed wire rope, the strands are given their helical

shape in the process of manufacture, which serves to reduce metallic fatigue and friction between the parts.

The type and construction of wire rope is indicated by two figures, the first giving the number of strands, and the second the number of individual wires per strand. A 2-in., 6 × 7 rope has a maximum diameter of 2 in., as illustrated in Fig. 17-22, and has six strands of seven wires each. For a given diameter, ropes made up of many wires are more flexible than those with few wires, and may therefore be used on sheaves of smaller diameter. The common rope types are 6 × 7 coarse, 6 × 19 flexible, and 6 × 37 and 8 × 19, extra-flexible. The 6 × 7 rope is used for mine and yard haulage and guy wires; 6 × 19 is the standard hoisting rope, and is used for mine and ore hoists, car pullers, cranes and elevators. Extra-flexible rope is used for small diameter sheave applications, and for steel mill ladles, cranes, and high-speed elevators.

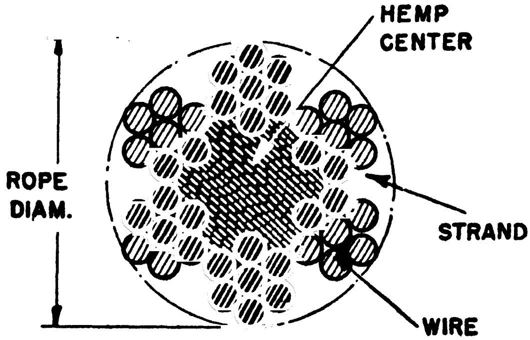


FIG. 17-22. 6 × 7 Wire Rope.

Commercial wire rope is usually selected on the basis of the breaking strength, as shown in Fig. 17-23. On the plot, *HS* represents extra-high-strength steel wire rope, *PS* plow steel wire rope, and *CS* cast steel wire rope. The figure also shows the list price and weight per foot of various types and grades.

17-18. Stresses in Wire Rope. A wire rope under load is subjected to stresses caused by the load lifted (which includes the weight of the carrier and the rope length between the carrier and the drum), to stresses caused by sudden starting and stopping and taking up slack in the rope, and to stresses caused by bending the rope around the sheave. If the load is lifted suddenly, the force *A*, induced by acceleration, is given by

$$A = Wa/32.2 \tag{17-3}$$

where *W* is the total load and *a* the acceleration in feet per second per second. Impact loads, caused by taking up rope slack suddenly, are difficult to evaluate but may cause an increase in the actual stress in the rope by as much as 100%. Impact stresses are usually accounted for by a reasonable increase in the factor of safety used for rope selection.

When a wire rope is bent around a sheave, the outer wires tend to increase in length. This action induces a tensile stress in the wires, over and above that caused by the direct load, and is dependent upon the wire size and rope construction. The total tensile force induced by bending, *B*, in tons, is given by

the following equation based upon Eq. 5-26 and using a modulus of elasticity of 12×10^6 for wire rope.

$$B = Cd^3/D \tag{17-4}$$

where d and D are diameters of rope and sheaves respectively, in inches, and C is a constant from Table 17-1. This shows that the bending stress of rope

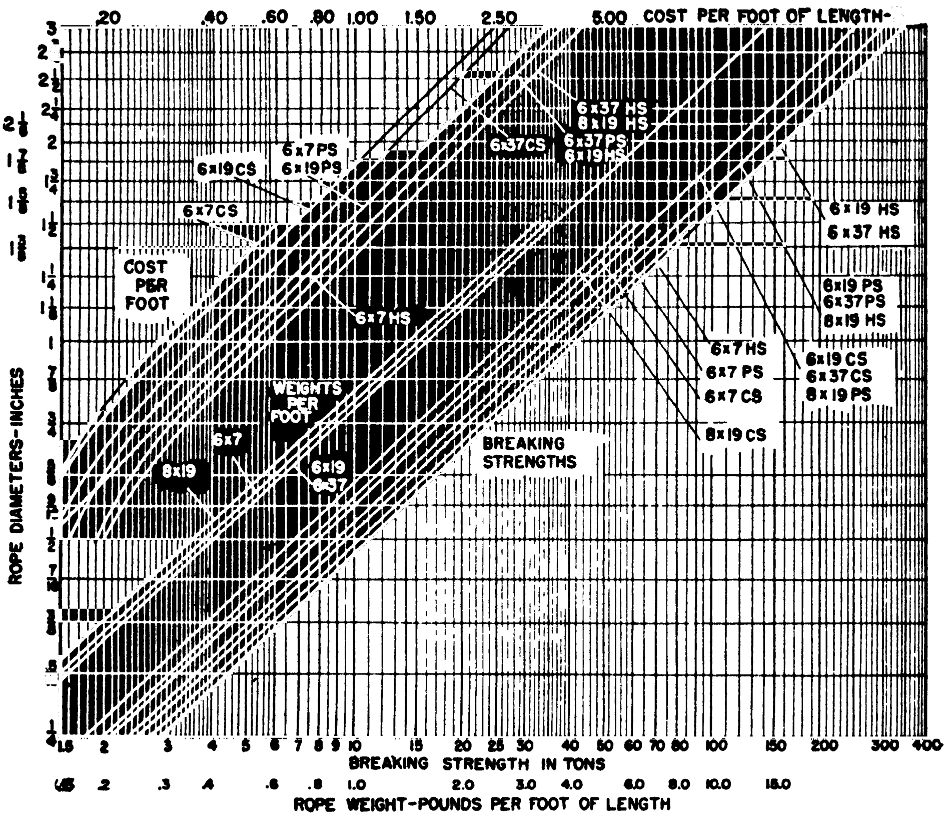


Fig. 17-23. Strength, Weight, and Cost of Wire Rope.

varies with the cube of the diameter and inversely as the diameter of the sheave. Recommended advisable and minimum sheave diameters are given in Table 17-1. Sheaves smaller than those indicated should not be used without the approval of the manufacturer.

The factor of safety of a wire rope is the ratio of the breaking strength to the sum of the stresses induced in the rope. The factor varies from 8 to 12 for elevator service, from 2½ to 5 for mine hoists, derrick service, and hand-operated cranes, and from 4 to 6 for power-actuated cranes. Wire rope which

is joined by splicing should not be used for severe or important service. Spliced rope is considered to have about 75% of the breaking strength of unspliced rope.

TABLE 17-1.—WIRE ROPE SELECTION DATA

Rope	Advisable Sheave Diam.	Minimum Sheave Diam.	Constant C
6×7	$72\,d$	$42\,d$	270
6×19	$45\,d$	$30\,d$	162
6×37	$27\,d$	$18\,d$	148
8×19	$31\,d$	$21\,d$	114

Various types of attachments are used for fastening wire rope to crane hooks and other devices; details and necessary dimensions may be found in suppliers' and manufacturers' catalogs. The rope socket attachment is the only one that will develop 100% of the full rope strength. It is made of forged steel with a tapered socket into which the separated wires of the rope are anchored by means of high-grade zinc poured into the socket in molten state. Another attachment uses a steel thimble for the loop or eye of the rope; the rope itself is fastened by clips, clamps, or by splicing. Rope connections made with clips or clamps are not recommended as permanent fastenings since they develop only 50 to 75% of the strength of the rope.

Example 17-2. Find the safe load that can be lifted by a ⅞-in., 6×19 plow steel wire rope using a factor of safety of 4. The sheave diameter is 70 in., and the total distance through which the load is to be lifted is 1000 feet. The rope speed is 600 f.p.m., and the load is to attain its maximum velocity in 20 feet.

Solution. From Fig. 17-23, the breaking strength and weight per foot of a ⅞-in., 6×19 wire rope are 28 tons and 1.25 lbs. The total load to which the rope can be subjected is equal to the breaking strength divided by the factor of safety, or 28/4, or 7 tons. The total load due to bending around the sheave is obtained from Eq. 17-4, with constant C from Table 17-1, and is

$$B = 162 \times 0.875^3/70 = 1.55 \text{ tons}$$

The net permissible load is $7 - 1.55$, or 5.45 tons. The stress in the rope induced by acceleration is obtained from Eq. 17-3. The acceleration a in terms of the velocity v, feet per second, and the distance H in feet, is

$$a = v^2/2H = (600/60)^2/(2 \times 20) = 2.5 \text{ ft. per sec. per sec.}$$

The acceleration force is

$$A = 5.45 \times 2.5/32.2 = 0.423 \text{ ton}$$

The weight of the rope is 1000×1.25 lbs., or 0.625 ton. The useful load that can be lifted is $5.45 - 0.423 - 0.625$, or 4.4 tons.

Example 17-3. **A hand-operated winch has a drum 8 in. in diameter. Find the size and type of wire rope for a load of 1500 lbs.**

Solution. Because of the comparatively slow speed of operation and the relatively short lift, neither the acceleration stresses nor the weight of the rope need be considered. The sheave diameter will probably have an important influence on the size, and consequently on the type of rope. If a 6 × 19 rope is selected, tentatively, the maximum rope diameter d, based upon the minimum sheave diameter from Table 17-1, will be 8/30, or 0.267 in., giving a ¼-in. rope. The bending stress in this rope will be

$$B = 162 \times 0.25^2/8 = 0.316 \text{ ton}$$

The sum of the useful and bending loads is 0.75 + 0.316, or 1.066 tons; with a factor of safety of 3, the breaking strength must be at least 1.066 × 3, or 3.198 tons. The breaking strength of ¼-in., 6 × 19 cast steel, plow steel, and high-strength steel ropes, from Fig. 17-21, are 2.1, 2.5, and 2.9 tons, respectively. Although a ¼-in., 6 × 19 HS rope would probably serve, it will be better to redesign for a more flexible rope. If an 8 × 19 rope be selected, the maximum rope diameter d will be 8/21, or 0.381 in. If a ⅜-in. rope is chosen, the bending stress, from Eq. 17-4, with constant C from Table 17-1, will be

$$B = 114 \times 0.375^2/8 = 0.752 \text{ ton}$$

The sum of the useful and bending loads is 0.75 + 0.752, or 1.5 tons, and the required breaking strength is 1.5 × 3, or 4.5 tons. A plow steel rope of this size and construction has a breaking strength of 4.6 tons, and is satisfactory.

A trial may also be made of a 5⁄16-in., 8 × 19 rope. The bending stress is

$$B = 114 \times 0.313^2/8 = 0.434 \text{ ton}$$

The required breaking strength is (0.75 + 0.434)3, or 3.55 tons. A 5⁄16-in., 8 × 19 plow steel rope has a breaking strength of 3.2 tons, which would probably be satisfactory under the circumstances.

PROBLEMS—CHAPTER 17

1. This problem relates to the winch of Fig. 17-2, which is designed for a 1 ton load.

a. Assuming an efficiency of 60%, which is a fair value for this gear set since the lubrication and attention are likely to be intermittent or indifferent, determine the force that must be exerted at the crank handle to lift the maximum load. The load radius should include the effect of a ⅜-in. diameter wire rope.

b. In what direction should the crank be turned—clockwise or counter-clockwise, looking at the front of the winch—to lift the load? How fast will the load be lifted if the crank is turned 30 times per minute? What effective horsepower is developed in so turning the crank? Is it necessary to hold the crank to prevent the load dropping? Explain and prove

c. The worm wheel is made of cast iron with hobbed teeth. Determine the load-carrying capacity.

d. Check the stress in the driving pins on the worm shaft.

e. Considering the weight of the winch concentrated at the centerline of the drum shaft, find the maximum stresses in the bolts for the maximum load on the winch. Assume ⅝-in. diameter through bolts with nuts and washers and disregard the pull on the crank.

f. Check the stresses in the worm wheel shaft and specify a suitable material.

g. Select a suitable type and size of wire rope.

2. A high-speed mine hoist has a lift capacity of 2 tons and a rope speed of 500 FPM. The hoist drum is 38 in. in diameter and is driven through some medium by a 1200-RPM DC motor. The hoist, driving medium, and motor should form as compact a group as possible, with a high efficiency. Make recommendations to the design department as to a suitable driving medium and motor for this unit, giving any necessary figures.

3. A college testing laboratory has a 40-KW, 900-RPM DC generator and a 50-HP, 900-RPM AC motor. It is desired to couple these units so as to have an MG set for testing purposes. The units are to be mounted on a common arc welded base.

a. Determine the necessary mounting dimensions of the motor and generator from electrical suppliers' catalogs or from the NEMA standards.

b. Select three suitable commercial couplings for this unit.

c. Design a suitable steel base, to be mounted on a concrete footing.

d. Design the concrete footing.

4. In a coal mine the weight of the cage and coal lifted per trip is 20,000 lbs. The height of the lift is 500 ft. The hoist drum diameter is 5 ft. and the rope is accelerated from rest to a speed of 1200 ft. per min. in 5 seconds.

a. Determine whether one, two, or three 1¼-in., 6 × 19 Plow Steel wire ropes should be employed for an overall safety factor of from 5.5 to 6.

5. Select a suitable wire rope for the hoist of Problem 2. The hoist lift is 2000 ft., and the hoist attains full speed in 30 seconds.

6. The runway of a crane consists of 30-ft. lengths of an I beam, each length being supported at its ends. The crane wheels are 6 ft. apart, and the anxious load on each wheel is 4 tons. Find a suitable I beam, neglecting the localized stress concentration due to the effect of the trolley wheels on the beam flange.

7. Design an arc welded jib crane with an effective load radius of 16 ft. and a mast length of 7 ft., center to center of bearings, for a net load of 1 ton.

8. Like Problem 7, for a radius of 12 ft. and a net load of 3000 lbs.

CHAPTER 18

SPECIAL STRESS APPLICATIONS

External Pressures

18-1. The design of a pressure vessel or tube to withstand external pressures differs fundamentally in basic concept from that required for the design of vessels subjected to internal pressure. This difference in principle is somewhat analogous to that between tension rod and column member design. The cross-sectional shape of tension members is of no great importance provided there is no appreciable eccentricity of the load but, if the same member were subjected to axial loading and thus acted as a column, it would be essential to take into account the disposition of its cross-section area about its centroid. By analogy, cylindrical vessels subjected to internal pressure require sufficient wall and joint strength to resist rupture, whereas vessels designed to resist external pressures must resist collapse. Internal pressure vessels do not require consideration of such features as length-diameter ratio L/D, shell thickness-diameter ratio t/D, or irregularities incident to the fabrication of the vessel, such as out-of-roundness, dents or bumps, all of which factors are of major importance in the design of externally loaded vessels.

18-2. Theoretical Relations. The design and construction of pressure vessels subjected to external pressures is governed by theoretical analysis, experimental observation, and empirical data based upon experience. Theoretical analysis is complicated by the actual size and ratios of length, diameter, and thickness of the fabricated pieces, as well as consideration of the physical properties of the materiáls used. Three types of cylindrical vessels are recognized with respect to linear dimensions. Short vessels are those in which the ends are close enough to stiffen the shell radially and thus maintain the true cylindrical form without eccentricity or dents. Such vessels will yield when the stresses are sufficiently high to overcome the circumferential strength, and the stress capacity is thus dependent principally on the relation between material strength, shell thickness and diameter. For these vessels the effect of length is negligible provided it is small enough to be defined as short. Long vessels have ends far apart; the stiffening effect of the ends is not noticeable at the mid-sections of the cylinder. In such designs initial irregularities will induce bending and, finally, buckling which is practically independent of the compressive or tensile strength of the material, and the stress capacity is influenced by the wall thickness, vessel diameter, and a minimum length. Such vessels will fail at stresses well below the yield point. Intermediate length vessels are between the extremes

of the first two types and thus both material strength and flexural stresses due to vessel length may be of varying importance with relation to the thickness and diameter. Either long or intermediate vessels can be stiffened by circumferential construction and thereby made to approximate vessels of short cylinder design.

Much of the theoretical analysis for the effect of external pressure on tubes and vessels is based upon consideration of thin-walled tubes. These analyses include two important factors: a critical length L_c, which is defined as the minimum length above which resistance to collapse is independent of length, and the number n of lobes or bumps in a circumference at the time of collapse. Both of these factors are related to the outer diameter D and the thickness t, of the tube, and to Poisson's ratio μ; the following expression has been approximately verified by experiment.

$$n = \sqrt[4]{\frac{0.75\,\pi^2(1-\mu)^{1/2}}{(L/D)^2(t/D)}} \tag{18-1}$$

and

$$L_c = KD\sqrt{\frac{D}{t}} \tag{18-2}$$

where

$$K = 0.642\sqrt[4]{1-\mu^2} \tag{18-3}$$

Von Mises derived a formula for thin-walled tubes and vessels shorter than the critical length, subjected to external radial pressure only. It is accurate for short lengths and checks experimental results within a few per cent even near the critical length L_c. This simplified formula is

$$p_c = \frac{n^2}{3}\left[1 + \frac{2}{n^2\left(\frac{2L}{D}\right)^2 - 1}\right]\frac{2E}{1-\mu^2} + \frac{2E(t/D)}{n^2\left[m^2\left(\frac{2L}{D}\right)^2 + 1\right]^2} \tag{18-4}$$

where p_c is the collapsing pressure, n the number of lobes at that pressure, L the length of the tube, and the other symbols as defined above.

Carman has developed a relation for seamless and lap-welded steel tubes, which is useful for lengths beyond the critical:

$$p_c = 50.2 \times 10^6(t/D)^3 \tag{18-5}$$

When axial loads are superimposed upon radial loads, column effects enter and the equations become more complex; their use is not yet justified by test and are beyond the scope of this text.

18-3. ASME-UPV Code. The ASME-UPV Code has been set up on the basis of the above and other formulae, and upon experimental and experience data for unfired vessels subjected to external pressure. Three equations are

used which cover the three types of vessels referred to in section 18-2. The working pressure P is specified to be one fifth of the collapsing pressure p_c. For short cylinders

$$p_c = 5P = \frac{2S_y t}{1.05\,D} \tag{18-6}$$

where S_y is the yield point of the material, psi.

For cylinders of intermediate depth

$$p_c = 5P = \frac{2.60\,E(t/D)^{2.5}}{(L/D) - 0.45(t/D)^{0.5}} \tag{18-7}$$

where L is the effective length and E is the elastic modulus.

For long vessels where t/D is equal to or less than 0.023, Carman's formula (Eq. 18-5) is used,

Where t/D is greater than 0.023

$$p_c = 5P = \frac{86{,}670\,t}{D} - 1386 \tag{18-8}$$

The range of application of each of these equations is dictated by experience; that is, a factor of experience indicates when the vessel can be considered short, intermediate, or long. The effect of such experience factors is shown graphically in the curves of Fig. 18-1, which are taken from the ASME-UPV Code. This particular set of curves apply to a steel with a tensile yield strength of 27,500 psi. and a modulus of elasticity of 29×10^6 (specifications S-1 and S-2. Table 3-1). Similar curves can be drawn for other steel alloys and for non-ferrous materials. The horizontal lines at the left of the chart represent short cylinder conditions and are calculated by Eq. 18-6. The sloping lines are for the intermediate cylinders for which Eq. 18-7 applies. The horizontal lines at the right are for long cylinders and Eqs. 18-5 and 18-8 were used. The break points (change in slope) of these curves show the distinction between short, intermediate, and long cylinders for various t/D ratios. The curves are plotted with the L/D ratios, as abscissae where L is the length between centers of head seams or circumferential stiffeners and D is the outer diameter of the vessel. The ordinates represents working pressures, psi., which are equal to one fifth the collapsing pressure. Each curve is for one t/D ratio, or ratio of shell thickness to outside diameter. The data and curves specified by the ASME-UPV Code are not applicable to tubes expanded, rolled, or screwed into tube sheets, or to vessels used in petroleum processing.

In using the curves of Fig. 18-1, the L/D ratio is computed, and the value of the t/D ratio corresponding to the L/D ratio and the working pressure determined. For L/D ratios greater than 20, the t/D ratio is considered constant. Fig. 18-1 may also be used to compute the collapsing pressures for vessels made of steels having different physical characteristics than the one for which the

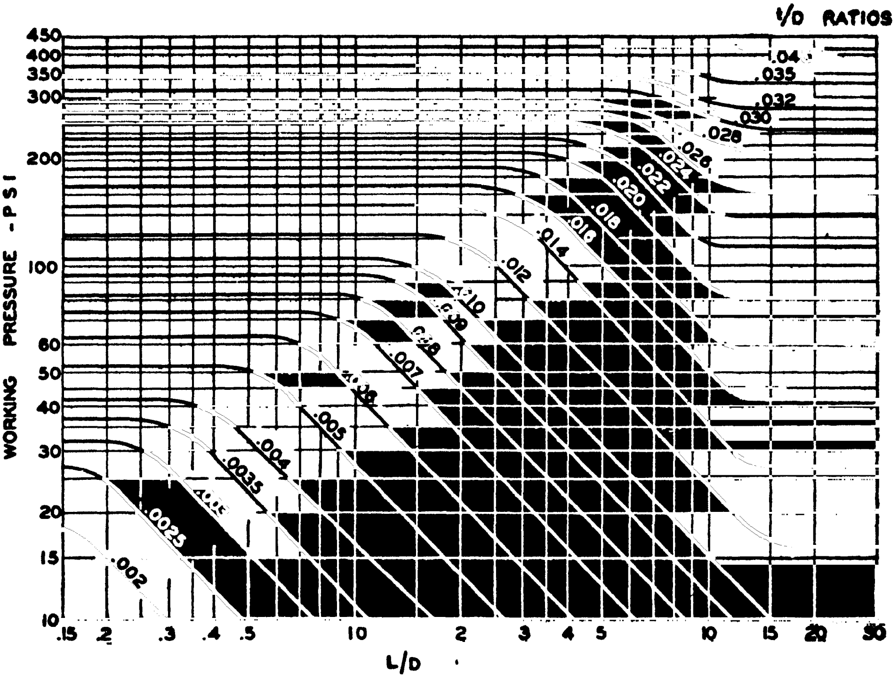


Fig. 18-1. Shell Thickness of External Pressure Vessels.

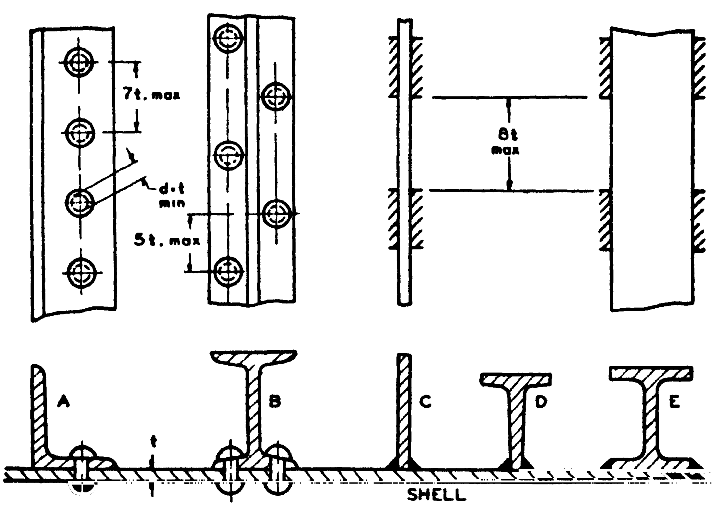


Fig. 18-2. Shell Stiffener Sections for External Pressure Vessels.

chart is constructed. For short vessels, the t/D ratio should be multiplied by a factor $S_y/27{,}500$ where S_y is the yield strength of the material. For vessels of intermediate length, the t/D ratio value should be multiplied by $E/(29 \times 10^6)$ where E is the modulus of elasticity of the material.

Example 18-1. A cylindrical vessel subjected to an external pressure of 20 psi. gage has an outer diameter of 90 in. and a length of 30 ft. between head seams. Determine the vessel thickness t for a steel of S-1 specification.

Solution. The length L is 30×12, or 360 in., and the L/D ratio is 360/90, or 4.0. From Fig. 18-1, at a working pressure of 20 and an L/D of 4 the t/D value is 0.0076. The thickness t is therefore 0.0076×90, or 0.684 in., for which an $^{11}\!/_{16}$-in. plate would be used.

18-4. Stiffening Rings. Cylindrical vessels subjected to external pressure are often reinforced by circumferential stiffening rings, which may be composed of bars or structural shapes riveted or welded to the exterior or interior surface of the shell of the vessel. Several representative stiffeners are shown in Fig. 18-2. Detail A shows a structural angle riveted in place; this type of stiffener is very common, but is subject to some eccentricity and may tend to turn over. Detail B shows an American Standard or I beam; this section affords symmetry, but is difficult to fabricate in circular form. Details C, D, and E represent a flat or rectangular bar, a T section, and a WF section welded in place. The flat bar is a very satisfactory stiffener, and is more easily fabricated than other types; the WF section is more readily bent than the I beam section.

Jacketed vessels, Fig. 18-3, are examples of equipment subjected to both internal and external pressures; the jacket is subjected to the former, while the vessel proper is subjected to external pressure from the fluid contained in the jacket. The detail at the right shows the application of a flat bar stiffening ring; that at the left shows how the jacket closure serves as a stiffener.

Exterior stiffening rings must extend completely around the vessel; interior stiffeners are usually made with gaps for drainage or to permit pipes or tubes to be carried, as shown in detail W, Fig. 18-3. (It should be noted, however, that the continuity of the ring must be kept.) Such gaps should not be larger than one quarter the length of a possible failure lobe, and the ASME-UPV Code specifies that the length M of the gap must not exceed the arc length obtained from Fig. 18-4. In this chart the solid line curves give various values of the gap length, outer diameter ratio, as referred to the L/D and t/D ratios. Should the length of the gap exceed the values permitted by Fig. 18-4, the stiffening rings must be fastened to the supporting cradle, indicated in detail X, Fig. 18-3, and the moment of inertia I_b at the weakest section of the cradle must be at least equal to the moment of inertia of the stiffening ring. The minimum arc of contact of the cradle should be 120°, and greater arcs are preferred.

Joints in stiffening rings, such as the fillet-welded plate shown at A, or the butt weld at B, Fig. 18-3, must be of sufficient size to develop the required moment of inertia for the stiffening ring. If the ring has openings as in detail *W*

for piping or tubes, or for vessel joints, the net moment of inertia I_a of the ring must be of sufficient magnitude to answer satisfactorily.

The required moment of inertia of circumferential stiffening rings is found by the following general formula, which is used as the basis of a chart in the ASME-UPV Code:

$$I_r = \frac{0.035\,D^3Lp}{E} \tag{18-9}$$

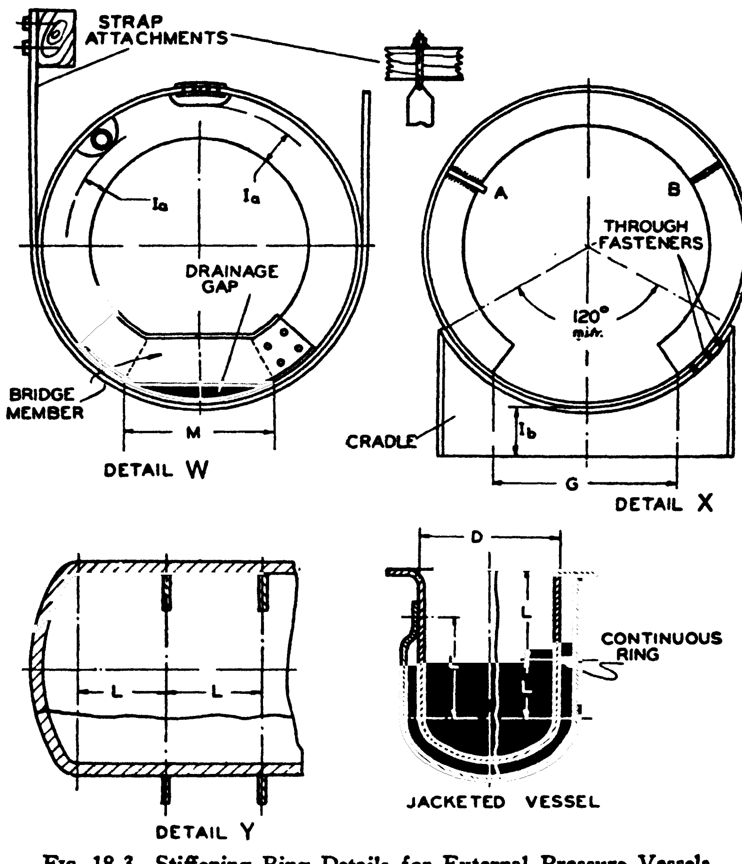


FIG. 18-3. Stiffening Ring Details for External Pressure Vessels.

where I_r is the moment of inertia of the stiffener about its centroidal axis parallel to the vessel axis, inches⁴, D the outer diameter and L the length between stiffeners, inches, p the following pressure, psi., and E the elastic modulus. If the working pressure P is taken as one fifth the collapsing pressure, and the elastic modulus is taken as 29×10^6, Eq. 18-9 becomes

$$I_r = \frac{6\,D^3LP}{10^9} \tag{18-10}$$

Stiffeners should not be stronger than required, since a vessel under external pressure contracts to a certain extent, and stress concentrations in those portions of the vessel adjacent to the stiffeners may cause failure.

It has been indicated previously that shape irregularity incident to manufacture is of secondary importance in the design of cylindrical vessels subjected to

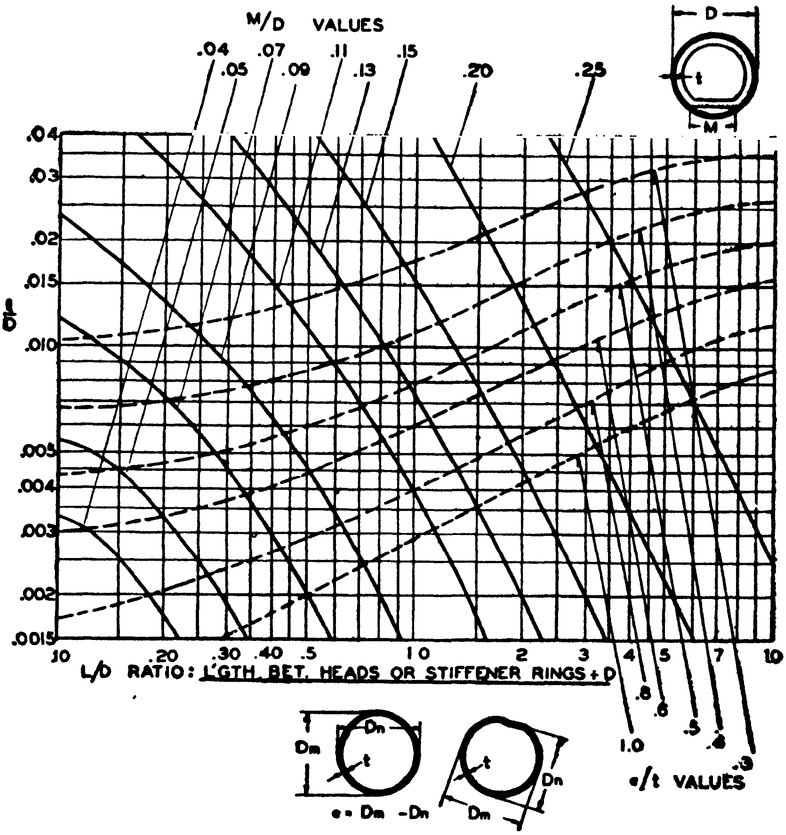


Fig. 18-4. Out-of-roundness Determination for External Pressure Vessels.

internal pressures, since the effect of the forces within the vessel is to expand the shell equally in all directions and thereby cause it to assume a cylindrical form. When a cylinder collapses under external pressure, however, characteristic lobes or bulges are formed, whose occurrence and frequency depend upon the L/D and t/D ratios (Fig. 18-1). The possibility of lobe formation is greatly increased by any initial irregularity in the cylindrical shape of the shell, and such variation must therefore be carefully limited in design and construction. The limits of eccentricity e, or difference between the maximum

and minimum diameters of the shell, as referred to the thickness and the L/D and t/D ratios, are indicated by the dotted-line curves in Fig. 18-4. For vessels with longitudinal lap joints, however, the allowable eccentricity obtained from Fig. 18-4 is exclusive of the diameter differences resulting from the joint. Vessels for external pressures are often re-rolled or re-formed after fabrication, to insure compliance with the data of Fig. 18-4.

It must be particularly noted that any arrangement of stiffeners that tends to localize restraint is worse than no restraint at all, and that internal stays or supports used for any purpose must not bear against the vessel shell except through the medium of a substantially continuous ring. Supporting cradles or straps, for example, must not be placed between supports, and should be arranged so as to permit uniform radial contraction of the vessel walls.

Example 18-2. Find the maximum out-of-roundness permitted in the vessel of Example 18-1.

Solution. The t/D ratio, using the actual shell thickness, is 0.6875/90, or 0.00764. From Fig. 18-4, for an L/D ratio of 4.0, the eccentricity-thickness ratio e/t is 0.85, and the permissible difference e between the maximum diameter D_m and the minimum diameter D_n is therefore $0.225t$, or 0.85×0.6875, or 0.585, say 19/32 in. This permissible out-of-roundness should be specified on the construction drawing of the vessel, and the measurements on the completed shell should be made in a sufficient number of planes to make sure that the entire surface is within this limit.

Example 18-3. Redesign the vessel of Example 18-1, employing circumferential stiffeners, and determine the most economical proportions for this vessel.

Solution. A superficial consideration of this problem might indicate that the most economical design is one in which the thinnest shell is used. From Fig. 18-1, for a working pressure of 20 psi., the smallest permissible value of t/D is 0.0021, with corresponding L/D value of 0.15. This gives a plate thickness of 0.0021×90, or 0.189 in., and a stiffener ring spacing of 0.15×90, or 13.5 in., giving a total of $30 \times 12/13.5$, or 26.6 spaces, necessitating 25 stiffener rings. In addition, the maximum out-of-roundness ratio e/t, from Fig. 18-4, is 0.76 and the permissible variation in diameter will then be 0.76×0.189, or 0.144 in., which is somewhat small for a diameter of 90 in. It may therefore be advisable to investigate several alternate design possibilities as follows:

Construction Possibility	t/D	t	L/D	L	No. of Rings	e/t	e
A	0.0021	0.189	0.15	13.5	25	0.76	0.144
B	0.0025	0.225	0.25	22.5	15	0.75	0.169
C	0.0030	0.27	0.40	36.0	9	0.75	0.202
D	0.0035	0.315	0.56	50.4	6	0.72	0.227
E	0.0040	0.36	0.80	72.0	4	0.74	0.266
F	0.0050	0.45	1.4	126.0	2	0.77	0.346
G	0.0060	0.54	2.2	192.0	1	0.79	0.426
Ex. 18-1	0.0076	0.684	4.0	360.0	0	0.85	0.585

The tabulation shows that construction *B*, as contrasted with *D*, will permit a reduction in plate thickness of (0.315 — 0.225)/0.225, or 40%, at the expense of 15 — 6, or 9 additional stiffening rings. Construction *D*, however, will permit an additional tolerance of $\frac{1}{16}$ in. on the out-of-roundness, which may have considerable effect upon the fabrication cost. The probabilities of most economical construction are obviously types *D* and *E*; further differentiation between these will depend largely upon fabricating and material costs, which cannot be adequately treated in these pages.

If a design based upon the construction possibility *D* is used, the plate thickness will be $\frac{11}{32}$ or 0.3438 in. The theoretical stiffener spacing is 50.4 in., which corresponds to about seven spaces, or an actual spacing of 360/7, or 51.4 in., say 51½ in. The required moment of inertia I_r of a stiffening ring for an outer diameter of 90 in., a length *L* of 51½ in., and a pressure *P* of 20 psi., is found from Eq. 18-10 to be

$$I_r = \frac{6 \times 90^3 \times 51.5 \times 20}{10^9} = 4.5 \text{ in.}^4$$

From Table 7-6, either a 4 × 3 × ½-in. or a 3½ × 3 × ¾-in. angle is acceptable. Since the former has the least weight, it will be selected.

If the stiffeners are placed inside the shell, the maximum length *M* of a drainage gap is found from Fig. 18-4. For t/D and L/D values of 0.0035 and 0.56, respectively, the M/D value is approximately 0.09, which will permit a value of *M* of 0.09 × 90, or 8.1 in., say 8 in.

18-5. Design Details. The design of longitudinal and circumferential joints, heads, and reinforced and unreinforced openings in the shell for cylindrical vessels subjected to external pressures should in general conform to the rules given for vessels subjected to internal pressures. Furthermore, shell thickness should be checked by Eq. 3-3. Riveted longitudinal joint efficiency should not be less than 50%. It should also be noted that for the usual head position—concave to the shell interior—the external pressure will act upon the convex surface of the head.

18-6. Tube Design. Ferrous and non-ferrous tubes and pipe subjected to external pressure are selected on the basis of the data of Fig. 18-5, using allowable working stresses from Tables 9-4 and 9-5. These data are applicable only to outer diameters between ½ and 6 in., and for wall thicknesses not less than 0.049 in. Additional wall thickness must be provided whenever corrosion or wear due to cleaning operations is anticipated, or when tubes are rolled or otherwise set into tube sheets. When tube ends are threaded, a wall thickness of 0.8 divided by the number of threads per inch must be added to the thickness obtained from Fig. 18-5.

Cast Iron Construction

18-7. Pressure vessels constructed wholly or partly of cast iron may be employed for containers constructed in accordance with the provisions of the ASME-UPV Code, within certain limitations of temperature and pressure. Such vessels must not be used as containers for lethal gases or liquids; the maximum pressure must not exceed 160 psi.; the maximum temperature must not exceed 450° F. for steam or other gases, or 375° F. for liquids. Vessels employed for liquid circulation or storage at temperatures less than 120° F.

may be used for pressures up to 200 psi. Cast iron pipe fittings conforming to the ASA standards can be used as a whole or a part of such pressure vessels, for temperatures and pressures in accordance with the Code and the ASA ratings.

Permissible material for pressure vessel use in this category is given in Specification S-13 of the ASME Materials Specification Code and Section II of the ASME Boiler Construction Code. This specification refers to gray iron

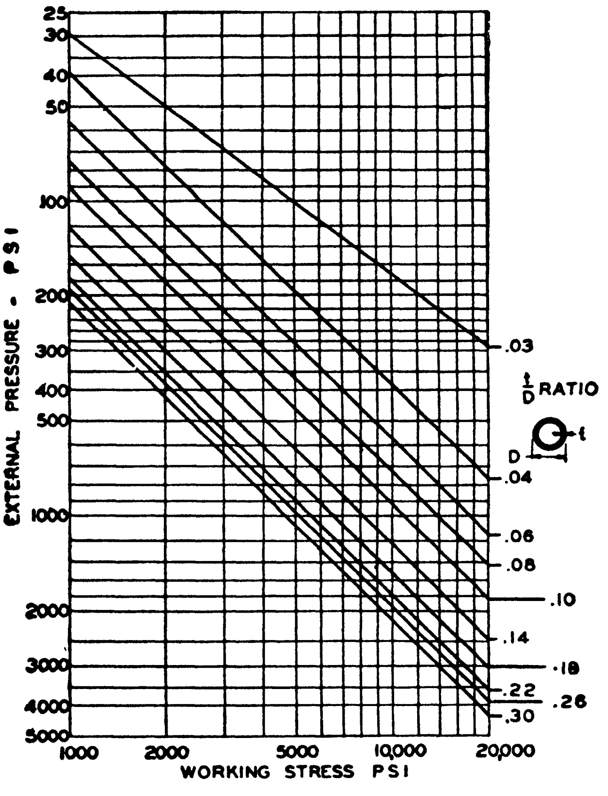


Fig. 18-5. Design of Tubes for External Pressure.

castings and is adapted from ASTM Spec. A48-36. It recognizes five classes of gray iron castings, designated numbers 20, 25, 30, 35, and 40. The allowable working stresses for those classes of cast iron are given by:

$$S = 100C \tag{18-11}$$

where S is the allowable tensile stress, psi., and C is the class number of the material. A class 30 iron, for example, has a permissible tensile design strength of 3000 psi. Allowable stresses in flexure are 50% greater than the tensile stresses given by the above; allowable compressive stresses are 100% greater.

STAYED HEADS AND STAYBOLT APPLICATIONS

18-8. Staybolts are used to brace or "stay" flat or slightly curved heads and plates. They are used to some extent in jacketed vessels, and are particularly important in vessels with flat heads that would require an uneconomical or excessive head thickness if left unstayed. Several representative staybolt constructions are shown in Figs. 18-6 and 18-9. The screwed and riveted staybolt of Fig. 18-6 is threaded at both ends and is screwed through both heads so that at least two threads project beyond each side of the plate; the outer ends are riveted over or upset without scoring or deforming the plate.

Staybolts must be drilled through the center with a "tell-tale" hole at least $\frac{3}{16}$ in. in diameter, as shown in Fig. 18-6. A tell-tale hole is required by the

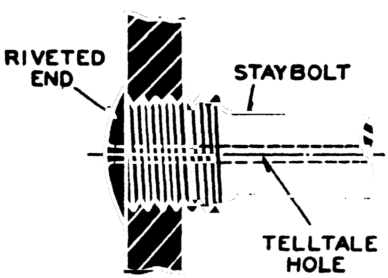


FIG. 18-6. Screwed and Riveted Staybolt with Telltale Hole.

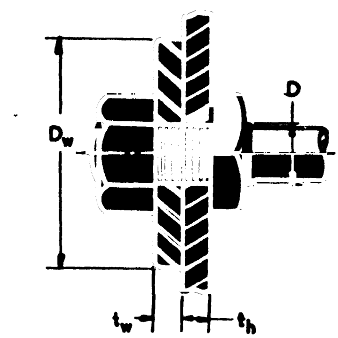


FIG. 18-7. Through-stay with Inside Nut and Outside Nut and Washer.

codes as a safety feature, so that if a crack or flaw develops in the bolt, the leak can be detected before damage is done. The hole need not extend entirely through the bolt, but must reach ½ in. inside the plate. The through-stay of Fig. 18-7 has upset threaded ends with a heavy standard nut and a special washer to resist the internal pressure. For through-stays with nuts on both sides, the diameter D_w of the washer must not be less than 0.4 times the pitch of the stays; the washer thickness t_w must be at least equal to the head thickness t_h. For through-stays with formed heads, or with nut and washer, where the stays are screwed into the heads, D_w must be at least equal to 2.5 D, and t_w must be at least equal to 0.5 t_h.

Acceptable forms of welded stays are shown in Fig. 18-8; their application is permissible if the stay diameter does not exceed ⅞ in., or if the plate thickness does not exceed ¾ in.

Flexible staybolts, shown in Fig. 18-9, and at D, Fig. 18-8, are preferred to ordinary or rigid members because the latter are often broken at or near stiff corners or edges of the stayed surfaces. Anchor blocks or sleeves for

flexible staybolts may be screwed into the sheets, but welded attachment is usually recommended.

Tubes which are beaded or flared into tube sheets have a very considerable holding power, and are recognized as stay-tubes if they project at least $\frac{1}{4}$ in. beyond the surface of the plate and are slightly flared, or if they are screwed into

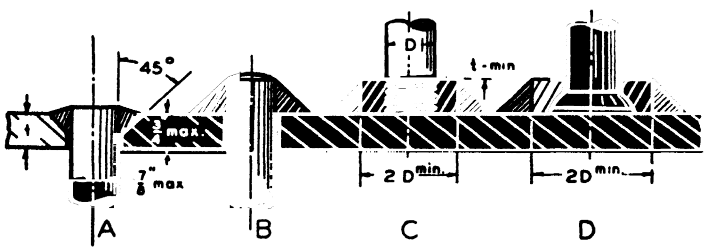


FIG. 18-8. Welded Staybolts and Proportions.

place. The tube ends may be upset to give sufficient metal under the thread without requiring an unnecessary increase in the tube wall thickness. Nuts on stay-tubes are not recommended.

18-9. Staybolt Design. The thickness t of a stayed flat head is given by

$$t = \frac{P}{C}\sqrt{\frac{p}{S}} \tag{18-12}$$

where p is the operating pressure and S the allowable stress in the head, psi., P the maximum pitch or center-to-center distance of the staybolts, and C is a constant for various types of staybolts, values of which are obtained from Table 18-1.

Flat heads not less than $\frac{3}{8}$ in. thick may be strengthened with a riveted doubling plate covering the full area of the stayed surface. The thickness of this reinforcing plate must not be less than two thirds of the head thickness. For such conditions, the thickness t in Eq. 18-12 can have a minimum value of three fourths the combined thickness of the head and doubling plate; in no case, however, may the combined thickness exceed one and one half times the head thickness. If two heads are connected by stays and only one of these requires staying, the value of the constant C is governed by the thickness of the head requiring staying.

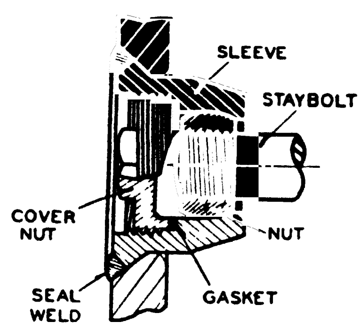


FIG. 18-9. Flexible Staybolt.

The maximum allowable pitch P of screwed staybolts with riveted ends is given in Table 18-2. For conditions beyond the range of this table, the design

Table 18-1

Type of Stay	Shown in Fig.	Plate Thickness Limitation	Constant C
Screwed through plate	18-6	7/16" max.	1.61
Screwed through plate	18-6	over 7/16"	1.67
Welded staybolts	18-8	3/4" max.	1.67
Screwed through plates, with single nuts outside of plates		1.3 D min.	1.77
Screwed through plates with formed heads		1.3 D min.	1.87
With inside nut and outside nut with washer	18-7	1.3 D min.	2.02

Table 18-2.—Maximum Allowable Pitch on Screwed Staybolts, Ends Riveted Over

Pressure, lb. per sq. in.	Thickness of Plate, in.						
	5/16	3/8	7/16	1/2	9/16	5/8	11/16
	Maximum Pitch of Staybolts, in.						
100	5¼	6⅜	7⅜	...	...	...	...
110	5	6	7	8⅜	. .	...	...
120	4¾	5¾	6¾	8	...	...	...
125	4¾	5⅝	6⅝	7¾	...	...	...
130	4⅝	5½	6½	7⅝	...	...	...
140	4½	5⅜	6¼	7⅜	8⅛	...	...
150	4¼	5⅛	6	7⅛	8	...	...
160	4⅛	5	5⅞	6⅞	7¾	...	...
170	4	4⅞	5⅝	6¾	7½	8⅜	...
180	.	4¾	5½	6½	7⅜	8⅛	...
190	...	4⅝	5⅜	6⅜	7⅛	7⅞	...
200	...	4½	5¼	6⅛	7	7¾	8½
225	...	4¼	4⅞	5⅞	6½	7¼	8
250	...	4	4⅝	5½	6¼	6⅞	7⅝
300	...	...	4¼	5	5⅝	6¼	7

may be based upon Eq. 18-12, if the pitch does not exceed 8½ in. The distance from the edge of a staybolt hole to the edge of an adjacent rivet hole is substituted for P in Eq. 18-12 for staybolts adjacent to the riveted holes bounding a stayed surface. For flanged heads with riveted or welded seams, the distance from the edge of the staybolt hole to the beginning of the flange curvature should not exceed the maximum permissible pitch P. Maximum permissible staybolt stresses are given in Table 18-3. Welded-in staybolts, shown in Fig. 18-8, can be used for pressures not exceeding 150 psi., for plate thicknesses not exceeding ¾ in., and for bolt diameters not exceeding ⅞ in. The maximum allowable stress in the weld throat should not exceed 6000 psi.

TABLE 18-3.—MAXIMUM ALLOWABLE STRESSES FOR STAYBOLTS AND STAYS OR BRACES

Description of Staybolts and Stays or Braces	Stresses, lb. per sq. in.	
	For Lengths Between Supports Not Exceeding 120 Diameters	For Lengths Between Supports Exceeding 120 Diameters
Unwelded or flexible staybolts less than 20 diameters long, screwed through plates with ends riveted over	7500	—
Hollow steel staybolts less than 20 diameters long, screwed through plates with ends riveted over	8000	—
Unwelded stays or braces and unwelded portions of welded stays or braces	9500	8500
Steel through stays or braces exceeding 1½ in. diameter	10,400	9000
Welded portions of stays or braces	6000	6000

Example 18-3. The lower jacket of a flat-bottomed cylindrical mixing kettle has a diameter of approximately 3 ft. and a height of 4 in., and is subjected to a steam pressure of 50 psi. Using steel with an allowable unit stress of 10,000 psi., and a welded joint efficiency of 80%, determine the shell thickness and design flat heads for the jacket.

Solution. The vessel shell thickness, from Eq. 4-3, is

$$t = \frac{50 \times 18}{10{,}000 \times 0.80 \times 2} = 0.0562 \text{ in. corroded.}$$

By Eq. 4-4, however, this thickness cannot be less than

$$t = \frac{100 + 36}{1000} = 0.136 \text{ in.}$$

A $\frac{3}{16}$-in. shell would probably be selected.

The thickness of an unstayed flat head, from Eq. 4-11, is

$$t = 36\sqrt{\frac{0.50 \times 50}{10{,}000}} = 1.80 \text{ in.}$$

This head thickness is obviously out of proportion to the shell thickness, and a stayed head is indicated. If six staybolts equally spaced on an 18-in. diameter circle are used, with one staybolt at the center, the staybolt pitch is equal to 9 in. From Table 18-1, the factor C for welded staybolts is 1.67. From Eq. 18-12, the head thickness is

$$t = \frac{9}{1.67}\sqrt{\frac{50}{10{,}000}} = 0.381 \text{ in.}$$

and a 13/32-in. plate would be required for both the vessel and jacket bottom.

The total pressure on the central staybolt is considered equivalent to the area of a 9-in. diameter circle, or 63.62 sq. in. The total area supported by the central staybolt is considered equivalent to the area of a 9-in. diameter circle, or 63.6 sq. in. The total area supported by the six staybolts on the 18-in. circle is considered equivalent to the difference between the areas of 27-in. diameter and 9-in. diameter circles, or 572.6 — 63.6, or 509 sq. in., giving an area of 509/6, or 51 sq. in. per bolt. The maximum bolt load is therefor at the center, and is 63.6 × 50, or 3180 lbs. Staybolts similar to those of Fig. 18-8 can be used; for an allowable unit shearing stress of 6000 psi., a shear area across the throat of 3180/6000, or 0.54 sq. in., is required. For a plate thickness of 13/32 in., and an estimated weld leg of 3/8 in., the throat dimension will be 0.375 × 0.7, or 0.263 in. The minimum mean diameter of the weld is 0.54/0.263 π, or 0.653 in. From Table 18-3, the allowable unit stress for staybolts less than 20 diameters long is 7500 psi.; the required area will be 3180/7500, or 0.424 sq. in., resulting in a diameter of 3/4 in. Since the bolt diameter is less than 7/8 in., and greater than the minimum mean diameter of the weld, this staybolt design is satisfactory.

Stuffing Boxes and Packings

18-10. A stuffing box is a device for preventing leakage or transfer of fluid between moving parts, and usually consists of a relatively soft packing which is compressed or confined by an adjustable member called a gland. Stuffing box packings differ from gaskets principally in that they are used in confined spaces, and do not of themselves withstand stresses due to fluid pressure.

The general arrangement and proportions of a stuffing box for moderate pressures and temperatures are shown in Fig. 18-10A. The gland holds the packing in the box, serves to apply the necessary confining pressure, and allows for wear of the packing. The force on the gland studs consists of the pressure against the ring area of the gland and the frictional force exerted by the moving rod against the packing. For a stationary or rotating rod, the total force F is

$$F = \pi P(D^2 - d^2)/4 \qquad (18\text{-}13)$$

where D is the diameter of the skirt or cylindrical portion of the gland, d is the diameter of the rod, as shown in Fig. 18-12A, and P is the internal pressure, psi.

If the rod is subjected to axial motion, the axial friction is assumed to be equal to 30% of the stud forces, and the total load F_a is given by

$$F_a = 1.3\,F \qquad (18\text{-}14)$$

Gland bolts and studs are often subjected to initial or tightening stresses, and to load fluctuations caused by variations in pressure and uneveness of packing. For these reasons, design stresses of 10,000 psi. or less are used for soft steel bolts. In high pressure work, however, larger stresses at the roots of the threads must be used to keep the bolt and gland proportions within reasonable

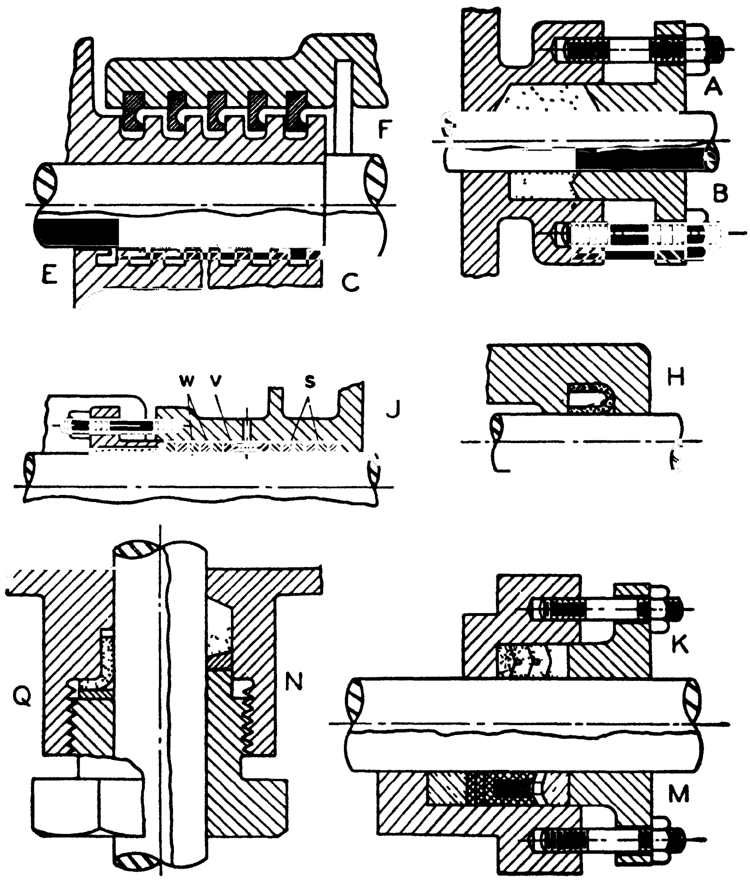


FIG. 18-10. Stuffing Boxes.

size. Since high pressure operations are usually accompanied by slowly moving parts, the stresses induced by friction are comparatively small. The flange thickness t of the gland is usually made 75% greater than the bolt or stud diameter. The thickness may be computed on the basis that the flange is a cantilever built in at the skirt, with a concentrated load equal to the force on the bolt. The thickness of the skirt wall varies from $0.25\sqrt{d}$ to $0.5\sqrt{d}$, the former value

being used for hydraulic stuffing boxes. Threaded packing glands, Fig. 18-10 N and Q, are very frequently used on tubular equipment.

18-11. Packings. Packings often consist of relatively plastic material, bonding such substances as cotton fabric or rope, asbestos, shredded metal, and the like. Packings of flexible materials such as rubber and leather, or more rigid substances such as pressed graphite, soft metals, or molded plastics, are also used. The chemical properties of the material usually govern its selection and use, but such physical properties as the melting or softening point, and the comparative rigidity, must be considered, particularly for elevated temperature or high pressure service.

Packings are usually preformed to size and shape so that they fit snugly in the space in the stuffing box, but strip, coil, or rope form packings are also available for use. Packings of ring form with rectangular cross sections are used for cylindrical rods and shafts for moderate pressures and temperatures; for pressures above 2000 psi. it is customary to use molded rings, as shown in Fig. 18-10K, which interlock and expand against the walls due to the pressure of the fluid. U-shaped or spring-type packings, as shown in Figs. 18-10H and 18-10M, are used for the same purpose. For low pressures and smaller parts, U-shaped packings may be replaced by cup or flanged packings ("hat leathers"), as shown in Fig. 18-10Q.

A stuffing box with a multiple group of packings is shown in Fig. 18-10J. Any leakage of fluid past the first set of packing rings *S* must pass through an open space caused by a lantern ring *V* before coming in contact with the second set of packing rings *W*. Leakage into the lantern ring *V* is withdrawn by connecting the lantern space with some lower pressure portion of the system as, for example, the suction side of a compressor which is used for maintaining pressure in the body of the equipment. This arrangement can be used in an inverse sense to prevent air from being drawn into the suction of a centrifugal pump, by connecting the pump discharge line to the lantern ring space. The lantern rings can be used both as a seal against infiltration and as a device for minimizing leakage.

18-12. Labyrinths. In high-speed machines, such as gas turbines and rotary compressors where no adequate cooling is available, and in high pressure equipment where small clearances are required, but for which packing materials are inadequate because of very low or very high temperatures, devices known as labyrinths are used instead of packings.

Several forms of labyrinths are shown at C, E, and F, Fig. 18-10. The principle of operation of a labyrinth requires some flow to enable the device to function as a seal. The fluid first passes a restriction and then expands into a chamber which consumes a certain amount of energy. After a series of expansions and repeated changes of direction of flow, a considerable pressure drop will exist between the initial and terminal points of the labyrinth. The magnitude of the pressure drop is dependent upon the clearance and the shape

of the expansion chambers, but there must always be some leakage at the terminal point in order to render the labyrinth operable. The effective operation is less marked for liquids than for vapors. Several labyrinths in series, separated by a lantern ring or its equivalent, are commonly used in steam turbines and similar machines. Fig. 18-10F shows a turbine labyrinth, in which dummy packing rings are used to form the expansion chambers.

HIGH PRESSURES

18-13. Vessels intended for operation at internal pressures up to approximately 2000 psi. are usually designed on the basis of the "thin cylinder" equations given in Chaps. 3 and 4. When pressures are encountered which require shell thicknesses in excess of 10% of the inside radius (ASME-UPV Code), or 10% of the inside diameter (API-ASME Code), the thin cylinder formulae are not applicable since they result in unsafe design. Cylinders and tubes for use at high pressures are seamless, unless of welded construction, and are fabricated by forging or drawing; riveted construction is never used. High pressure design may be considered in two classifications: ordinary temperature equipment of a few hundred degrees, and high temperature construction where metal creep and chemical penetration determine the allowable working stresses of the metals.

High pressure design for ordinary temperature work falls into two subdivisions: for pressures below 3000 atmospheres, and for pressures above 3000 atmospheres. The basic theory for design below 3000 atmospheres depends upon the validity of Hooke's Law, and thus will apply to stresses below the elastic limit of the material. Very high pressures will produce stresses in excess of the elastic limit and special treatment is required to determine the necessary wall thickness to withstand pressures much in excess of 3000 atmospheres.[44,45,53]

18-14. Pressures Below 3000 Atmospheres. The strength of steels available for work at this pressure makes it possible to design cylinders by forging and so-called monobloc or single-piece construction, but other methods of construction useful for very high pressure applications are often advantageously employed at these lower pressures. Although autofrettage and multi-layer construction are especially desirable at elevated temperatures, and will be discussed later, it is sometimes advisable to use such methods of construction even where their application is not essential.

Lamé first proposed a theory for the equilibrium existing between the internal pressure in a cylinder and the stresses induced in the walls of the cylinder, which is still the basis for all rigorous high pressure design. The basic assumptions on which the Lamé equations were developed should be clearly understood so that their application and limitations may be recognized. Lamé's principal assumption were: (*a*) A perfectly elastic material. (*b*) A monobloc cylinder of isotropic material. (*c*) The material free from stress prior to the application of any external forces. (*d*) Right sections through the cylinder walls

remain undistorted and plane under stress. (*e*) The cylinder closed by heads and arranged so that the cylinder walls are subjected to a uniform tension parallel to the axis, of the same intensity at all points on the area of any annular right section. (*f*) The internal pressure greater than the external pressure.

Assumption (*a*) limits the relation to those materials whose stress-strain characteristics obey Hooke's Law. The Lamé equations are not applicable when the elastic limit is exceeded and must not be used to calculate wall thickness to prevent bursting. Their primary application is in computing wall thickness necessary to maintain the metal within its elastic range. The assumption (*b*) of a monobloc cylinder of homogeneous composition and elastically uniform throughput can probably never be realized. That the metal is free from all stress prior to the pressure load exerted on the inside of the cylinder (*c*) is likewise unattainable because of the unavoidable strains induced by the method of fabrication. The assumption of plane strain (*d*) is valid only for a cylinder of infinite length; for long cylinders this condition is approximated except near the enclosing ends.

A uniform distribution of longitudinal tension transferred by the heads (*e*) is not possible but can be approximated by proper design. Since the longitudinal stress is never the controlling stress, the deviation from this ideal is relatively unimportant. However, since longitudinal stress does exist, it is not possible to have all right sections remain plane and undistorted. Therefore, Lamé's equations are expected to deviate from actuality for short cylinders, and to approach actuality when end effects diminish as cylinder lengths increase. End effect apparently becomes inconsequential when the ratio of length to diameter exceeds 8.

From an analysis similar to that used in Chap. 3 and Fig. 3-10, except that the cylinder wall is thick, Lamé derived the following:

For radial or normal compression stress S_r at a distance r from the axis of the cylinder

$$S_r = a - \frac{b}{r}$$

For a circumferential tension S_c at a distance from the axis of the cylinder

$$S_c = a + \frac{b}{r}$$

a and b are constants, and can be shown to have the following values when p_1 is greater than p_2.

$$a = \frac{p_1 r_1^2 - p_2 r_2^2}{r_2^2 - r_1^2}$$

and

$$b = \frac{(p_1 - p_2) r_1^2 r_2^2}{r_2^2 - r_1^2}$$

where r_1 and r_2 are the inner and outer cylinder radii of the cylinder and p_1 and p_2 are the internal and external pressures.

From these expressions it is evident that the maximum stress occurs in circumferential tension and thus the expression for S_c is of major importance. If an allowable working stress S is assumed, so that S is equal to S_c, then

$$\frac{r_2^2}{r_1^2} = 1 + \frac{2(p_1 - p_2)}{p_1 + S} \quad (18\text{-}15)$$

In most cases the external pressure p_2 is atmospheric and negligible compared to p_1. When p_2 is neglected, the expression simplifies to

$$\frac{r_2^2}{r_1^2} = \frac{S + p}{S - p} \quad (18\text{-}16)$$

which is often referred to as Lamé's Formula. S is the maximum stress at the inner surface or bore of a tube. Rearranging and solving for p,

$$p = S\frac{r_2^2 - r_1^2}{r_2^2 + r_1^2}$$

If the ratio of the outer and inner radii r_2 and r_1 is set equal to a factor K, then Eq. 18-16 becomes

$$p = S\frac{K^2 - 1}{K^2 + 1} \quad (18\text{-}17)$$

Further, let

$$f = \frac{K^2 - 1}{K^2 + 1} \quad (18\text{-}18)$$

then

$$p = fS \quad (18\text{-}19)$$

which is a very convenient form of the Lamé equaton. This development is consistent with Eq. 3-4 given in section 3-9.

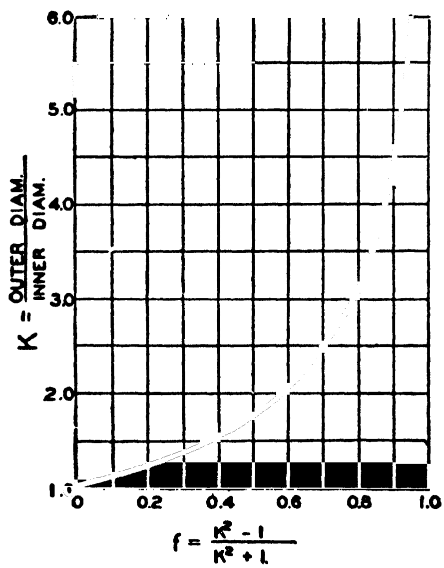


Fig. 18-11. Design Curve for Use with the Lamé Equation (18-18).

Since the outer and inner radii r_2 and r_1 determine the factor f, the relationship between the internal pressure and working stress (for any given ratio of radii) can be found by the use of Fig. 18-11. Allowable working stresses are usually taken as about one third of the elastic limit.

From Fig. 18-11, it is evident that the shell thickness increases as K increases. At high values of K the factor f increases but cannot be greater than unity since it approaches unity asymptotically. The Lamé analysis thus limits the permissible pressure for the allowable working stress selected (within the elastic

limit). The nature of the relation of K and f, however, is such that as the cylinder thickness increases there is relatively little strength added to the walls. For example, for a steel tube with a bore of 2 in. and a wall thickness of 1 in., K would be equal to 2, and f to 0.6; a design stress of 30,000 psi. would allow a maximum working pressure of 18,000 psi. If the wall thickness is increased from 1 to 3 in., then K is 4, f is 0.88 and the working pressure becomes 26,400; a 47% increase in strength for a 300% increase in metal wall thickness. In fact even a wall thickness approaching infinity as a limit would permit the allowable stress to be exceeded when the internal pressure is numerically greater.

While it is not possible to calculate the relation of ultimate strength to internal pressure, many modifications of the Lamé formula have been proposed for this purpose, using constants resulting from experimental work. One of the simplest of these is the use of the ultimate strength in place of the working stress S. Such an approximation has been found to hold reasonably well for the mild steel cylinders of about 2 in. outside diameter. Such relations must however be justified by experiments on bursting strengths of cylinders of similar material, shape, and proportions.

Example 18-4. A solid drawn copper tube ½-in. O.D. and ¼-in. bore is subjected to an internal gas pressure of 3000 psi. What is the hoop stress in the tube wall?

Solution. The ratio K is 0.5/0.25, or 2. From Fig. 18-11, f is equal to 0.6. Substituting in Eq. 18-19, S is equal to 3000/0.6, or 5000 psi. This stress is well below the yield point of such tubing.

Example 18-6. A nickel manganese steel whose yield point is 96,000 psi. and whose ultimate tensile strength is 116,000 psi., is used for fabricating a drawn cylinder 9 in. I.D. and 4 ft. long, which is to contain a fluid at an internal pressure of 750 atmospheres. What wall thickness should be used so that the hoop stress in the walls will not exceed 60% of the yield point of the steel.

Solution. From Eq. 18-16,

$$\frac{r_2^2}{4.5^2} = \frac{0.6[96{,}000 + 750(14.7)]}{0.6[96{,}000 - 750(14.7)]}$$

$$r_2^2 = 20.3\left(\frac{68{,}650}{46{,}550}\right) = 30.0$$

$$r_2 = 5.46 \text{ in., or } 5\tfrac{1}{2} \text{ in.}$$

Thus the wall thickness should be 1 in.

While pressures up to 3000 atmospheres can be handled in equipment of forged, welded, and so-called monobloc construction, it is often more economical and safer to employ pre-stressed construction. This latter is essential for very high pressure work, and as design data are developed it will become more useful for the control of the moderately high pressures just discussed.

18-15. Pressures Above 3000 Atmospheres. For pressures in the neighborhood of 3000 atmospheres it is evident that for a very thick wall the factor f (Eq. 18-19) approaches unity and p approaches S. In this way pressures of

3000 atmospheres (or 44,100 psi.) and above will induce stresses approximately equal to the yield point of many alloy steels, and the Lamé formulae cannot be used because the material will not behave as a true elastic body above such stresses. Experiments show, however, that cylinders can be made to withstand pressures of as high as 20,000 atmospheres, resulting in an apparent stress of 294,000 psi., which is considerably higher than even the ultimate strengths of the steels used. Obviously the Lamé relations do not hold and should not be used directly as the basis for computations of strength in this range of stresses.

It has been found that in very thick cylinders, subject to very high pressures, the material at the inner surface is stressed beyond its elastic limit and plastic flow results. Strain-hardening results from such flow (as described in Chap. 2) and the inside layer of metal thus develops a much higher elastic limit if the pressure is subsequently relieved. By successive application and release of pressure it is possible to develop work-hardened inner layers of considerable thickness. These inner layers must then themselves be overstrained (past their newly developed elastic limit) in order to subject the outer layers of the wall to stresses which would exceed their elastic limit. Very high pressures can therefore be contained by properly worked thick walls as long as there is sufficient thickness in the outer layers so that they will remain in the elastic state. Failure of such cylinders first occurs by rupture of the outside layers, and begins when the elastic layers have become too thin to prevent plastic flow of the more highly stressed inner layers. The process whereby a cylinder is exposed to internal pressure until plastic flow occurs, a permanent set achieved, and strain-hardening results is known as autofrettage. (The term autofrettage means "self-hooping" in French.) The amount of permanent deformation required to produce safe working strengths depends upon the working pressure of the cylinder and experimental determination. It should be noted that when test bars are cut from such cylinders for strength determinations, they must be properly selected and cut so as to be tested for hoop strength or longitudinal strength, since the stresses are developed in different manner in the two planes; hoop stresses are ordinarily the greater of the two.

Some form of pre-stressing is used for all high pressure vessels of appreciable size. Only small experimental vessels and those at relatively moderate high pressures (several thousand psi.) are now made of monobloc construction. There are a number of ways in which pre-stressing of inner layers can be accomplished. Autofrettage is widely used although its control is empirical at the present time. Another method of pre-stressing is to employ a compound or multi-layer cylinder, in which a tube with the desired inner diameter has a concentric outer tube or hoop shrunk onto it, resulting in an initial compression of the inner tube. This composite cylinder will withstand considerable internal pressure even before the compressive stresses are overcome and the tensile stresses are developed in the inner tube, and can thus be used for very high pressures. Compound or multi-layer construction is very useful and is a direct outgrowth of the hoop shrinking process used in gun manufacture. Stresses for

this type of construction may be obtained from extension of the Lamé formulas and close design control is possible.

The preceding methods produce pre-strain by employing reinforcing or outer layer material to carry longitudinal stress. A third pre-strain method consists of winding wire or ribbon under tension around a tube or cylinder. Since wire winding will only increase hoop strength, sufficient longitudinal strength must be built into the solid inner wall to handle the longitudinal stress developed by the final load. Wire winding offers many possibilities for making equipment to withstand very high pressures. Wire of 3000 psi. is readily available and can be wrapped on a cylinder in successive layers to withstand almost any radial pressure; the limiting factor in such construction is the longitudinal load. Proper construction consists in winding one layer of wire upon another and varying the tension from layer to layer in such a way that when the inner tube is subjected to pressure each layer of wire will be under the same tension. The design of wire wound tubes requires determination of the inner tube thickness, thickness of wire layers, and the tension to be applied to each wire layer.

Comstock [24] has developed an expression for hoop and radial stresses in wire wound and multi-layer construction, based upon the Lamé formulae. The allowable tensile stress S_t at the inner wall of the tube is

$$S_t = \frac{p(r_2^2 + r_1^2) - 2Tr_2(r_3 - r_2)}{r_2^2 - r_1^2} \qquad (18\text{-}20)$$

where r_1 and r_2 are the internal and external radii of the tube, and r_2 and r_3 the internal and external radii of a surrounding layer or coil; p is the internal pressure in the tube, psi.; and T is the uniform tension in the surrounding layer or coil of wire under full load. (It is assumed that all stresses are below the elastic limit, and that the modulus of elasticity of the tube and layer materials is the same.) From the above, it may be seen that for a given pressure p, and a stress S_t within the elastic range, T will be a function of the three radii. If S_c is the allowable compressive stress at the inner wall of the tube, then

$$-S_c = S_t - p\left(\frac{r_3^2 + r_1^2}{r_3^2 - r_1^2}\right) \qquad (18\text{-}21)$$

From these expressions, the ratio between the outer radius r_3 of the layer and the inner radius r_1 of the tube is given by

$$\frac{r_3}{r_1} = \sqrt{\frac{S_t + S_c + p}{S_t + S_c - p}} \qquad (18\text{-}22)$$

For a given or selected value of T and r_1, radii r_3 and r_2 can be obtained from Eqs. 18-20 and 18-21. The necessary tension T_r required in winding any coil or layer of wire is

$$T_r = T\left[\frac{r_3(r_2^2 + r_1^2) - 2rr_1^2}{r(r^2 - r_1^2)} - \frac{2pr_1^2}{(r^2 - r_1^2)}\right] \qquad (18\text{-}23)$$

where r is any radius between r_3 and r_2, and is the inner radius of the wire layer. When there is no internal pressure the tension in each layer or coil will be

$$T_o = T - \frac{pr_1^2}{r_3^2 - r_1^2}\left(1 + \frac{r_3^2}{r^2}\right) \tag{18-24}$$

Longitudinal stresses in the tube are computed as described in Chap. 3.

Example 18-5. A long cylindrical wire-wound vessel with an inner tube of 2 in. I.D. is to withstand 15,000 psi. working pressure. The allowable tensile and compressive stresses in the tube are not to exceed 10,000 and 20,000 psi., respectively. Wire will be wound so that successive layers are 0.05 in. greater in radius, and the wire has an allowable tensile stress of 40,000 psi. Design such a tube.

Solution. From Eq. 18-22

$$\frac{r_3}{r_1} = \frac{r_3}{1} = \sqrt{\frac{10{,}000 + 20{,}000 + 15{,}000}{10{,}000 + 20{,}000 - 15{,}000}} = 1.732 \text{ in.}$$

Substituting in Eq. 18-20

$$10{,}000 = \frac{15{,}000(r_2^2 + 1) - 2(40{,}000)(1.732 - r_2)}{r_2^2 - 1}$$

whence r_2 is equal to $\sqrt{2.030}$, or 1.422 in. Thus the tube should have an O.D. of 2.844 in. or a wall thickness of 0.422 in. The first layer of wire will have a radius r_a of 1.422 in. The tension of application of the first coil must be:

From Eq. 18-23

$$T_{r_a} = 40{,}000\left[\frac{1.732(2.03 + 1)}{1.422(2.03 - 1)} - \frac{2(1.422)1}{(2.03 - 1)}\right] = 36{,}450$$

Repeating for coils $r_b, r_c \ldots r_g$, and calculating T_o from Eq. 18-24, the results are

Radius	Radius	T_r Application Tension	T_o Tension at Zero Internal Pressure
r_a	1.422	36,450	21,370
r_b	1.450	34,690	21,800
r_c	1.500	32,080	22,500
r_d	1.550	30,000	23,130
r_e	1.600	28,300	23,700
r_f	1.650	26,800	24,250
r_g	1.732	25,000	25,000

Each layer should be wrapped under the corresponding tension T_r shown above. When full pressure is applied the tension in each wire coil will be 40,000 psi.

18-16. Temperature Stresses in Tubes and Cylinders. In thin-walled vessels used for heat transfer, the temperature gradient through the wall is

usually small and causes very little difference in the stresses at the inner and outer wall surfaces. Thick-walled vessels, however, may be subjected to high temperature differences between inner and outer wall surfaces, and heat stresses may be set up in both axial and circumferential directions. The cold wall surface will develop tensile stress in relation to the warm wall, and maximum tensile and compressive stresses will occur at the respective surfaces. These temperature stresses will be imposed upon whatever pressure stresses are present, and the resultants will be in effect during any heat transfer process.

Lorenz has developed a relation for tubes transferring heat, based upon the assumptions of operation within the elastic range of the material and uniform strength throughout the walls. Since circumferential stresses control, the hoop stress S_1 at the inner surface, is given by

$$S_1 = \frac{GE(t_1 - t_2)}{2}\left(\frac{m}{m-1}\right)\left(\frac{2K^2}{K^2-1} - \frac{1}{\log K}\right) \qquad (18\text{-}25)$$

The hoop stress at the outer surface is given by

$$S_2 = \frac{GE(t_2 - t_1)}{2}\left(\frac{m}{m-1}\right)\left(\frac{2}{K^2-1} - \frac{1}{\log K}\right) \qquad (18\text{-}26)$$

In these expressions, S_1 and S_2 are tensile (cold and value positive) or compressive stresses, psi., depending upon which surface is colder; t_1 and t_2 are the temperatures, ° F., of the inner and outer surfaces; G is the linear coefficient of expansion at the temperature; E is the modulus of elasticity, psi.; m is the reciprocal of Poisson's Ratio; and K is the ratio of the outer to the inner diameter of the tube or vessel.

For situations where the Lamé formula applies, the pressure stresses can be computed by it and the Lorenz formulae (Eqs. 18-25 and 18-26) used to compute the temperature stresses. The resultant stress during heat transfer will be equal to the algebraic sum of the heat and pressure stresses.

18-17. High Pressures at Elevated Temperatures. Metals and other materials of construction suffer loss of strength at elevated temperatures, and the stress-strain curves change with changing temperature. The allowable working stress for high temperatures must therefore be determined by experiment, and no general relation now exists for relating working stresses at different temperatures. If a given working stress is desired and this stress is known by experiment to be less than the elastic limit at the desired temperature, then stress calculations and wall thickness can be made as described in preceding sections. There are many applications at present where steels are used at temperatures up to 700° F. for which design can be based upon the thick-cylinder formula of Lamé.

When the pressure is high enough to preclude the use of the Lamé assumptions, or when the temperature is such that the desired working stresses are above the elastic limit at that temperature, it is evident that the metal will be

stressed in its plastic region and plastic flow or creep, described in section 2-16, will result.

For design under conditions of creep, the working stress should be such that the expected life of the structure should lie within the second or constant rate stage of creep. A creep rate of 0.1% per 100,000 hours is considered low and is often used in design. The total strain or deformation resulting from creep during the working life must be allowed for in the design, so that fittings, attachments, and the vessel itself will not develop stress concentration and cause failure. In many applications of tubing and vessels where equipment is used for heat transfer, both temperature stresses and creep considerations affect the design. In these cases the Lorenz formulae will not apply, and the relations developed by Bailey can be used.[17] From Bailey's analysis the relationship between heat flow, working pressure, tube proportions, and stress can be expressed as:

$$K^{2/N} = \frac{S}{S - P/AN} \tag{18-27}$$

where K is the ratio of O.D. to I.D., N is a function of heat flow through the tube wall, S is the circumferential working stress at the inner wall at the allowable creep rate and temperature, psi., P is the internal pressure, psi., and A is a function of the material. The constants A and N for several steels are shown in Table 18-4, where H is the heat input, B.t.u.'s per foot of length per hour.

TABLE 18-4.—CONSTANTS FOR USE IN EQ. 18-27

Material	A	N	k
$KA2S$	0.555	$\dfrac{12}{1 + \dfrac{H}{2820}}$	150
4-6% Cr-Mo Steel	0.600	$\dfrac{6}{1 + \dfrac{H}{4320}}$	230
Steel	0.600	$\dfrac{6}{1 + \dfrac{H}{5900}}$	310

In tube design problems of this sort it is necessary to know inside and outside wall temperatures (t_1 and t_2 respectively, corresponding to I.D. and O.D.), or having such temperatures to determine the heat throughput per foot of tubing. From the familiar heat transfer relations, a convenient form can be derived for use with Eq. 18-27, for heat flowing toward the bore, based upon internal tube pressures of approximately 1000 psi.

$$H = \frac{0.524k(t_2 - t_1)}{\log K} \tag{18-28}$$

where k is the thermal conductivity of the metal in B.t.u. per sq. ft. per ° F. per hour per inch of thickness. For convenience a few values of k in these units are given in Table 18-4.

Example 18-6. Oil is to be heated in a tube bank of 4-6 Cr-Mo-Steel to a temperature of 1000° F. at a pressure of 1200 psi. and it is desired to maintain a heat flux of approximately 25,000 B.t.u. per hour per foot of tube. Flows through the tubes are such that the Δt across the oil film may be assumed to be 50° F. What tube wall thickness should be used and what temperature must be maintained on the outside surface of the tubes?

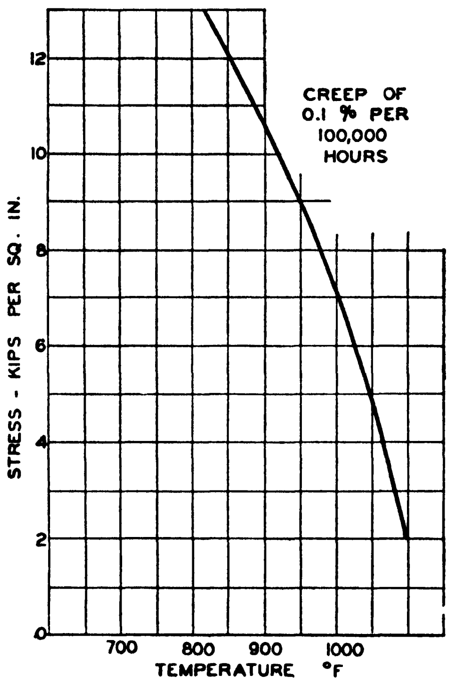


FIG. 18-12. Creep Data for 4-6% Cr. Mo. Steel.

Solution. From Table 18-4,

$$N = \frac{6}{1 + \frac{25,000}{\text{43[illegible]0}}}$$

$$N = 0.887$$

and $$A = 0.6$$

thus $$AN = 0.6 \times 0.887 = 0.532$$

and $$2/N = 2/0.887 = 2.25$$

From creep data at 1050° F. (Fig. 18-12) for this steel a working stress of 5000 psi. will result in a creep of 0.1% per 100,000 hours. This is a satisfactory low creep and 5000 psi. will be used as the allowable working stress S at the bore of the tube. Substituting in Eq. 18-27

$$K^{2.25} = \frac{5000}{5000 - \frac{1200}{0.532}} = 1.82$$

$$K = 1.31 \text{ or the ratio of O.D. to I.D.}$$

If the tube bore is 3 in., the O.D. will be 3.93 in. and the wall thickness would be taken as ½ in.

For substituting in Eq. 18-28, k from Table 18-4 is 230, and

$$t_2 - t_1 = \frac{25,000 \log 1.31}{0.524 \times 230} = 24.2$$

$$t_2 = 24.2 + 1050 = 1074° \text{ F.}$$

This temperature must be maintained on the outside wall of the tube.

In addition to wall thickness computed for high pressure vessels from any of the foregoing considerations, allowance must be made for corrosion, embrittlement by gases or other chemicals, and changes in alloy structure due to sustained high temperatures. These factors can be taken care of by increasing the metal thickness, lowering the working stresses, or reducing the useful life of the equipment.

Non-ferrous metals at high temperature exhibit creep characteristics, but even more dangerous is the development of intercrystalline cracks and crystal growth in many alloys. Copper and nickel alloys in general have unfavorable creep characteristics at temperatures above 400° F. The strength of most non-ferrous metals and alloys drops very rapidly above this temperature, but some of the Cu-Ni alloys can be used in higher temperature work.

18-18. High Pressure Pipe. Two rather distinct types of high pressure apparatus are of interest to the designer; i.e., equipment designed for small size experimental work and that intended for large scale industrial applications. Equipment of small size to withstand either relatively low pressures around 1000 psi., or pressures up to 50,000 psi. and more, are employed in experimental work. It is desirable to keep such equipment as compact as possible, hence there are few actual pipe lines used, but rather short connections to adjacent pieces. Very few industrial processes call for pressures over 15,000 psi., but at such pressures the densities of the fluids being handled are so great that reaction vessels and piping need only be of very small size. Thus the inside passages and reactor spaces permit different technique in fabrication of equipment for very high pressure experimental purposes than for the larger equipment used in the lower pressure (1000 psi. and less) commercial installations. Equipment for very high pressure and for experimental work often consists of vessels of only an inch or two in diameter, and fittings with 1/8- or 1/4-in. bore. Equipment for commercial high pressure reactions may be seen in the form of autoclaves and reactors of 3 to 4 ft. in inside diameter, with tubes and fittings of 1 in. or more in bore.

Aside from the necessity of special alloys to withstand particular corrosion problems, piping used in high pressure work is usually of medium carbon or alloy steel, or solid copper. These materials may be fabricated into pipe by drawing, while for very high pressure applications holes are drilled in bar and block stock. Copper is often convenient for pressures up to 7000 psi., but steels should be used for higher pressure work. For experimental and small equipment work several rule-of-thumb diameter ratios are recommended. With pressures up to 4000 psi., the ratio of outside diameter to inside diameter O.D./I.D. should be not less than 2 for copper or steels. For pressures from 4000 to 7000 psi., a ratio of 3 is satisfactory for steel; for pressures up to 20,000 psi., a ratio of 4 is usually used for Cr-Mo steel. For pressures above 10,000 psi., small holes should be drilled in solid pieces. The highest pressures obtained have been contained in cylinders 1½-in. outside diameter, with passages 1/8 in. in diameter. For any pressure work over 30,000 psi. some strengthening medium like autofrettage or multi-layer construction should be employed. At lower pressures accompanied by high temperatures all equipment should be subjected to autofrettage or other strengthening techniques.

18-19. Pipe Fittings. Connections for high pressure piping may be effected by screwed, flanged, and welded fittings.[47] Fig. 18-13 shows two forms

of screwed connections; the one at the left is similar to a screwed union for ordinary pressure service. The cylindrical nut *N* is screwed on the pipe *P* by

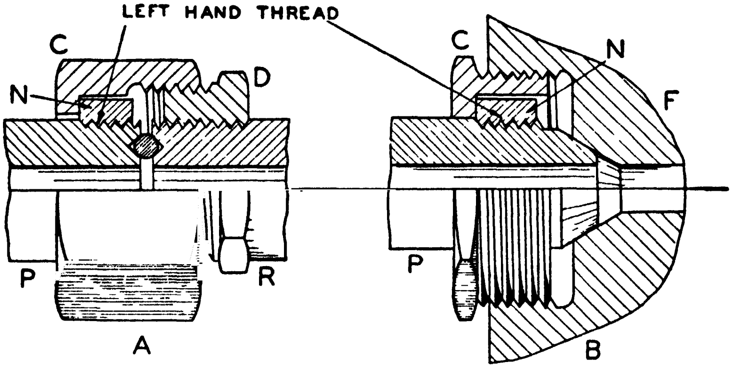


FIG. 18-13. High-pressure Screwed Fittings.

left-hand threads, and a pressure seal is effected by a ring gasket *G* fitting in vee-shaped grooves in the ends of the pipes. The fitting at the right has a pipe with a conical end, which is finished by lapping, so that a perfect seal is obtained. In some cases, the pipe end and the conical seat are lapped together; in others, each element is lapped separately. (A suitable lapping ring for finishing the end of the pipe is shown in Fig. 18-14.) The gasketed joint is superior to the conical joint, in that it permits ease of alignment and some degree of flexibility.

Fig. 18-15 shows a typical high pressure tee connection, made from a solid block, for conical pipe end connection similar to Fig. 18-13 (right). Elbows,

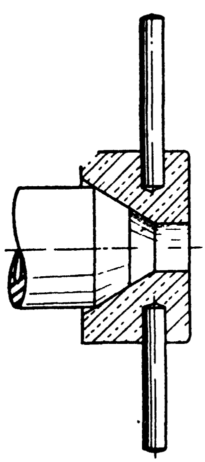


FIG. 18-14. Lapping Ring for Pipe Ends.

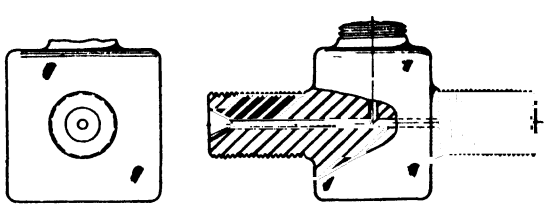


FIG. 18-15. High Pressure Screwed Tee.

couplings, crosses, and the bodies of high pressure needle valves, as shown in Fig. 18-16, are constructed in a similar manner. Since the connection threads are not tapered, a metal seal is necessary, which is usually obtained by heating the fittings and permitting solder to flow into the threads as the joint is made. Soft solder is usually used for copper fittings, hard solder for steel.

Ground-joint flanged unions similar to Fig. 18-17 are commercially available for cold working pressures up to 6000 psi., in pipe sizes from ½ to 2 in. Four and six bolt flanges, up to 2½-in. pipe size, may be obtained with tongue-and-groove faces for gasket application in the same pressure range.

When a flanged joint becomes too bulky for convenient use, the Vickers-Anderson clamp joint shown in Fig. 18-18 can be used. The pipes or tubes have integral shoulders with slightly conical faces; the face angle is less than the angle of friction. Three clamp ring sections, with two studs at each joint, are used to draw the pipe ends together. Any desired form of gasket or gasket seat can be incorporated in the joint.

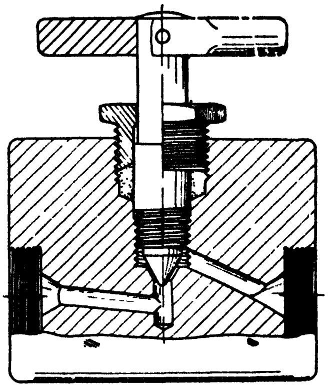


FIG. 18-16. High Pressure Needle Valve.

Valves for high pressure work embody essentially the same principles as those described in Chap. 9. Stainless steel is used as a construction material wherever possible, although bronze valves may be used for clean gases, such as hydrogen and nitrogen, for pressure service up to 4500 psi. The needle valve shown in Fig. 18-16 is a commercial product for pressures up to 15,000 psi. for use with ¼-in. O.D. high pressure tubing. The tubing connection is similar to that shown in Fig. 18-13 (right). Leakage past the valve stem is eliminated by using shredded metallic packing. Fig. 18-19 shows a commercial 6000-lb. drop forged steel angle check valve, which is available in pipe sizes from ¼ to 1½ in. Angle globe valves, similar to Fig. 9-14, are available for 900-lb. oil service; angle needle valves, similar in principle to Fig. 9-15, are available for 6000-lb. service.

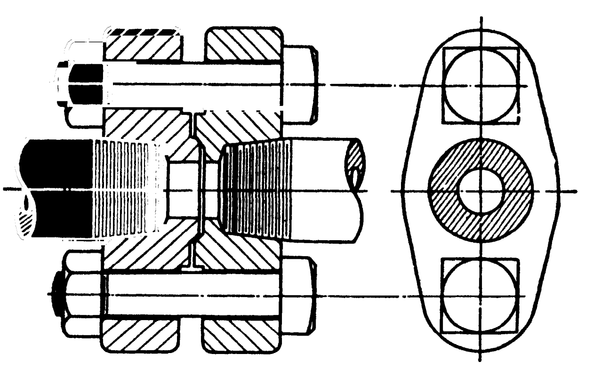


FIG. 18-17. High Pressure Flanged Joint.

18-20. Closures and Attachments. Closures for high pressure vessels are usually quite massive, especially in the case of autoclaves, which are heated pressure vessels, and are thus often subjected to severe pressure and corrosive conditions.[89] One form of closure or head construction for an autoclave (for pressures of around 3500 psi. at elevated temperatures) is shown in Fig. 18-21. The head is fastened by eight studs, and an appreciable bolt hole area necessitates extra head thickness. The actual joint between the end plate and the body is effected by a copper gasket, softened and annealed before being placed in its seat. Shallow vee grooves are turned in both seating faces. The studs must be pulled up evenly so that pressure is applied to the whole gasket surface uniformly. Studs are usually designed to withstand a tightening up stress of approximately 50% of the full pressure load, or in other words to handle 1.5 times the working stress due to autoclave pressure.

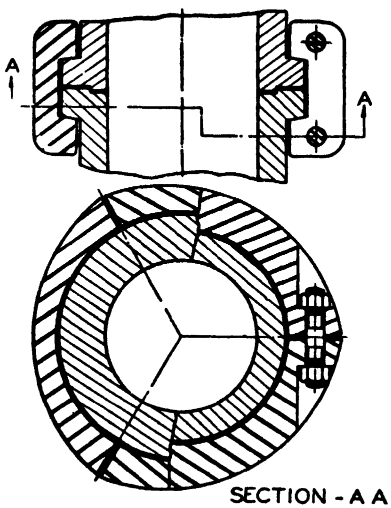


Fig. 18-18. High Pressure Clamp Joint.

Larger pressure vessels of 3 ft. or more in diameter are usually made of cylindrical shape with one integral hemispherical end, and the other end removable and flat; or with two removable flat ends. Fig. 18-22 is given to illustrate one desirable and useful type of closure and an attachment based upon the same principle. At the right section of this figure the flat head is connected to the straight wall of a cylindrical vessel, while at the left, the end is attached to an end section of the wall partly formed. This type of connection is self-sealing and is easy to assemble and maintain.

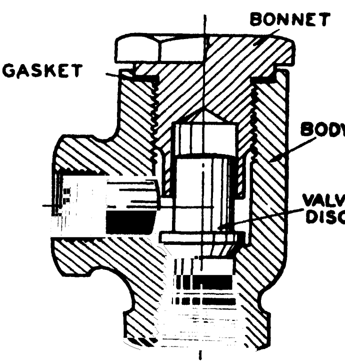


Fig. 18-19. High Pressure Check Valve.

The principle of the joint is that the internal pressure forces the cover or pressure plate against a metal gasket of tapered cross section. The gasket should have a high modulus of elasticity so that it will not be stressed beyond its elastic limit. The retainer ring is of split construction so that it can be put in position in the recess of the vessel wall. A stud plate serves to hold all parts in position and to draw up on the gasket for tightness before working pressure is applied. There is no load on the stud plate when working pressure develops

The advantages of this construction are that the joint is self-sealing and excellent tightness can be realized even for fluctuating pressures, the assembly and maintenance are very easy and relatively few studs are required, the construction is inexpensive. The principal limitation or disadvantage in its use is that the bolt hole circle is relatively small and thus does not allow much room for attachments to be made in heads of small diameter.

On the left section of Fig. 18-22 is shown a pipe attachment of the same general principle of self-sealing as for the end plate closure. It should be noted that in this case the bolts resist the internal

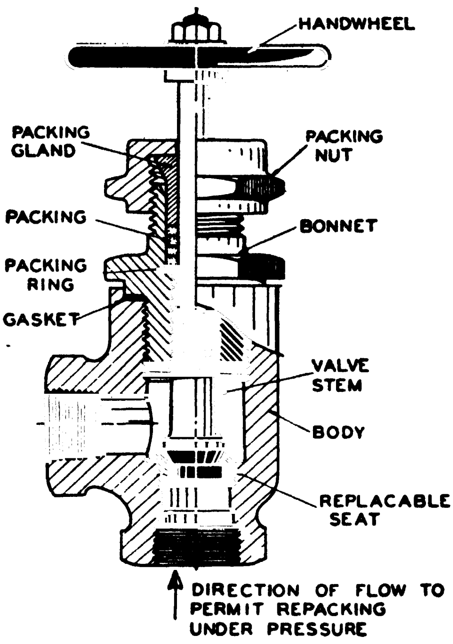


Fig. 18-20. High Pressure Angle Valve.

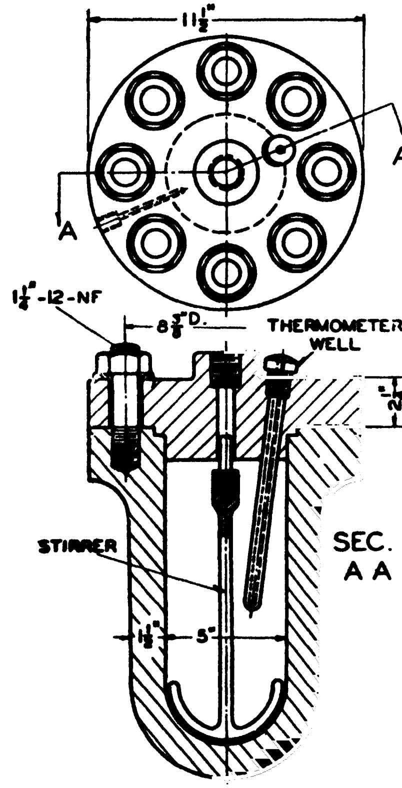


Fig. 18-21. 3,500 Lb. Autoclave.

pressure and must be designed with this in mind. Heavy connection nozzles similar to those shown in Chap. 10 are also used where pressures are not too high, by welding them into place.

18-21. Hydraulic Test. All high pressure vessels and other auxiliary piping and equipment should be given a thorough hydraulic test at an appreciably higher pressure than the intended use. Care must be taken, however, that the hydraulic test does not stress the metal above its yield point. For relatively low pressure work it is advisable to refrain from too close an approach to the

yield point in the test. The hydraulic test consists of filling the vessel or system completely with water, removing all fixed gases, and then increasing the water pressure by means of a suitable high pressure pump. The equipment is held under pressure for some time and the stretch, lengthening, or distortion determined. In the case of cylindrical vessels (e.g., commercial oxygen cylinders) where it is possible to immerse them completely in water, the stretch or expansion can be measured easily by noting the displacement of water in this outer jacket. The outer water jacket or test tank is arranged so that the test cylinder is completely immersed with only the pressure tube connection passing out through the test container. The test tank is completely filled with water, all air removed, and connection made to a water burette. This burette should be so arranged

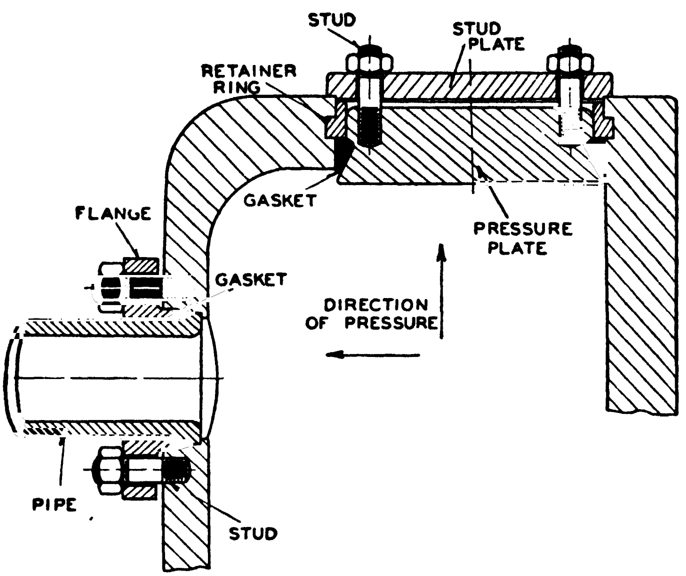


Fig. 18-22. Self-sealing High Pressure Closures.

that it will read zero when the jacket water pressure and the cylinder water pressure are atmospheric. Pressure is then applied to the cylinder hydraulically to a point approximately 100 lbs. below desired test pressure and held there until equilibrium is reached. The water level in the burette will rise, and when it reaches a steady value the pressure is increased until test pressure is attained. The burette reading at this point gives the total expansion of the vessel under test. Pressure in the cylinder is then released to atmospheric and the burette is read again when conditions are steady; this reading gives the permanent expansion of the cylinder. If none of the metal has been stressed beyond its elastic limit there should be no permanent expansion.

The Interstate Commerce Commission has stringent specifications covering all cylindrical vessels used to transport compressed gases and vapors. These specifications call for hydraulic tests to be made at 5/3 of the rated working pressure, and the permanent expansion must not exceed 10% of the total expansion. Thus a standard commercial oxygen cylinder rated at 2015 psi. is tested at 3360 psi., and if its permanent expansion is less than 10% of total expansion the cylinder is considered to be safe. Actually, new cylinders for commercial gases are held by the manufacturers to a permanent expansion of from zero to only a few per cent. The I.C.C. specifications further call for the hydraulic test to be made each 5 years.

Large cylinders (one ton and more capacity) used for transportation, storage, or chemical reactions of compressed gases like chlorine, ammonia, etc., are hydraulically tested at more frequent intervals by the users, so that the effect of any corrosion or embrittlement can be noted. The I.C.C. specifications are usually the criteria for such vessels as well as for those used for transportation only. All reaction vessels for high pressure work, and especially those operating at elevated temperatures, should be tested at frequent intervals to insure safe operation and to follow the fatigue characteristics of the assembly.

PROBLEMS—CHAPTER 18

1. A welded cylindrical vessel is subjected to an external pressure of 40 psi. gage, has an outer diameter of 72 in., and has a length of 20 ft. between head seams.

a. Determine the vessel thickness for S-2 grade A steel.

b. Determine the permissible out-of-roundness for this vessel.

c. Determine the most economical proportions for the vessel if circumferential stiffeners are used.

d. Select suitable stiffener sections for the most economical design.

e. Select dished heads, with convex exteriors, for this vessel.

2. Like Problem 1, for a vessel subjected to an external pressure of 50 psi., with a diameter of 78 in., and a length of 40 ft. between heads. Temperature 800° F.

3. Find the necessary wall thickness and specify an S-22 copper tube with a flow area of 6 sq. in. for an external pressure of 125 psi. The tube is 7 ft. long and both ends are threaded.

4. Specify S-47 cupro-nickel tubes, with an inner diameter of 4 in., for an external pressure of 220 psi. and a temperature of 380° F. The tubes are 4 ft. long, rolled into tube sheets, and 0.01 in. should be allowed for corrosion.

5. What maximum external pressure may a 4-in., Schedule 160 steel pipe 3 ft. long, with threaded ends, be subjected to?

6. Design heads cut from flat plate for the vessel of Problem 6, Chapter 4, using staybolts to keep the head thickness equal to or less than twice the vessel plate thickness.

7. Like Problem 6, but for a head thickness equal to or less than 1½ times the plate thickness.

8. Find the theoretical thickness of a tank for CO_2 gas shipment. The tank is to be made of seamless steel tubing with an ultimate strength of 50,000 psi. The outer diameter of the tank is 8 in., the length is 40 in., and the internal pressure is 1500 psi. Corrosion allowance need not be considered.

9 An autoclave like Fig. 18-21 is to be used at 500° F. and at 4500 psi. internal pressure. Specify the stud bolt size.

10. Lay out and give complete specifications for a vertical cylindrical container 6 in. I.D. and 36 in. inside length to operate at 3000 psi. air pressure and 120° F. The vessel is to have removable ends with one high-pressure connection (¼-in. I.D.) in each and one high-pressure connection for drainage in the bottom end. All fittings and gaskets are to be of copper or brass. A support is to be made 3 in. from the bottom on which caustic potash flakes may rest and be packed to within 5 in. of the top.

11. Lay out and give complete specifications for a tubular heat exchanger to handle 100 standard (68° F., 30 in. Hg.) cu. ft. per min. of compressed air at 2500 psi. The air is to be heated from 60° F. to 140° F. by steam at 50 psi. gage. Pressure drop must be less than 25 psi. at the air flow given.

REFERENCES

1. Almen, J. O., "Improving Fatigue Strength of Machine Parts," *Mechanical Engineering,* 533 (1943).
2. Aluminum Co. of America, "Structural Aluminum Handbook," Pittsburgh, Pa. (1940).
3. American Chain and Cable Co., "Modern Wire Rope Digest," Wilkes-Barre, Pa. (1941).
4. American Gear Manufacturers Association, "AGMA" Standards," Wilkinsburg, Pa.
5. American Iron & Steel Institute, "American Iron & Steel Institute, Year Book," New York (1930).
6. American Petroleum Institute, and the American Society of Mechanical Engineers, "Code for the Design, Construction, and Repair of Unfired Pressure Vessels for Petroleum Liquids and Gases," (API-ASME Code) New York (1938).
7. American Society for Metals, "Metals Handbook," Cleveland (1939).
8. American Society for Testing Materials, "A.S.T.M. Standards," Philadelphia, Pa.
9. American Society for Testing Materials, "A.S.T.M. Standards on Plastics," Philadelphia, Pa. (1943).
10. American Society for Testing Materials, "Creep Data," Philadelphia, Pa. (1938).
11. American Society for Testing Materials, "Impact Resistance and Tensile Properties of Metals at Subatmospheric Temperatures," Philadelphia, Pa. (1941).
12. American Society of Mechanical Engineers, section VIII of A.S.M.E. Boiler Construction Code, "A.S.M.E. Code; Unfired Pressure Vessels," (ASME-UPV Code) New York (1943).
13. American Society of Mechanical Engineers, "Code for the Design of Power Transmission Shafting," New York (1927).
14. The American Society of Mechanical Engineers, "Material Specifications, A.S.M.E. Boiler Construction Code," 1940 Edition, New York.
15. American Standards Association and American Society of Mechanical Engineers, "American Tentative Standard Code for Pressure Piping," New York (1935).
16. American Welding Society, "Welding Journal," Monthly, New York.
17. Bailey, R. W., "The Utilization of Creep-Stress Data in Engineering Design," *Proc. Inst. Mech. Eng.*, London (1935).
18. Berard, S. J., and Waters, E. O., "Machine Design," second edition, D. Van Nostrand Co., New York (1932).
19. Bliss, H., "The Cost of Process Equipment and Accessories," *Trans. Am. Inst. Chem. Engrs.,* **37,** 763 (1941).
20. Carswell, T. S., and Haslanger, R. U., "Chemical Engineering Applications for Plastics," *Trans. Am. Inst. Chem. Engrs.,* **39,** 755 (1943).
21. Chemical and Metallurgical Engineering, "Corrosion, Heat, Abrasion-Resistant Materials for the Construction of Chemical Engineering Equipment," *Chem. and Met.,* **47,** 597 (1940).

22. Chemical and Metallurgical Engineering, "Materials of Construction for Chemical Engineering Equipment," eleventh edition, Sept., 1944; *Chem. and Met.*, **51**, 96 (1944).
23. Cleveland Worm and Gear Co., "Catalog," Cleveland, Ohio (1937).
24. Comstock, C. W., "Some Considerations in the Design of Thick-Wall Pressure Cylinders," *Trans. Am. Inst. Chem. Engrs.*, **39**, 299 (1943).
25. Davis, H. E., Troxell, G. E., and Wiskocil, C. T., "Testing and Inspection of Engineering Materials," McGraw-Hill Book Co., New York (1941).
26. Faires, V. M., "Design of Machine Elements," The Macmillan Co., New York (1941).
27. Fowler, F. C., and Brown, G. G., "Cost of Pressure Vessels and Pressure Fractionating Columns," *Trans. Am. Inst. Chem. Engrs.*, **39**, 241 (1943).
28. Hesse, H. C., "Engineering Tools and Processes," D. Van Nostrand Co., New York (1941).
29. Heyer, R. H., "Engineering Physical Metallurgy," D. Van Nostrand Co., New York (1939).
30. Johnson, J. B., Bryan, C. W., and Turneaure, F. E., "Modern Framed Structures," John Wiley and Sons, New York (1929).
31. Kellogg, M. W., Co., "Design of Piping Systems, Expansion Stresses in Piping Systems," New York (1941).
32. Koshkin, S. J., "Modern Materials Handling," John Wiley and Sons, New York (1932).
33. Lincoln Electric Co., "Procedure Handbook of Arc Welding Design and Practice," fifth edition, Cleveland, Ohio.
34. Linde Air Products Co., "The Oxy-Acetylene Handbook—A Manual on Oxy-Acetylene Welding and Cutting Procedures," New York (1943).
35. Link-Belt Co., "General Catalog," Chicago, Ill. (1939).
36. "Machinery's Handbook," Industrial Press, New York (1942).
37. Marin, J., "Mechanical Properties of Materials and Design," McGraw-Hill Book Co., New York (1942).
38. McKay, R. J., and Worthington, R., "Corrosion Resistance of Metals and Alloys," Reinhold Publishing Co., New York (1936).
39. Meigs, D., "Closures for High Pressure Vessels," *Trans. Am. Inst. Chem. Engrs.*, **39**, 769 (1943).
40. Moore, H. F., "Textbook of the Materials of Engineering," McGraw-Hill Book Co., New York (1941).
41. Morris, C. T., and Carpenter, S. T., "Structural Frameworks," John Wiley and Sons, New York (1943).
42. Morse, F. T., "Power Plant Engineering and Design," second edition, D. Van Nostrand Co., New York (1942).
43. National Electrical Manufacturers' Association, "Handbook," New York (1940).
44. Newitt, D. M., "The Design of High Pressure Plant and the Properties of Fluids at High Pressures," Oxford Press, London (1940).
45. Newitt, D. M., "The Design of Vessels to Withstand High Internal Pressures," *Trans. Inst. Chem. Engrs.*, **14**, 85 (1936).
46. Nordenholt, G. F., Kerr, J., and Sasso, J., "Handbook on Mechanical Design," McGraw-Hill Book Co., New York (1942).
47. Perry, J. H., "Chemical Engineers Handbook," second edition, McGraw-Hill Book Co., New York (1941).
48. Sasso, J., "Plastics for Industrial Use," McGraw-Hill Book Co., New York (1942).

49. Society of Automotive Engineers, "Society of Automotive Engineers Handbook," New York (1936).
50. Speller, F. N., "Corrosion, Causes and Prevention," second edition, McGraw-Hill Book Co., New York (1935).
51. Spielvogel, S. W., "Piping Stress Calculations Simplified," McGraw-Hill Book Co., New York (1943).
52. Taylor Forge and Pipe Works, "Modern Flange Design," second edition, Chicago, Ill. (1941).
53. Tongue, H., "The Design and Construction of High Pressure Chemical Plant," Chapman and Hall, Ltd., London (1934).
54. Walker, J. H., and Crocker, S., "Piping Handbook," McGraw-Hill Book Co., New York (1939).
55. Wert, E. A., and Smith, F., "Design of Piping for Flexibility," third edition, Blaw-Knox Co., Pittsburgh, Pa. (1943).
56. Whiting Co., The, "Crane Engineering," Chicago, Ill. (1930).
57. Williams, R. S., and Homerberg, V. O., "Principles of Metallography," fourth edition, McGraw-Hill Book Co., New York (1939).
58. American Institute of Steel Construction, "Steel Construction," third edition, New York (1940).

INDEX